OPTIMAL PARAMETERS OF EXPERIMENT FOR DETERMINING DIELECTRIC CONSTANT OF A LAYER IN A RECTANGULAR WAVEGUIDE

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URSI GASS in Rome, Italy

29 August – 5 September 2020

EXPERIMENTAL SETUP

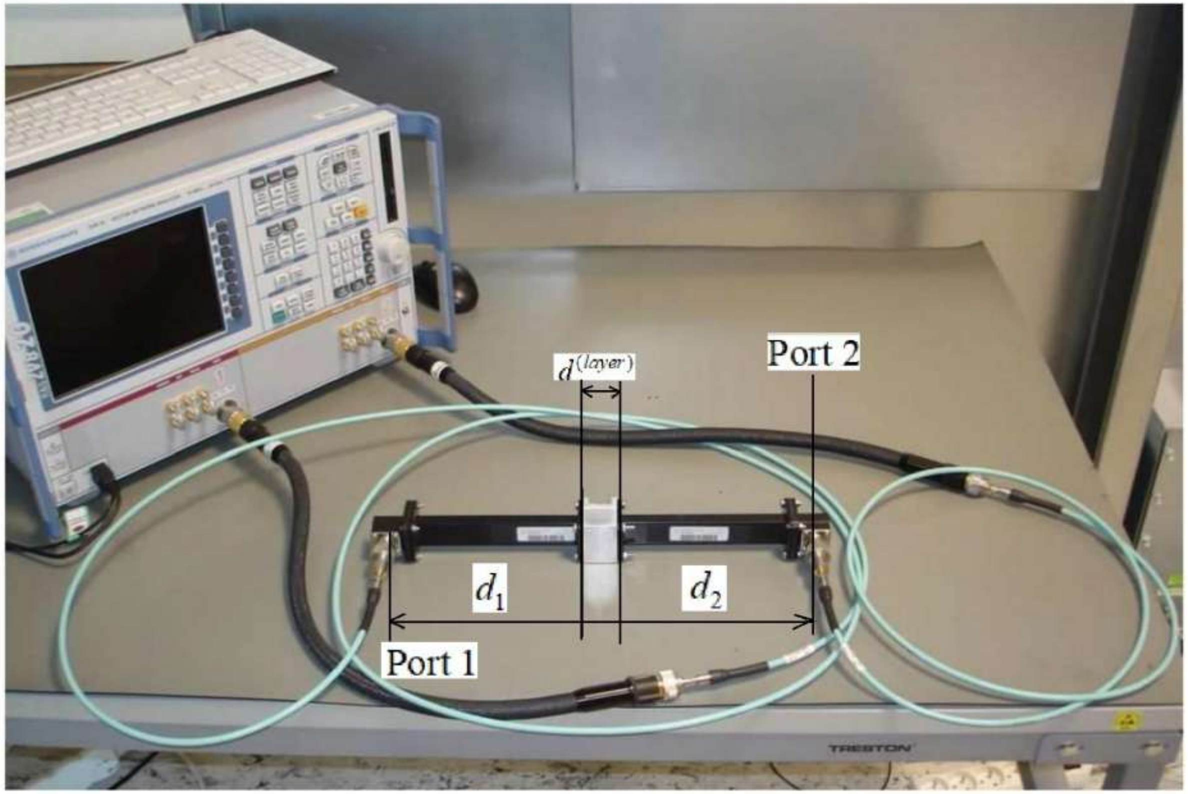
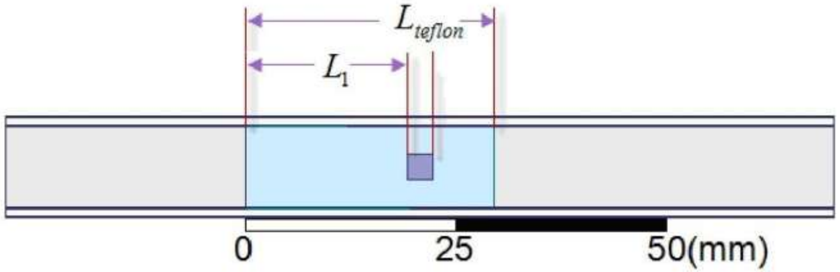


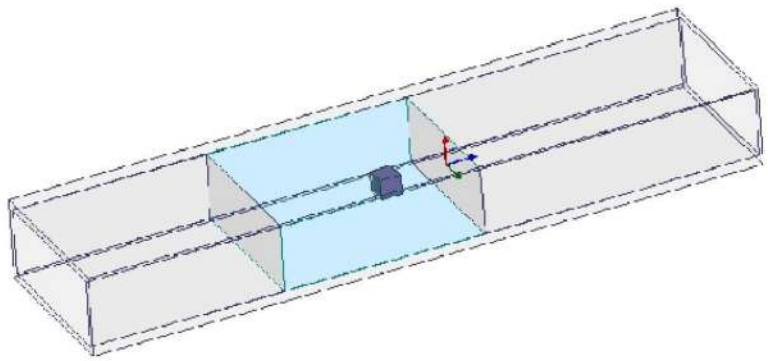
Fig.: Vector network analyzer connected to the waveguide with a multilayered diaphragm*.

* (Tomasek et al., 2015)

PROBLEM STATEMENT

Geometry





Maxwell's equations

$$\varepsilon(\mathbf{r})\frac{\partial \mathbf{E}}{\partial t} = rot\mathbf{H}, \quad \mu_0 \frac{\partial \mathbf{H}}{\partial t} = -rot\mathbf{E}, \quad t>0, \quad \mathbf{r}\in\Omega,$$

$$[\mathbf{E}(\mathbf{r})\times\mathbf{n}] = 0, \quad \mathbf{r}\in\partial\Omega,$$

$\mathbf{n}$ the normal unit vector to $\partial\Omega$

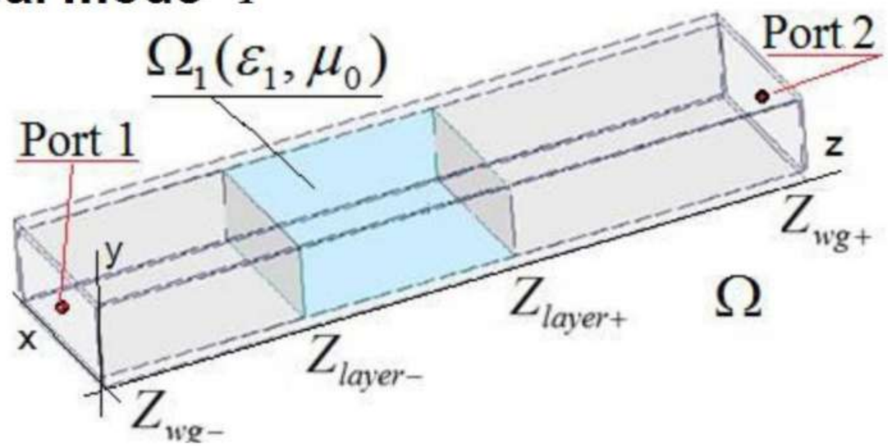
$$\mathbf{E}(\mathbf{r},0) = \mathbf{E}^0(\mathbf{r}), \quad \mathbf{H}(\mathbf{r},0) = \mathbf{H}^0(\mathbf{r}), \quad \mathbf{r}\in\Omega$$

$$\Omega = (0,a)\times(0,b)\times(-\infty,\infty)$$

Fig.: A waveguide Ω with a diaphragm and an inclusion.

The aim of solving forward and inverse scattering problems is a determination of the dielectric media parameters of inclusions in a waveguide of rectangular cross-section using the transmission coefficient F.

Vector Network Analyzer measurements, scattering matrix S and transmission coefficient of the principal mode F



Port 1, $z = Z_{wg-}$ source input

Port 2, $z = Z_{wg+}$, transmitted field output

Port 2, the transmitted field in the empty part of single-mode waveguide

$$\hat{E}_{y,trans}(x_0, y_0, Z_{wg+}) = (A + R^{+}_{1,0})e^{ik^{(z)}_{1,0} Z_{wg+}} + \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}(1-\delta_{n,1}\delta_{m,0})R^{+}_{n,m}Y_m(y_0)e^{-|k^{(z)}_{n,m}|Z_{wg+}} \approx (A + R^{+}_{1,0})e^{ik^{(z)}_{1,0} Z_{wg+}}$$

principal mode — evanescent modes — principal mode

Port 1, Port 2, the scattering matrix S

empty waveguide:

$$S^{(wg)}_{12} = \frac{\hat{E}_{y,inc}(x_0, y_0, Z_{wg+})}{\hat{E}_{y,inc}(x_0, y_0, Z_{wg-})}$$

waveguide with the diaphragm:

$$S^{(wg,layer)}_{12} = \frac{\hat{E}_{y,trans}(x_0, y_0, Z_{wg+})}{\hat{E}_{y,inc}(x_0, y_0, Z_{wg-})}$$

the transmission coefficient of the principal waveguide mode:

$$F = 1 + R^{+}_{1,0} / A = \frac{\hat{E}_{1,0,y,trans}(x_0, y_0, Z_{layer+})}{\hat{E}_{1,0,y,inc}(x_0, y_0, Z_{layer+})} \approx \frac{\hat{E}_{y,trans}(x_0, y_0, Z_{wg+})}{\hat{E}_{y,inc}(x_0, y_0, Z_{wg+})} = \frac{S^{(wg,layer)}_{12}}{S^{(wg)}_{12}}$$

TRANSMISSION COEFFICIENT OF THE PRINCIPAL MODE OF AN ELECTROMAGNETIC WAVE FOR A DIELECTRIC LAYER IN A WAVEGUIDE

definition $$F = \frac{\hat{E}_{y,trans}(x_0, y_0, Z_{wg+})}{\hat{E}_{y,inc}(x_0, y_0, Z_{wg+})} = \frac{S_{12}^{(wg,layer)}}{S_{12}^{(wg)}}$$

close-form solution $$F(\varepsilon, f) = \frac{Z_0^{(layer)}(f)}{g(\varepsilon, f)}$$

$$g(\varepsilon, f) = c_\varepsilon(f) + i\,H(t_\varepsilon(f))\, s_\varepsilon(f)$$

the transmission elements of the scattering matrix S :

$S_{12}^{(wg)}$ for empty waveguide

$S_{12}^{(wg,layer)}$ for waveguide with dielectric layer

$Z_0^{(layer)} = e^{ik_1^{(z)}(f)d^{(layer)}}$ for empty layer

$$s_\varepsilon(f) = \sin\left(k_\varepsilon^{(z)}(f) d^{(layer)}\right)$$

$$c_\varepsilon(f) = \cos\left(k_\varepsilon^{(z)}(f) d^{(layer)}\right)$$

$$t_\varepsilon(f) = k_\varepsilon^{(z)}(f) / k_1^{(z)}(f)$$

$$H(x) = 0.5(x + 1/x)$$

$$k_\varepsilon^{(z)}(f) = (k_\varepsilon^2(f) - (k^{(x)})^2)^{1/2}$$

$$k_1^{(z)}(f) = (k_1^{\,2}(f) - (k^{(x)})^2)^{1/2}$$

$$k_\varepsilon(f) = \varepsilon^{1/2} k_1(f)$$

$$k_1(f) = 2\pi f / c$$

$$k^{(x)} = \pi / a$$

a the waveguide width

$d^{(layer)}$ the layer width

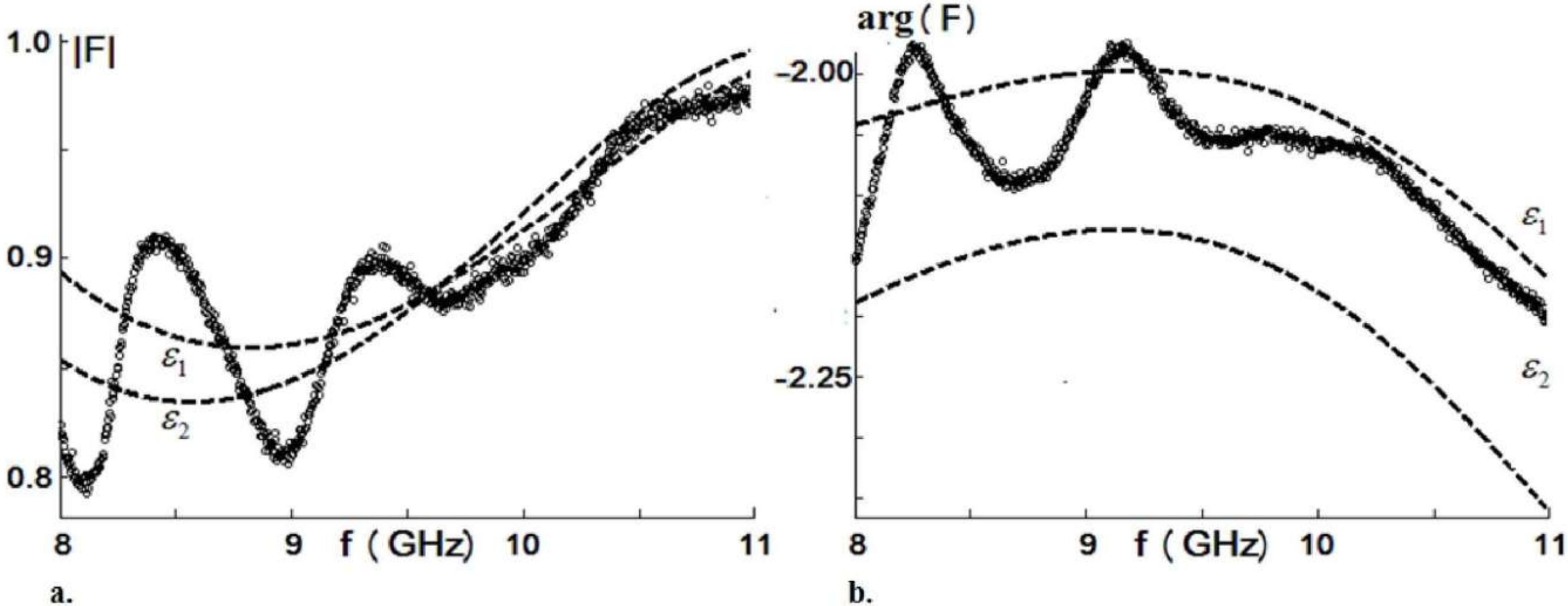
$\varepsilon = \varepsilon^{(layer)}$ the relative layer dielectric constant

ε_0 the dielectric constant of vacuum

$\mu = \mu_0$ the vacuum magnetic permeability

THE EXPERIMENT QUALITY

Comparison of experimental multifrequency data with exact solution for F in a waveguide with Teflon diaphragm



ºººº : experimental data for F, ---: exact data for F,

ε_1 =2.0, ε_2 =2.1 - test values of the Teflon permittivity,

$L_{wg} = 0.2$ **m,** $d^{(layer)} = 0.02$ **m,** $N^{(\exp)}$ **=801.**

AN INCORRECT INVERSE PROBLEM WITH INACCURATE EXPERIMENTAL DATA

Let $\mathbf{f} = \left(f_1, \ldots, f_{N^{(\exp)}}\right) \in \mathbb{R}^{N^{(\exp)}}$ frequency vector

$\mathbf{F}^{(\exp)} = \left(F_n^{(\exp)}, \ldots, F_{N^{(\exp)}}^{(\exp)}\right) \in \mathbb{C}^{N^{(\exp)}}$ complex-valued measurement data of $N^{(\exp)}$ experiments

Consider the equation

$$\mathbf{g}\left(\varepsilon^{(layer)}, \mathbf{f}\right) = \mathbf{g}^{(\exp)}, \qquad (1)$$

$$\mathbf{g}(\varepsilon, \mathbf{f}) = \left(g(\varepsilon, f_1), \ldots, g(\varepsilon, f_{N^{(\exp)}})\right) \in \mathbb{C}^{N^{(\exp)}},$$

$$\mathbf{g}^{(\exp)} = (g_1^{(\exp)}, \ldots, g_{N_{\exp}}^{(\exp)}) \in \mathbb{C}^{N^{(\exp)}}, \quad g_n^{(\exp)} = Z_0^{(layer)}(f_n) / F_n^{(\exp)}, \; n = 1, \ldots, N^{(\exp)}, \; N^{(\exp)} \geq 1$$

Let $\Omega^{(\varepsilon)} = \{\varepsilon : \varepsilon \geq 1\}$, $G\left(\mathbf{f}, \Omega^{(\varepsilon)}\right) = \left\{ \mathbf{g}(\varepsilon, \mathbf{f}) \in \mathbb{C}^{N^{(\exp)}}, \varepsilon \in \Omega^{(\varepsilon)} \right\}$

Problem 1 Find a real $\varepsilon^{(layer)} \in \Omega^{(\varepsilon)}$ satisfying relation (1)

for a given complex vector $\mathbf{g}^{(\exp)} \in G\left(\mathbf{f}, \Omega^{(\varepsilon)}\right)$ with the selected frequency vector $\mathbf{f}$.

Problem 2 Find a real $\varepsilon^{(layer)} \in \Omega^{(\varepsilon)}$ satisfying relation (1)

for a given complex vector $\mathbf{g}^{(\exp)} \in \mathbb{C}^{N^{(\exp)}}$ with the selected frequency vector $\mathbf{f}$.

approximate solution	Problem 1	Problem 2
existence	✔	✘
uniqueness	see page 7	

PROBLEM 1:
UNIQUENESS OF THE SOLUTION

a priori estimate of the dielectric constant

$$\Omega_{\mathrm{E}}^{(\varepsilon)} = \{\varepsilon : 1 \le \varepsilon \le \mathrm{E}\},\ \ \mathrm{E} > 1$$

$$G\left(\mathbf{f}, \Omega_E^{(\varepsilon)}\right) = \left\{ \mathbf{g}(\varepsilon, \mathbf{f}) \in \mathbb{C}^{N^{(\exp)}},\ \ \varepsilon \in \Omega_E^{(\varepsilon)} \right\},\ \ \mathbf{f} = \left(f_1, \ldots, f_{N^{(\exp)}}\right) \in \mathbb{R}^{N^{(\exp)}}$$

one-to-one correspondence

$$\Omega_E^{(\varepsilon)} \Rightarrow G(\mathbf{f}, \Omega_E^{(\varepsilon)})$$

$N^{(\exp)} = 1$	if $\dfrac{d^{(layer)}}{0.5\lambda_E(f)} < 1$	✔
$N^{(\exp)} \ge 2$	if $\dfrac{d^{(layer)}}{0.5\lambda_E(f_{n+1})} - \dfrac{d^{(layer)}}{0.5\lambda_E(f_n)} < 1,\ \ n = 1, \ldots, N^{(\exp)} - 1$ or $h^{(f)} < h_E^{(f)} = \dfrac{c}{2d^{(layer)}} \dfrac{1}{E^{1/2}}$	✔
$N^{(\exp)} = 1$	if $\dfrac{d^{(layer)}}{0.5\lambda_E(f)} = 4$ $g(\varepsilon_m, f) = 1$ for $\varepsilon_m^{1/2} = E^{1/2} m/2 \le E^{1/2}, m = 1,2$	✘
$N^{(\exp)} \ge 2$	if $h^{(f)} = 4h_E^{(f)}, f_n = nh^{(f)}, n = 1, \ldots, N^{(\exp)},$ $g(\varepsilon_m, f_n) = 1$ for $\varepsilon_m^{1/2} = E^{1/2} m/2 \le E^{1/2}, m = 1,2$	✘

PROBLEM 1: UNIQUENESS OF THE SOLUTION

EXAMPLES

a priori estimate of the dielectric constant

$\Omega_{\mathrm{E}}^{(\varepsilon)} = \{\varepsilon : 1 \le \varepsilon \le \mathrm{E}\},\ \ \mathrm{E} > 1$

$N^{(\exp)} = 1$

$f = 9.25\ GHz$

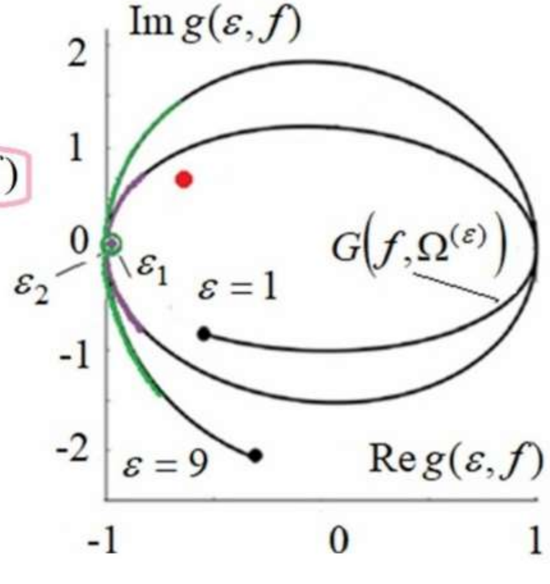
$G\left(f, \Omega_E^{(\varepsilon)}\right) = \left\{\, g(\varepsilon, f) \in \mathbb{C}^{N^{(\exp)}},\ \ \varepsilon \in \Omega_E^{(\varepsilon)} \,\right\}$

✗ no one-to-one correspondence

$\Omega_{\mathrm{E}}^{(\varepsilon)} \Rightarrow G(f, \Omega_{\mathrm{E}}^{(\varepsilon)})$

self-intersection points of $G(f, \Omega_{\mathrm{E}}^{(\varepsilon)})$

$g(\varepsilon_1, f) = g(\varepsilon_2, f)$



$N^{(\exp)} = 5$

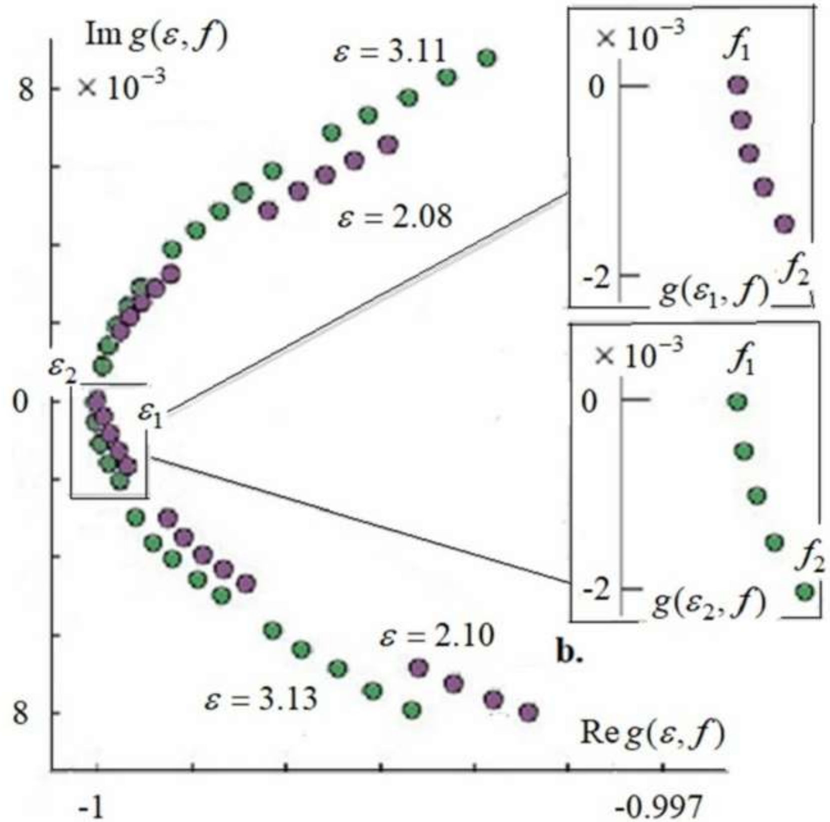
$\mathbf{f} = (9.25, 9.2501, 9.2502, 9.2503, 9.2504)\ GHz$

$\mathbf{g}(\varepsilon, \mathbf{f}) = \left(g(\varepsilon, f_1), \dots, g(\varepsilon, f_{N^{(\exp)}})\right) \in \mathbb{C}^{N^{(\exp)}}$

$G\left(\mathbf{f}, \Omega_E^{(\varepsilon)}\right) = \left\{\, \mathbf{g}(\varepsilon, \mathbf{f}) \in \mathbb{C}^{N^{(\exp)}},\ \ \varepsilon \in \Omega_E^{(\varepsilon)} \,\right\}$

✔ one-to-one correspondence

$\Omega_{\mathrm{E}}^{(\varepsilon)} \Leftrightarrow G(\mathbf{f}, \Omega_{\mathrm{E}}^{(\varepsilon)})$ $\quad \mathbf{g}(\varepsilon_1, \mathbf{f}) \ne \mathbf{g}(\varepsilon_2, \mathbf{f})$



MULTI-FREQUENCY LEAST SQUARES APPROXIMATE METHOD

Let $\mathbf{f} = \left(f_1, \ldots, f_{N^{(\exp)}}\right) \in \mathbb{R}^{N^{(\exp)}}$ frequency vector

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Consider the equation

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$$\mathbf{g}^{(\exp)} = (g_1^{(\exp)}, \ldots, g_{N_{\exp}}^{(\exp)}) \in \mathbb{C}^{N^{(\exp)}}, \quad g_n^{(\exp)} = Z_0^{(layer)}(f_n) / F_n^{(\exp)}, \ n = 1, \ldots, N^{(\exp)}, \ N^{(\exp)} \geq 1$$

Let $\Omega_{\mathrm{E}}^{(\varepsilon)} = \{\varepsilon : 1 \leq \varepsilon \leq \mathrm{E}\}$, $G\left(\mathbf{f}, \Omega_{\mathrm{E}}^{(\varepsilon)}\right) = \left\{ \mathbf{g}(\varepsilon, \mathbf{f}) \in \mathbb{C}^{N^{(\exp)}}, \varepsilon \in \Omega_{\mathrm{E}}^{(\varepsilon)} \right\}$

Problem 3 (least squares method, LSM) Find a real value $\varepsilon^{(LS)}$ satisfying the condition $\left\| \mathbf{g}(\varepsilon^{(LS)}, \mathbf{f}) - \mathbf{g}^{(\exp)} \right\|^{(\exp)} = \min\left(\left\| \mathbf{g}(\varepsilon, \mathbf{f}) - \mathbf{g}^{(\exp)} \right\|^{(\exp)}, \ \varepsilon \in \Omega_E^{(\varepsilon)} \right)$ for a given complex vector $\mathbf{g}^{(\exp)} \in \mathbb{C}^{N^{(\exp)}}$ with the selected frequency vector $\mathbf{f}$.

MULTI-FREQUENCY LEAST SQUARES APPROXIMATE METHOD

Consider the equation $$\mathbf{g}\left(\varepsilon^{(layer)},\mathbf{f}\right)=\mathbf{g}^{(\exp)} \qquad (1)$$

LSM $$\left\|\mathbf{g}(\varepsilon^{(LS)},\mathbf{f})-\mathbf{g}^{(\exp)}\right\|^{(\exp)}=\min\left(\left\|\mathbf{g}(\varepsilon,\mathbf{f})-\mathbf{g}^{(\exp)}\right\|^{(\exp)},\ \varepsilon\in\Omega_E^{(\varepsilon)}\right)$$

Proposition If $\varepsilon^{(layer)}\in\Omega_E^{(\varepsilon)}$, $E\geq 1$, and $h^{(f)}$, $N^{(\exp)}$ satisfy the conditions

$$h^{(f)}<h_E^{(f)}=\frac{c}{2d^{(layer)}}\frac{1}{E^{1/2}},$$

$$N^{(\exp)}\geq\frac{c}{4d^{(layer)}h^{(f)}}\frac{1}{\alpha},\quad 0<\alpha<1,$$

then

$$|\,\varepsilon^{(LS)}-\varepsilon^{(layer)}\,|\leq\kappa(\alpha)\left\|\mathbf{g}(\varepsilon^{(layer)},\mathbf{f})-\mathbf{g}^{(\exp)}\right\|,$$

$$\kappa(\alpha)=16\,\frac{(\varepsilon^{(layer)}+1)^2}{\varepsilon^{(layer)}-1}\frac{1}{1-\alpha}$$

---

quality of the experiment:
- defects
  - measurement setup
  - material samples
- noise

approximate solution		convergence (see Proposition)	✔
existence	✔	to exact solution	$\varepsilon^{(LS)}\to\varepsilon^{(layer)}$
uniqueness	✘	if the quality of the experiment is improved	$\mathbf{g}(\varepsilon^{(layer)},\mathbf{f})\to\mathbf{g}^{(\exp)}$

MULTI-FREQUENCY LEAST SQUARES APPROXIMATE METHOD

actual experiment (Ivanchenko et al., 2016)

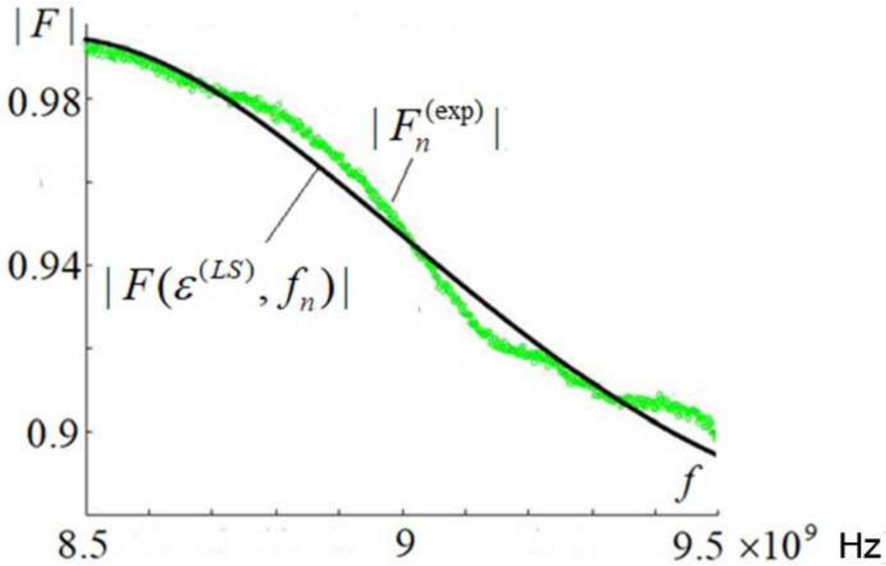

Fig. $\{|F_n^{(\exp)}|\}_{n=1,\dots N^{(\exp)}}$ data of experiment with a Teflon layer in waveguide

$\{|F(\varepsilon^{(LS)}, f_n)|\}_{n=1,\dots N^{(\exp)}}$ data calculated by LSM

the frequency range $[8.5-9.5]$ (GHz)

Conclusion

We have shown how to modify the traditional multi-frequency measurement technique in order to overcome incorrectness of reconstructing the layer permittivity in a rectangular waveguide and free space. The correctness can be achieved by reducing the frequency resolution taking into account an a priori given range of values of the dielectric constant.

The uniqueness of the resulting solution and well-posedness of the corresponding inverse problems are rigorously proved in a series of mathematical statements.

The developed approach leads to a practical algorithm of calculating the permittivity employing LSM. The solution obtained using this algorithm converges to the sought layer permittivity with the controlled rate under the condition that the quality of the experiment is improved.

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Thank you for your attention!