Embedded Element Patterns in Hierarchical Calibration of Large Distributed Arrays

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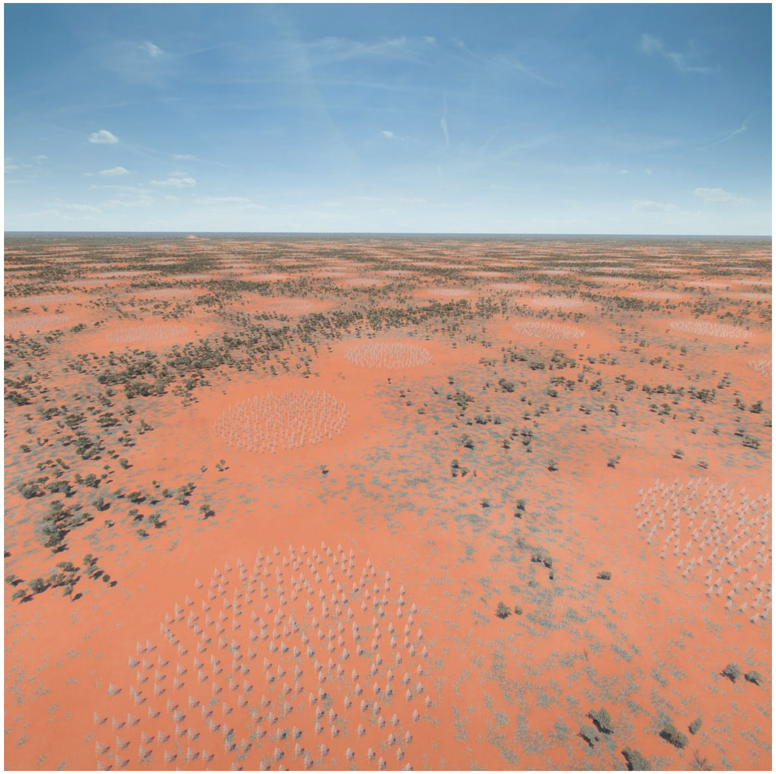
SKA-LOW and LOFAR

Low-frequency instrument of
Square **K**ilometre **A**rray

50 – 350 MHz

Array of subarrays (stations)



Low **F**requency **Ar**ray

10 – 250 MHz

Array of subarrays (stations)

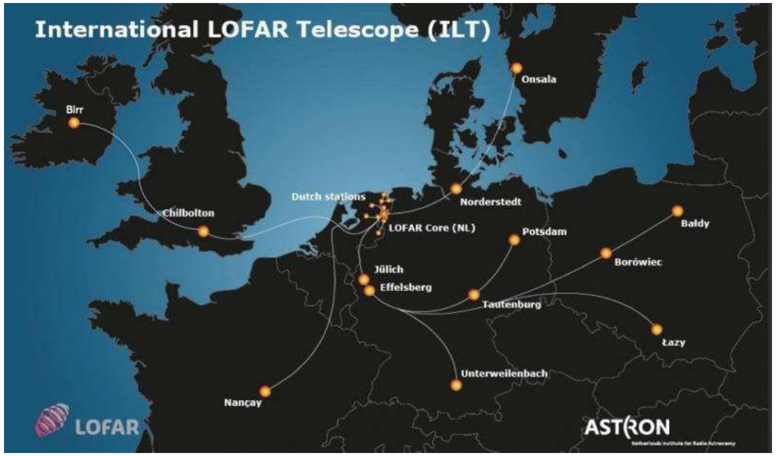
Hierarchical calibration

Calibration at station level needed for

- Accurate beamforming
  - Ensures station sensitivity
  - Allows beam shaping, e.g., nulling

Calibration at array level needed for

- High-dynamic range imaging
  - Needs station beam stability
- Absolute calibration
  - Flux transfer from flux calibrators



Embedded Element Patterns

Virone et al., IEEE TAP, 2018
Di Ninni et al., IJAP, 2019

EEPs can be simulated and validated in-situ using drones

Simulation can compute (in order of increasing costs)

- Isolated EEP: EEP of isolated antenna
- Average EEP: average EEP of all antennas in station
- Individual EEPs: different EEP for each antenna within station

Questions

- What is needed for station-level calibration?
- What is needed for array-level calibration?

Derived question

- What calibration accuracy is needed?

Gain accuracy and decorrelation

Beamforming efficiency with RMS phase error σ_φ

$$\eta_{BF} = \cos^2(\sigma_\varphi)$$

Implications: $\eta_{BF} \geq \{0.99, 0.98\}$ requires $\sigma_\varphi \leq \{5.7, 8.1\}$ degrees

Beamforming eff. with relative RMS error ε on real and imaginary part

$$\eta_{BF} = \frac{1}{1+2\varepsilon^2} \approx 1-2\varepsilon^2$$

Implications: $\eta_{BF} \geq \{0.99, 0.98\}$ requires $\varepsilon \leq \{0.071, 0.10\}$

Impact of beam (in)stability

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Array level calibration needs to be able to track dir. dep. gain changes

First order model for varying gain of *i*th station $g_i = g_{0,i} + \alpha_i t$

To keep errors below 20% of thermal noise, we need

$$\frac{|\alpha_i|}{|g_{0,i}|} \leq \sqrt{\frac{12}{5}} \frac{1}{\sqrt{\mathrm{SNR}}} \frac{1}{\tau}$$

where τ is the calibration interval in which the given SNR is achieved

Example: SNR = 10 and τ = 600 s allows rate of change of 0.082%/s

Note: time needed to achieve a certain SNR depends on SEFD

Hence: more sensitive instrument (lower SEFD) can keep up with faster gain changes

Impact of ignoring EEPs (1)

Wijnholds, SKA-LOW meeting, Florence, 2019
Haslam et al., A&A Suppl, 1982

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Simulation setup for 256-element SKA-LOW station

- Mock data based on simulated EEPs and Haslam map
- Calibration model assuming identical EEPs equal to average EEP
- Nominal gain equal to unity for each element
- 200 scenarios spread over 24 hours (one solution per 7.2 min)
- Simulation done for both SKALA4AL and EDA at 110 MHz
- Gain solutions used to calculate AF for each instant

$$\mathrm{AF}(\mathbf{l};\mathbf{l}_0) = \mathbf{w}^H(\mathbf{l}_0)(\mathbf{g}\odot\mathbf{a}(\mathbf{l})) = \sum_{p=1}^{P} g_p \exp\left(\frac{-2\pi i}{\lambda}\mathbf{x}_p(\mathbf{l}-\mathbf{l}_0)\right)$$

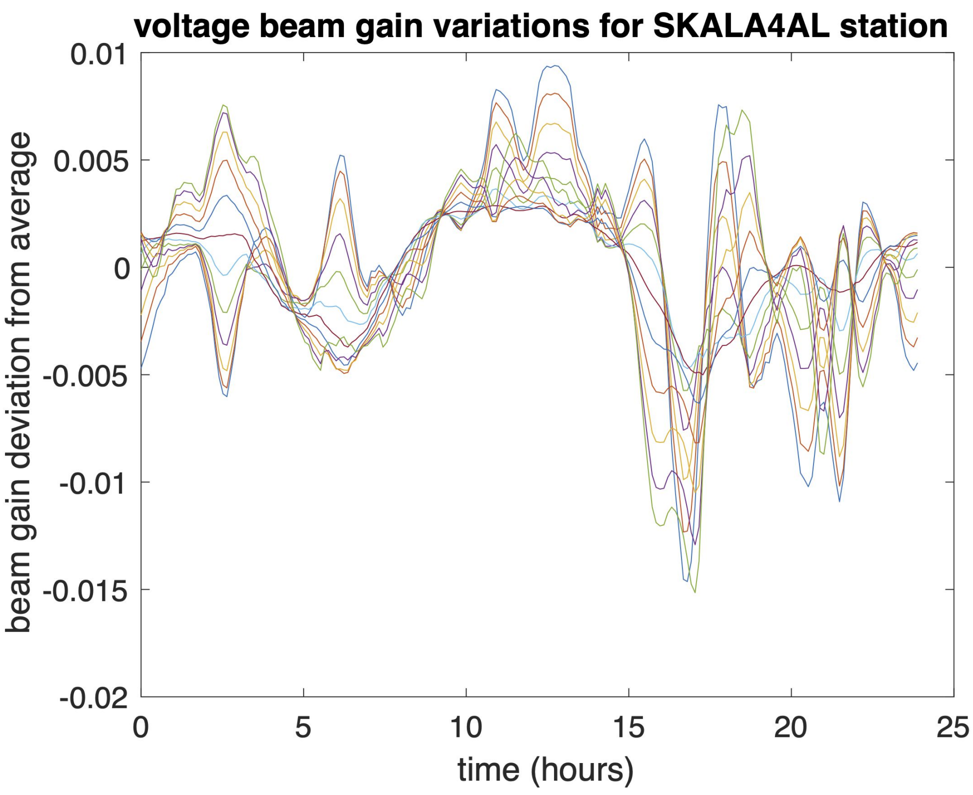
- Average AF normalized to have unit peak gain

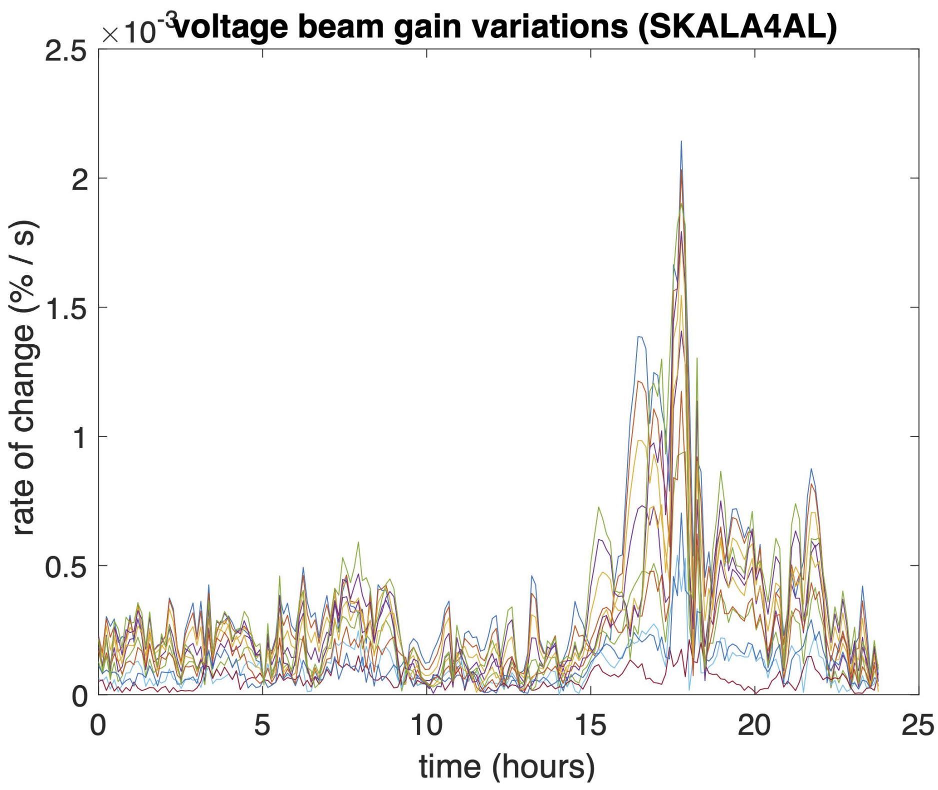
Impact of ignoring EEPs (2)

Left: beam gain variations along cross-section through station main beam with largest variations

Right: rate of change at each point of this cross-section

Conclusion: average EEP sufficient if sky model is correct





Flux transfer requirement

Balancing against absolute flux calibrator accuracy:

- Typical absolute flux accuracy of flux calibrators is ~5%
- Instrument should not be limiting, so LOFAR2.0 has set reproducibility of absolute flux calibration at 2%
- Here, reproducibility applies to the absolute flux calibration in the target field for different calibrators or the same calibrator at different sidereal times
- Assuming error towards calibrator and target field are uncorrelated gives tolerance of 1.4%

Compliance assessment flux transfer

Di Ninni et al., EuCAP 2019

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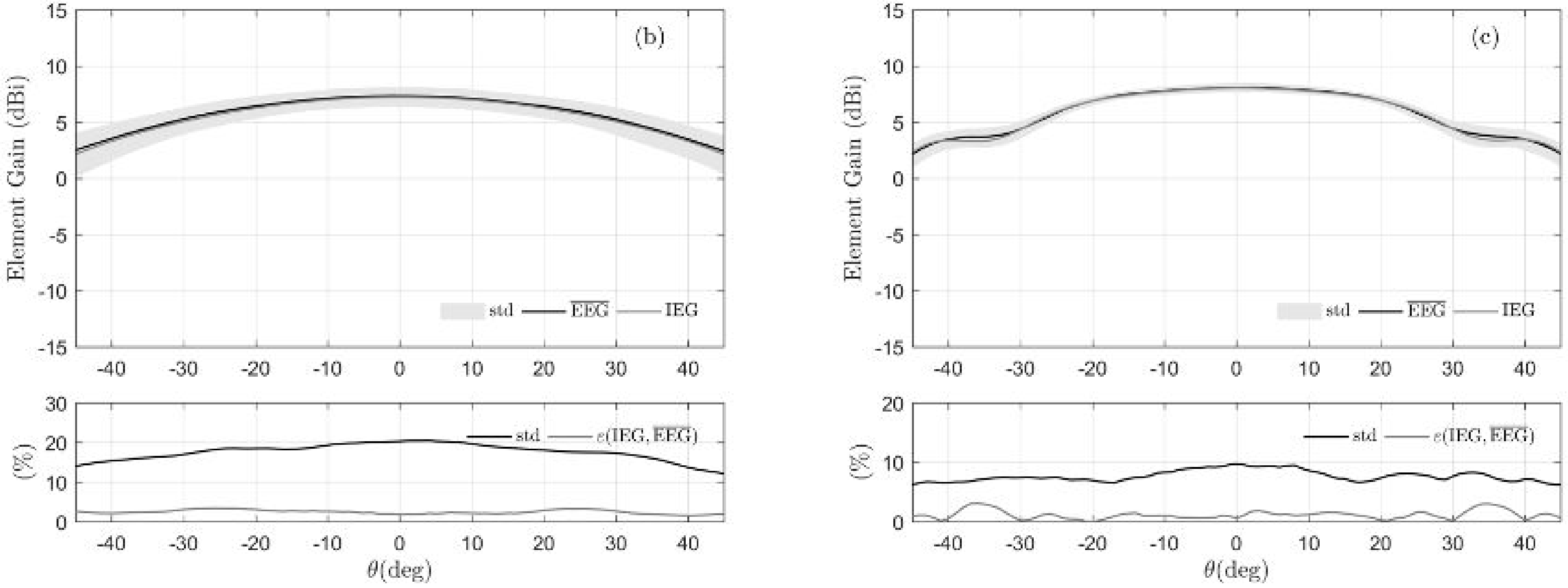
Comparison between average EEP and isolated EEP for SKA-LOW

Patterns (top) and difference (bottom) at 110 (l) and 350 (r) MHz

Differences up to about 3%, average EEP needed to meet requirement



Summary and conclusions

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Station level

- Requirement proposed on coherence during beamforming
- Requirement proposed on tolerable rate of change
- Both requirements can likely be met with an average EEP

Array level

- Requirement proposed on reproducibility of absolute flux calibration
- SKA-LOW needs average EEP to satisfy this requirement

Individual EEPs may (fortunately) not be necessary