Dyadic Green's Function Studies for the Three-shell Head Model

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Outline

1. Introduction
2. Mathematical formulation
3. Numerical results
4. Conclusions and future work

1 Introduction

Aims of the study

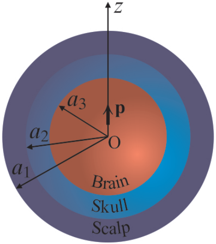
- Employ analytical expressions for the calculation of dyadic Green's function (DGF) of an isotropic three-layered head model
- Use DGF to compute the electric field due to infinitesimal electric dipole located in the internal core corresponding to the brain
- Numerically examine the electric field behavior due to placement of the dipole at various positions inside the head model

DBS–EEG–MEG applications

- Accurate calculation of the interaction of EM waves with biological tissues has direct applications in deep brain stimulation (DBS), used to treat Parkinson's disease and depression
- DBS is achieved through surgical implants of electrodes, which act as electrical sources and render local parts of the brain inactive for treatment needs. Measurement of the electric or magnetic activity of this internal source is carried out through electroencephalography (EEG) or magnetoencephalography (MEG)
- In biological imaging, EEG or MEG is performed via multilayered spherical models
- The most realistic head model routinely adopted is the **three-layered** one, consisting of the brain, the skull and the scalp

Configuration under study

- Spherical human head model consisting of three homogeneous layers, namely, the brain (layer V_3), the skull (layer V_2), and the scalp (layer V_1). Surrounding medium free space $\rightarrow$ unbounded outer layer V_0
- Each layer V_p has permittivity ϵ_p and permeability μ_p
- Excitation by an infinitesimal electric dipole with dipole moment **p**, located in V_3



- Layer V_3 (brain): $0 \leq r < a_3$, ϵ_3, μ_3
- Layer V_2 (skull): $a_3 < r < a_2$, ϵ_2, μ_2
- Layer V_1 (scalp): $a_2 < r < a_1$, ϵ_1, μ_1
- Layer V_0 (free space): $r > a_1$, ϵ_0, μ_0

DGF calculation

The component of the configuration's DGF in layer V_p, for $p = 0, 1, 2$, is expressed as an expansion of even/odd SVWF's [Prokopiou and Tsitsas, Studies Appl. Math., 2018]

$$\tilde{\mathbf{G}}^p(\mathbf{r};\mathbf{r}') = \frac{ik_3}{4\pi}\sum_{n=1}^{\infty}\sum_{m=0}^{n}\sum_{\sigma=e,o}\frac{2n+1}{n(n+1)}\epsilon_m\frac{(n-m)!}{(n+m)!}$$
$$\times\Big\{\mathbf{M}^{(1)}_{\sigma mn}(k_p,\mathbf{r})\Big[\alpha_n^p\mathbf{M}^{(3)}_{\sigma mn}(k_3,\mathbf{r}')+\beta_n^p\mathbf{M}^{(3)}_{\sigma mn}(k_3,\mathbf{r}')\Big]$$
$$+\mathbf{N}^{(1)}_{\sigma mn}(k_p,\mathbf{r})\Big[\gamma_n^p\mathbf{N}^{(3)}_{\sigma mn}(k_3,\mathbf{r}')+\delta_n^p\mathbf{N}^{(3)}_{\sigma mn}(k_3,\mathbf{r}')\Big]$$
$$+\mathbf{M}^{(3)}_{\sigma mn}(k_p,\mathbf{r})\Big[\tilde{\alpha}_n^p\mathbf{M}^{(1)}_{\sigma mn}(k_3,\mathbf{r}')+\tilde{\beta}_n^p\mathbf{M}^{(3)}_{\sigma mn}(k_3,\mathbf{r}')\Big]$$
$$+\mathbf{N}^{(3)}_{\sigma mn}(k_p,\mathbf{r})\Big[\tilde{\gamma}_n^p\mathbf{N}^{(1)}_{\sigma mn}(k_3,\mathbf{r}')+\tilde{\delta}_n^p\mathbf{N}^{(3)}_{\sigma mn}(k_3,\mathbf{r}')\Big]\Big\}.$$

$$k_p = \omega\sqrt{\epsilon_p\mu_p},\quad p = 0, 1, 2,\quad k_3 = \omega\sqrt{\epsilon_3\mu_3}$$

DGF calculation

In the case of layer V_3 which contains the dipole source, the DGF component is given by [Prokopiou and Tsitsas, Studies Appl. Math., 2018]

$$\tilde{\mathbf{G}}^3(\mathbf{r};\mathbf{r}') = \tilde{\mathbf{G}}^{\mathrm{pr}}(\mathbf{r};\mathbf{r}') + \tilde{\mathbf{G}}^{\mathrm{sec}}(\mathbf{r};\mathbf{r}').$$

- $\tilde{\mathbf{G}}^{\mathrm{pr}} \rightarrow$ primary contribution corresponding to the DGF of the unbounded medium filled with the material of layer V_3

$$\tilde{\mathbf{G}}^{\mathrm{pr}}(\mathbf{r};\mathbf{r}') = -\frac{\hat{\mathbf{r}}\hat{\mathbf{r}}}{k_3^2}\delta(\mathbf{r}-\mathbf{r}') + \frac{ik_3}{4\pi}\sum_{n=1}^{\infty}\sum_{m=0}^{n}\sum_{\sigma=e,o}\frac{2n+1}{n(n+1)}\epsilon_m\frac{(n-m)!}{(n+m)!}$$
$$\times\begin{cases}\left[\mathbf{M}^{(3)}_{\sigma mn}(k_3,\mathbf{r})\mathbf{M}^{(1)}_{\sigma mn}(k_3,\mathbf{r}') + \mathbf{N}^{(3)}_{\sigma mn}(k_3,\mathbf{r})\mathbf{N}^{(1)}_{\sigma mn}(k_3,\mathbf{r}')\right], & r > r' \\ \left[\mathbf{M}^{(1)}_{\sigma mn}(k_3,\mathbf{r})\mathbf{M}^{(3)}_{\sigma mn}(k_3,\mathbf{r}') + \mathbf{N}^{(1)}_{\sigma mn}(k_3,\mathbf{r})\mathbf{N}^{(3)}_{\sigma mn}(k_3,\mathbf{r}')\right], & r < r'\end{cases}.$$

- $\tilde{\mathbf{G}}^{\mathrm{sec}} \rightarrow$ secondary contribution due to scattering by the boundaries of the spherical shells, given by the DGF of the previous slide for $p = 3$

DGF calculation

- DGF expansion coefficients $\alpha_n^p, \beta_n^p, \tilde{\alpha}_n^p, \tilde{\beta}_n^p$ are determined by recursive relations [Prokopiou and Tsitsas, Studies Appl. Math., 2018]

$$\begin{bmatrix} \alpha_n^p & \beta_n^p \\ \tilde{\alpha}_n^p & \tilde{\beta}_n^p \end{bmatrix} = \mathbf{A}_n^p \begin{bmatrix} \alpha_n^{p-1} & \beta_n^{p-1} \\ \tilde{\alpha}_n^{p-1} & \tilde{\beta}_n^{p-1} \end{bmatrix},$$

$$\mathbf{A}_n^p = i x_p^2 \begin{bmatrix} u_p h_n(x_p)\tilde{j}_n(y_p) - \tilde{h}_n(x_p) j_n(y_p) & u_p h_n(x_p)\tilde{h}_n(y_p) - \tilde{h}_n(x_p) h_n(y_p) \\ \tilde{j}_n(x_p) j_n(y_p) - u_p j_n(x_p)\tilde{j}_n(y_p) & \tilde{j}_n(x_p) h_n(y_p) - u_p j_n(x_p)\tilde{h}_n(y_p) \end{bmatrix},$$

where $x_p = k_p a_p$, $y_p = k_{p-1} a_p$, $u_p = \zeta_p / \zeta_{p-1}$, $\zeta_p = \sqrt{\mu_p / \epsilon_p}$ and appropriate initial values for $\alpha_n^0, \beta_n^0, \tilde{\alpha}_n^0, \tilde{\beta}_n^0$ are used

- Similar recursive formulas hold for $\gamma_n^p, \delta_n^p, \tilde{\gamma}_n^p, \tilde{\delta}_n^p$

Electric field computation

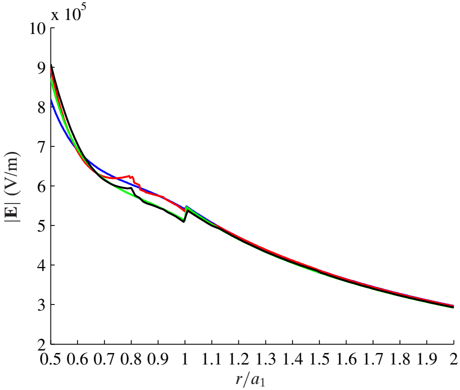
Electric field generated in layer V_p by the electric dipole located at $\mathbf{r} = \mathbf{r}_s$ in layer V_3 [Prokopiou and Tsitsas, Studies Appl. Math., 2018]

$$\mathbf{E}^p(\mathbf{r};\mathbf{r}_s) = i\omega\mu_p \tilde{\mathbf{G}}^p(\mathbf{r};\mathbf{r}_s)\cdot\mathbf{p}, \quad \mathbf{r}\in V_p.$$

Single sphere

Total electric field magnitude **E** on x-axis, for z-oriented dipole. Values of parameters: $k_0 a_1 = 0.6\pi$, $\epsilon_1 = 2.54\epsilon_0$, $\mu_1 = \mu_0$. Blue and red curves: dipole location on z-axis at $r/a_1 = r_s/a_1 = 0.1$. Green and black curves: dipole location on z-axis at $r/a_1 = r_s/a_1 = 0.2$.

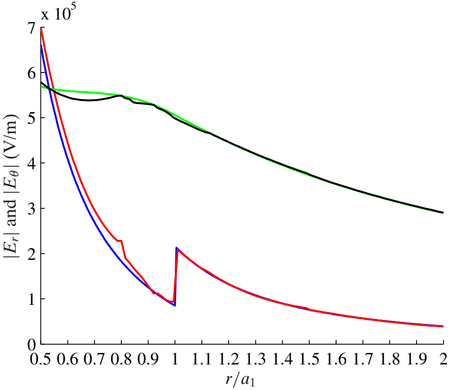
- Blue line: analytical solution
- Red line: HFSS
- Green line: analytical solution
- Black line: HFSS



Single sphere

Magnitude of electric field components E_r and E_θ on x-axis, for z-oriented dipole. Values of parameters: $k_0 a_1 = 0.6\pi$, $\epsilon_1 = 2.54\epsilon_0$, $\mu_1 = \mu_0$. Dipole's location on z-axis at $r/a_1 = r_s/a_1 = 0.2$. Blue and red curves: $|E_r|$. Green and black curves: $|E_\theta|$.

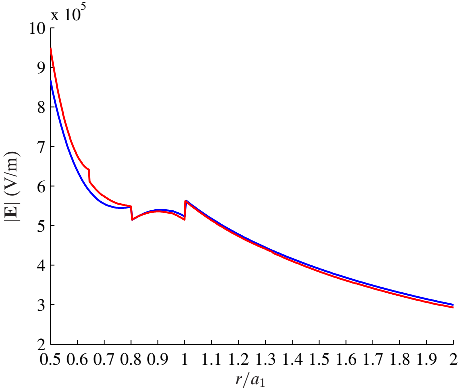
- Blue line: analytical solution
- Red line: HFSS
- Green line: analytical solution
- Black line: HFSS



Three-layered sphere

Total electric field magnitude **E** on *x*-axis, for *z*-oriented dipole located at $r/a_1 = r_s/a_1 = 0.2$. Values of parameters: $k_0 a_1 = 0.6\pi$, $a_2/a_1 = 0.95$, $a_3/a_1 = 0.8$, $\epsilon_1 = 4\epsilon_0$, $\epsilon_2 = 5\epsilon_0$, $\epsilon_3 = 2.54\epsilon_0$, $\mu_1 = \mu_2 = \mu_3 = \mu_0$.

- Blue line: analytical solution
- Red line: HFSS

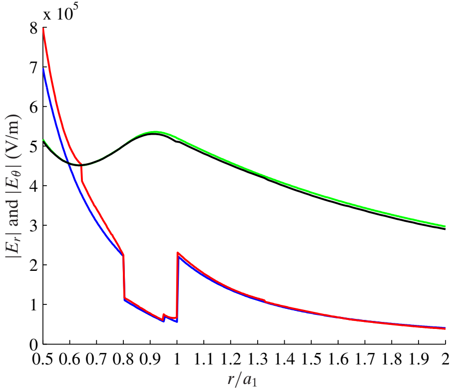


3 Numerical Results

Three-layered sphere

Magnitude of electric field components E_r and E_θ on x-axis, for z-oriented dipole located at $r/a_1 = r_s/a_1 = 0.2$. Values of parameters: $k_0 a_1 = 0.6\pi$, $a_2/a_1 = 0.95$, $a_3/a_1 = 0.8$, $\epsilon_1 = 4\epsilon_0$, $\epsilon_2 = 5\epsilon_0$, $\epsilon_3 = 2.54\epsilon_0$, $\mu_1 = \mu_2 = \mu_3 = \mu_0$. Blue and red curves: $|E_r|$. Green and black curves: $|E_\theta|$.

- Blue line: analytical solution
- Red line: HFSS
- Green line: analytical solution
- Black line: HFSS



4 Conclusions and future work

- Analytical calculation of the DGF in order to compute and examine the behavior of the electric field in a three-layered spherical head model, excited by an infinitesimal electric dipole
- Very good agreement between our analytical method and HFSS is observed
- Extension of the model to handle a greater number of layers and to include anisotropic conductivity properties
- Consideration of a volume integral equation method for the evaluation of the unknown DGF, in a multilayered anisotropic spherical head model

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Thank you for your attention!