Simplicial Homology Global Optimisation in the Problem of Point-to-Point Ionospheric Ray Tracing

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Ray Tracing Overview

- **Ionospheric ray tracing**
  - Based on Haselgrove’s equations, derived from Fermat’s principle
  - Allows to obtain ray trajectory in described medium given initial ray direction
- **Two types of problems:**
  - **Basic ray tracing** (initial value problem)

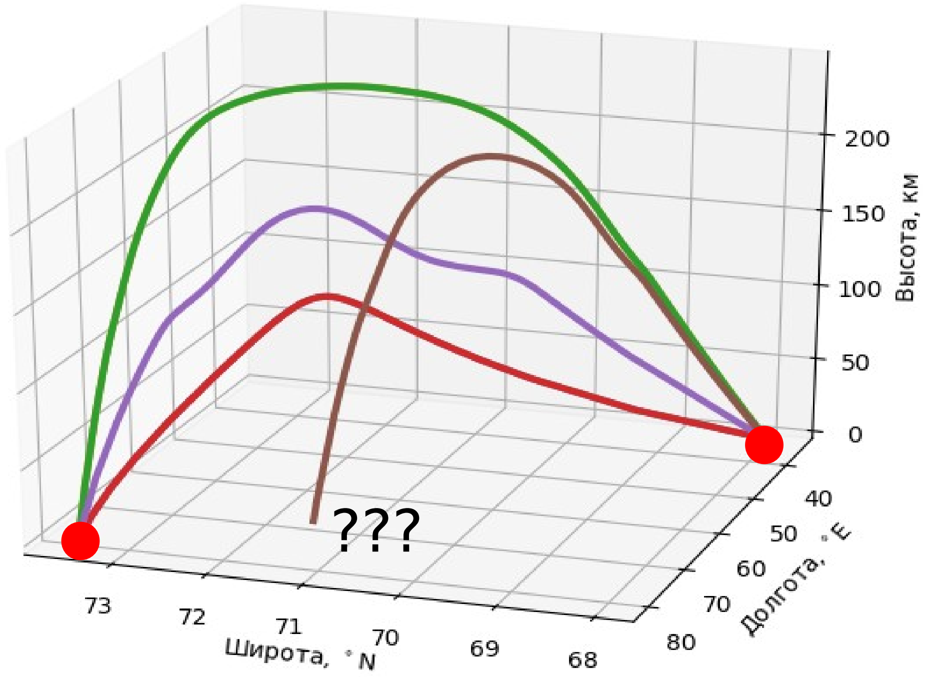
    Can be easily solved with raytracing, with a few caveats
  - **Point-to-point ray tracing** (boundary value problem)

    harder to solve with raytracing, requires guessing initial parameters

Point-to-Point Ray Tracing

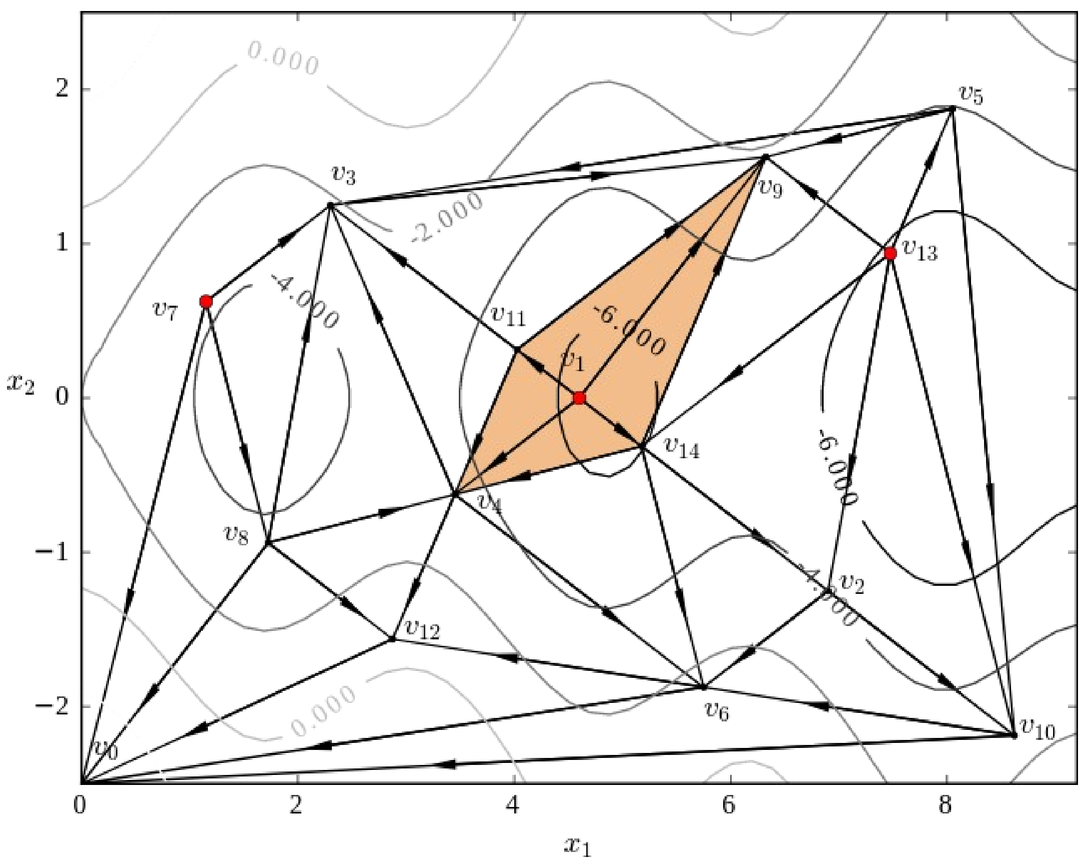
- An often-used approach for point-to-point ray tracing is to design **a cost function** that depends on initial ray elevation and azimuth when transmitted from source
- Cost function describes how close a particular ray would hit the receiver, making it a good candidate for **optimization**
- Derivative-free local optimization techniques such as **Nelder-Mead method** are often used due to computation costs incurred by evaluating the cost function.

Multipath Ray Tracing



- Depending on ionospheric structure and ray frequency, **multiple ray tracing solutions can occur**
- Local minimizers would only converge to a single solution (*which might even be an invalid one!*)
- **Global optimization is required for multipath solutions**

Simplicial Homology Global Optimization (SHGO)



[Endres, Sandrock, 2018]

Simplicial Homology Global Optimization

Main advantages:

- Derivative-free
- Attempts to find all local minima
- Converges to a fixed number of local minimizer points

Improving Convergence

- To improve convergence rate, a pseudo-random **Sobol sequence** of points in search space is used instead of uniform grid.
- When a “high ray” for the topmost layer is considered, an **additional cost penalty of *[elevation]*^{2}** is introduced when ray does not reach back to the surface, allowing local minimizers to converge in that case.

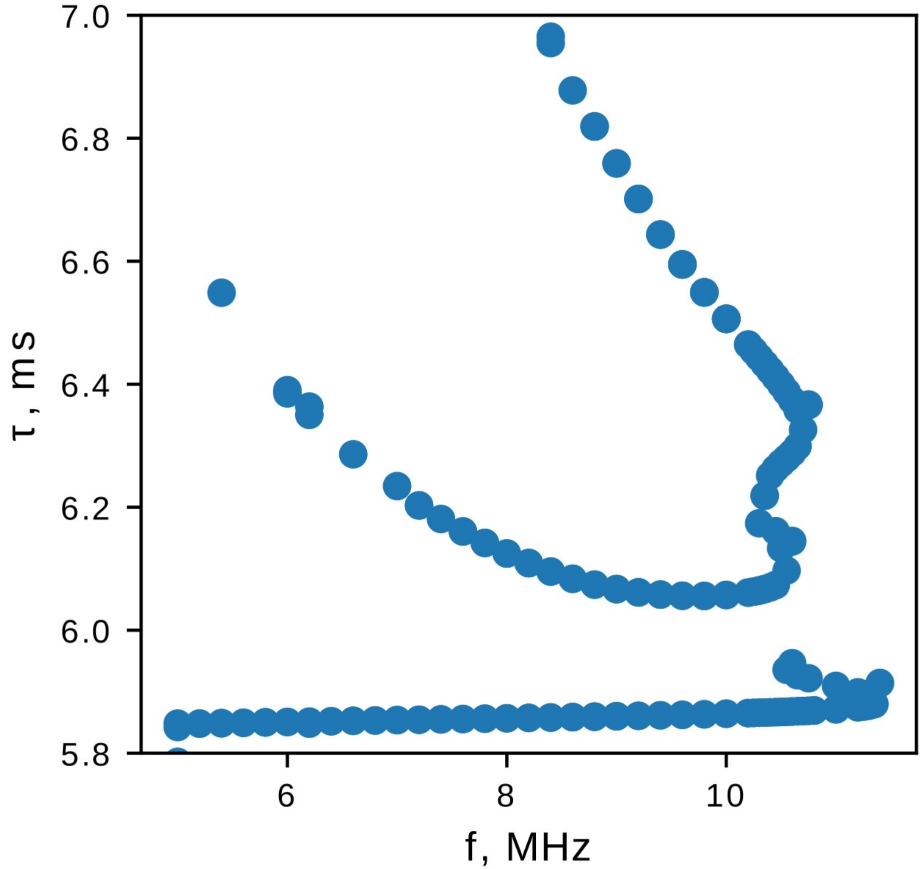
Ionospheric Model for Ray Tracing

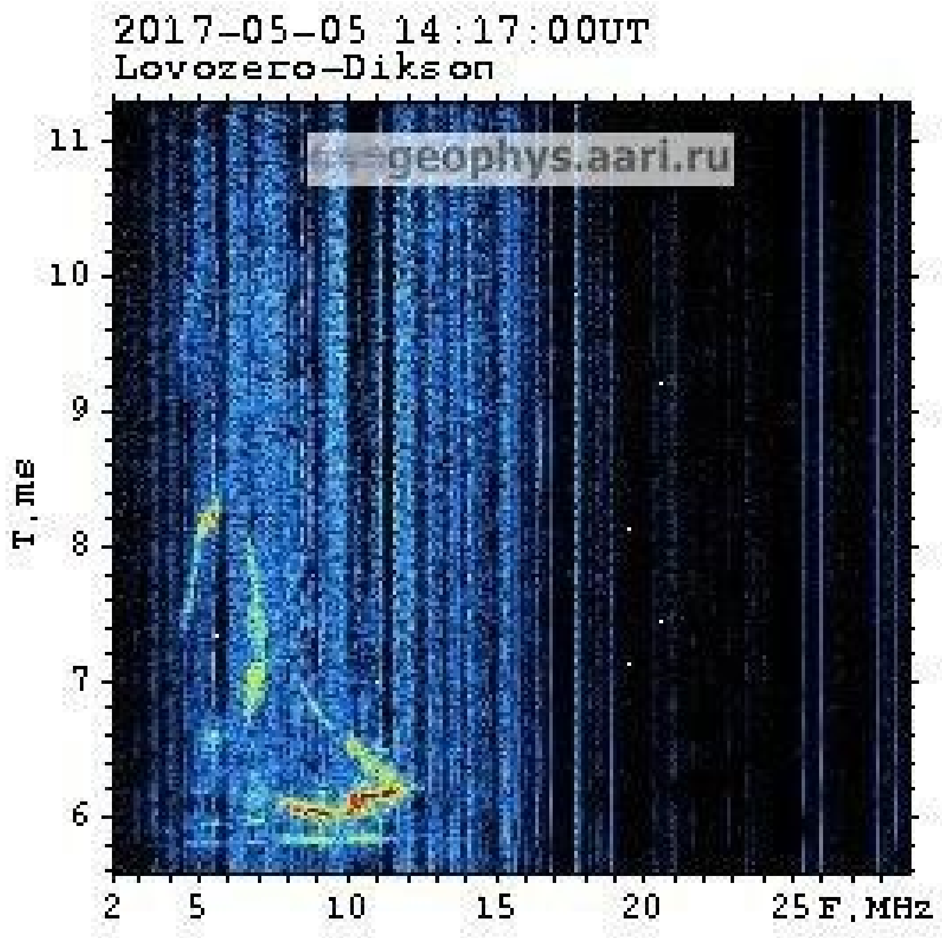
- **Due to a high number of required calculations, fast and robust ionospheric models are preferred**

In this work, data from the following models is used:

Model	Purpose
NeQuick2	Estimating plasma frequency
IGRF12	Accounting for magnetic field
NRL-MSISE-00	Estimating collision frequency from neutral particle concentration

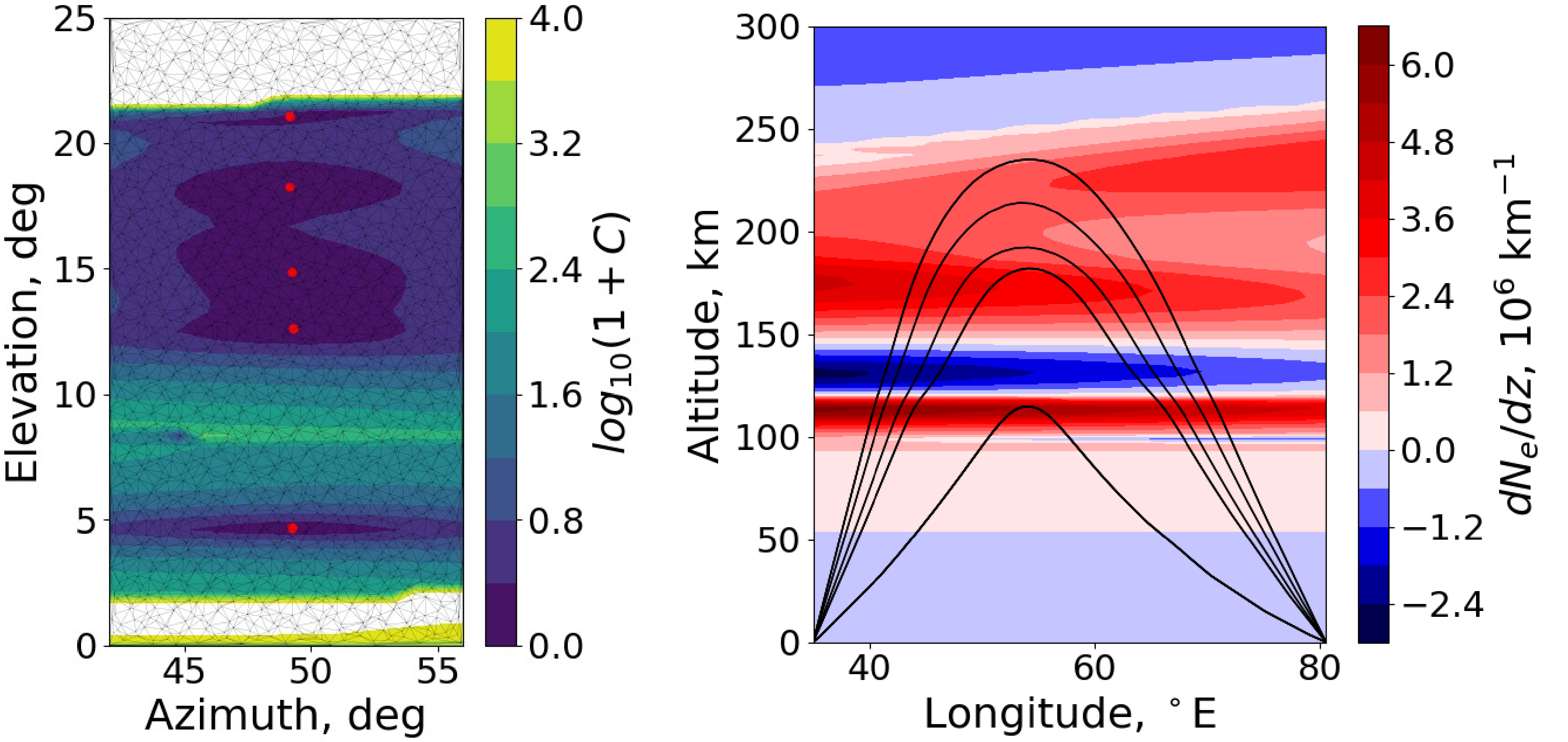
Oblique ionogram simulation





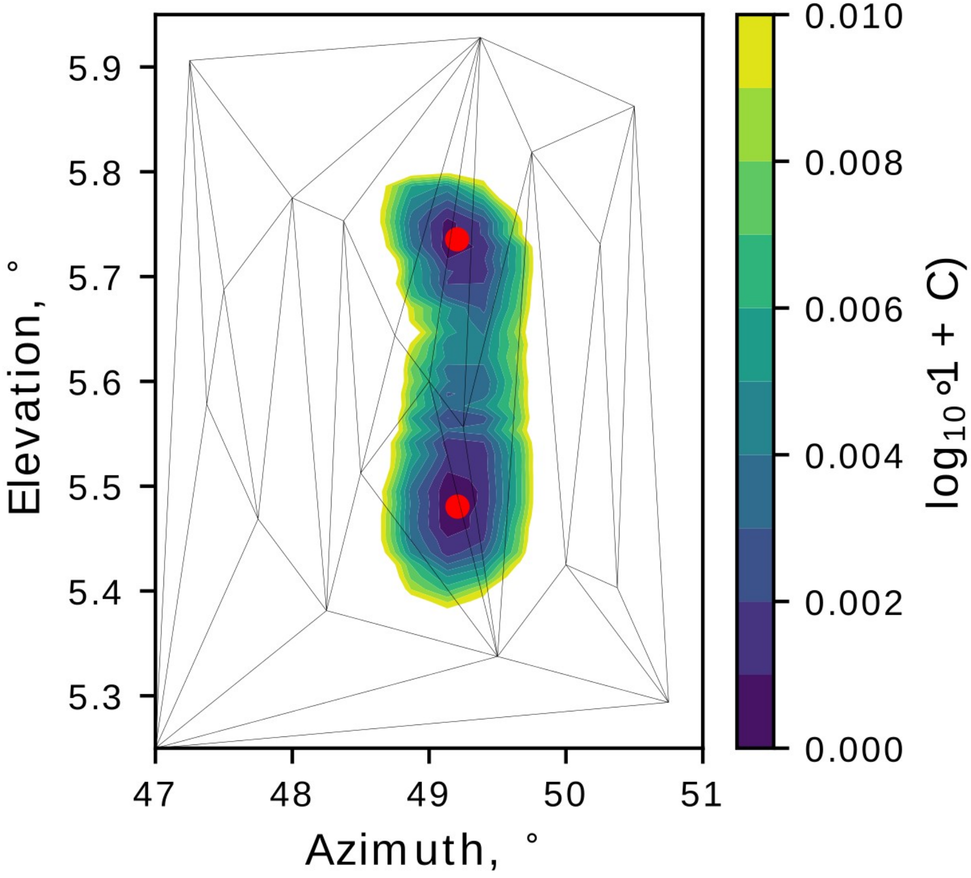
Simulated oblique ionogram [3-15 MHz, O-mode]
Using NeQuick2 model initialized with the corresponding F10.7 value for that day

Raytracing near MUF



Cost function and solutions for f = 10.48 MHz

Raytracing near MUF



- Even for low elevation separation between rays (~0.2 deg), a solution is found, provided with adequate point sampling density
- This makes this method suitable even for signal frequencies close to the MUF.

Conclusion

- **An improved raytracing algorithm has been developed that utilizes SHGO for autonomous finding of multipath ray solutions**
- **SHGO is demonstrated to be an efficient tool for point-to-point ray tracing, providing better and faster solutions than other global optimization alternatives**
- **An oblique ionogram simulated using the above algorithm is presented that agrees fairly well with experimental data even when near-MUF areas are considered**