DIFFRACTION BY A PLANAR JUNCTION BETWEEN DPS AND DNG MATERIAL SHEETS: THE UAPO SOLUTION FOR PLANE WAVES AT SKEW INCIDENCE

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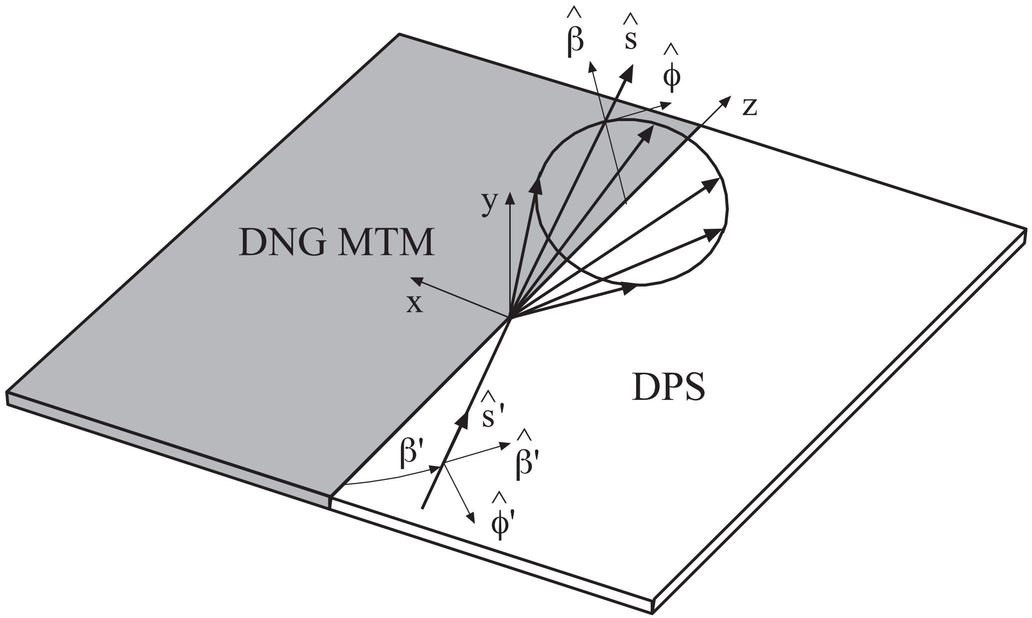
ABSTRACT

The plane wave diffraction by a planar junction between two lossy planar sheets is studied when the incidence direction is oblique to the rectilinear discontinuity of the structure. One slab consists of a standard double positive material, whereas an unusual double negative metamaterial is considered for the other slab. The Uniform Asymptotic Physical Optics approach is applied to find an efficient and user-friendly solution in the context of the Uniform Geometrical Theory of Diffraction. Such an approach is based on electric and magnetic equivalent sources radiating in the free space, and exploits an analytical process to obtain a closed form expression of the diffraction coefficients under the high-frequency assumption. A useful matrix representation is presented.

OUTLINE

- ✓ **DIFFRACTION PROBLEM**
- ✓ **THE UNIFORM ASYMPTOTIC PHYSICAL OPTICS APPROACH**
- ✓ **THE UAPO DIFFRACTION COEFFICIENTS**
- ✓ **NUMERICAL EXAMPLES**
- ✓ **CONCLUSIONS**

DIFFRACTION PROBLEM



$$\underline{E}^d = \begin{pmatrix} E^d_\beta \\ E^d_\phi \end{pmatrix} = \underline{\underline{D}} \frac{\exp(-jk_0 s)}{\sqrt{s}} \begin{pmatrix} E^i_{\beta'} \\ E^i_{\phi'} \end{pmatrix} = \underline{\underline{D}} \frac{\exp(-jk_0 s)}{\sqrt{s}} \underline{E}^i$$

Matrix of the diffraction coefficients

UNIFORM ASYMPTOTIC PO APPROACH

GO FIELD

↓

RADIATION INTEGRAL WITH A PO APPROXIMATION OF THE ELECTRIC AND MAGNETIC SURFACE CURRENTS

↓

USEFUL APPROXIMATION AND REPRESENTATION OF THE INTEGRALS

↓

STEEPEST DESCENT METHOD AND UNIFORM ASYMPTOTIC EVALUATION OF THE RESULTING INTEGRALS

↓

UAPO DIFFRACTED FIELD

UAPO DIFFRACTED FIELD (1)

RADIATION INTEGRAL WITH A PO APPROXIMATION OF THE ELECTRIC AND MAGNETIC SURFACE CURRENTS

$$\underline{E}^{s} \cong -jk_0 \iint_{S_{DNG}} \left[\left(\underline{\underline{I}} - \hat{R}\hat{R} \right) \left(\zeta_0 \underline{J}_s^{DNG} \right) + \underline{J}_{ms}^{DNG} \times \hat{R} \right] G\left(\underline{r}, \underline{r}' \right) dS - jk_0 \iint_{S_{DPS}} \left[\left(\underline{\underline{I}} - \hat{R}\hat{R} \right) \left(\zeta_0 \underline{J}_s^{DPS} \right) + \underline{J}_{ms}^{DPS} \times \hat{R} \right] G\left(\underline{r}, \underline{r}' \right) dS$$

$$\zeta_0 \underline{J}_s^{DNG} = \left[\left(1 - R_\perp - T_\perp \right) E_\perp^i \sin\beta' \sin\phi' \hat{e}_\perp + \left(1 + R_{||} - T_{||} \right) E_{||}^i \left(\hat{n} \times \hat{e}_\perp \right) \right] \exp\left(j\varphi(\underline{r}') \right) = \zeta_0 \underline{J}_s^{DNG} \Big|_{\varphi=0} \exp\left(j\varphi(\underline{r}') \right)$$

$$\underline{J}_{ms}^{DNG} = \left[\left(1 - R_{||} - T_{||} \right) E_{||}^i \sin\beta' \sin\phi' \hat{e}_\perp - \left(1 + R_\perp - T_\perp \right) E_\perp^i \left(\hat{n} \times \hat{e}_\perp \right) \right] \exp\left(j\varphi(\underline{r}') \right) = \underline{J}_{ms}^{DNG} \Big|_{\varphi=0} \exp\left(j\varphi(\underline{r}') \right)$$

$$\zeta_0 \underline{J}_s^{DPS} = \zeta_0 \underline{J}_s^{DPS} \Big|_{\varphi=0} \exp\left(j\varphi(\underline{r}') \right)$$

$$\underline{J}_{ms}^{DPS} = \underline{J}_{ms}^{DPS} \Big|_{\varphi=0} \exp\left(j\varphi(\underline{r}') \right)$$

UAPO DIFFRACTED FIELD (2)

USEFUL APPROXIMATION $\hat{R} \cong \hat{s}$

$$\underline{E}^S \cong -jk_0 \left[\left(\underline{\underline{I}} - \hat{s}\hat{s} \right) \zeta_0 \underline{J}_s^{DNG} \Big|_{\varphi=0} + \underline{J}_{ms}^{DNG} \Big|_{\varphi=0} \times \hat{s} \right] \iint_{S_{DNG}} \exp\left(j\varphi(\underline{r}') \right) G\left(\underline{r}, \underline{r}' \right) dS \; +$$

$$-jk_0 \left[\left(\underline{\underline{I}} - \hat{s}\hat{s} \right) \zeta_0 \underline{J}_s^{DPS} \Big|_{\varphi=0} + \underline{J}_{ms}^{DPS} \Big|_{\varphi=0} \times \hat{s} \right] \iint_{S_{DPS}} \exp\left(j\varphi(\underline{r}') \right) G\left(\underline{r}, \underline{r}' \right) dS$$

$$\underline{E}^s = \begin{pmatrix} E^s_\beta \\ E^s_\phi \end{pmatrix} \cong \left[\underline{\underline{M}}\, I^s_{DNG} + \underline{\underline{N}}\, I^s_{DPS} \right] \begin{pmatrix} E^i_{\beta'} \\ E^i_{\phi'} \end{pmatrix} = \underline{E}^s_{DNG} + \underline{E}^s_{DPS}$$

The matrices account for the expressions of the surface currents and

$$I^s_{DNG,DPS} = -jk_0 \iint_{S_{DNG,DPS}} \frac{\exp\left(j\varphi(\underline{r}') - jk_0 \left| \underline{r} - \underline{r}' \right| \right)}{4\pi \left| \underline{r} - \underline{r}' \right|} dS$$

UAPO DIFFRACTED FIELD (3)

The contribution of the DNG MTM sheet …

$$\underline{\underline{M}} = \underline{\underline{M}}_1 \left[\underline{\underline{M}}_2 \underline{\underline{M}}_4 \underline{\underline{M}}_5 + \underline{\underline{M}}_3 \underline{\underline{M}}_4 \underline{\underline{M}}_6 \right] \underline{\underline{M}}_7$$

$$\underline{\underline{M}}_1 = \begin{pmatrix} \cos\beta'\cos\phi & \cos\beta'\sin\phi & -\sin\beta' \\ -\sin\phi & \cos\phi & 0 \end{pmatrix}$$

$$\underline{\underline{M}}_2 = \begin{pmatrix} 1-\sin^2\beta'\cos^2\phi & -\cos\beta'\sin\beta'\cos\phi \\ -\sin^2\beta'\sin\phi\cos\phi & -\cos\beta'\sin\beta'\sin\phi \\ -\cos\beta'\sin\beta'\cos\phi & \sin^2\beta' \end{pmatrix}$$

$$\underline{\underline{M}}_3 = \begin{pmatrix} 0 & -\sin\beta'\sin\phi \\ -\cos\beta' & \sin\beta'\cos\phi \\ \sin\beta'\sin\phi & 0 \end{pmatrix}$$

$$\underline{\underline{M}}_4 = \frac{1}{\sqrt{1-\sin^2\beta'\sin^2\phi'}} \begin{pmatrix} -\cos\beta' & -\sin\beta'\cos\phi' \\ -\sin\beta'\cos\phi' & \cos\beta' \end{pmatrix}$$

$$\underline{\underline{M}}_5 = \begin{pmatrix} 0 & (1-R_\perp - T_\perp)\sin\beta'\sin\phi' \\ 1+R_{||}-T_{||} & 0 \end{pmatrix}$$

$$\underline{\underline{M}}_6 = \begin{pmatrix} (1-R_{||}-T_{||})\sin\beta'\sin\phi' & 0 \\ 0 & -1-R_\perp + T_\perp \end{pmatrix}$$

$$\underline{\underline{M}}_7 = \frac{1}{\sqrt{1-\sin^2\beta'\sin^2\phi'}} \begin{pmatrix} \cos\beta'\sin\phi' & \cos\phi' \\ -\cos\phi' & \cos\beta'\sin\phi' \end{pmatrix}$$

UAPO DIFFRACTED FIELD (4)

$$I_{DNG}^{S}=\frac{-jk_0}{4\pi}\int_{0}^{\infty}\exp\left(jk_0(x'\sin\beta'\cos\phi')\right)\int_{-\infty}^{\infty}\exp\left(-jk_0 z'\cos\beta'\right)\frac{\exp\left(-jk_0\sqrt{\left|\underline{\rho}-\underline{\rho}'\right|^2+\left(z-z'\right)^2}\right)}{\sqrt{\left|\underline{\rho}-\underline{\rho}'\right|^2+\left(z-z'\right)^2}}\,dz'\,dx'$$

where

$$\int_{-\infty}^{\infty}\exp\left(-jk_0 z'\cos\beta'\right)\frac{\exp\left(-jk_0\sqrt{\left|\underline{\rho}-\underline{\rho}'\right|^2+\left(z-z'\right)^2}\right)}{\sqrt{\left|\underline{\rho}-\underline{\rho}'\right|^2+\left(z-z'\right)^2}}\,dz'=-j\pi\exp\left(-jk_0 z\cos\beta'\right)H_0^{(2)}\left(k_0\left|\underline{\rho}-\underline{\rho}'\right|\sin\beta'\right)$$

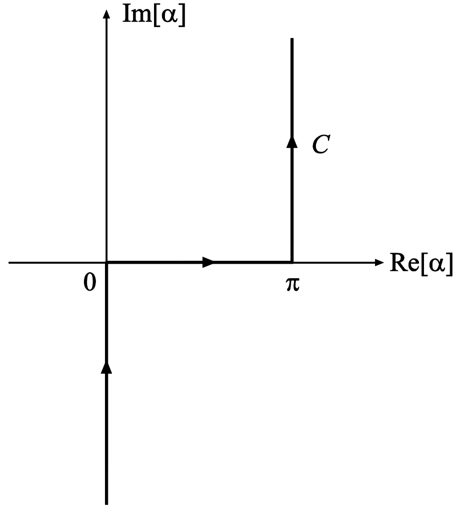
An integral representation of the zeroth order Hankel function of second kind and the application of the Sommerfeld-Maliuzhinets inversion formula provide:

$$I_{DNG}^{S}=\frac{\exp\left(-jk_0 z\cos\beta'\right)}{2\sin\beta'}\frac{1}{2\pi j}\int_{C}\frac{\exp\left(-jk_0\rho\sin\beta'\cos\left(\alpha\mp\phi\right)\right)}{\cos\alpha+\cos\phi'}\,\mathrm{d}\alpha$$

UAPO DIFFRACTED FIELD (5)

Accordingly, the integral contribution is represented by

$$I^s(\Omega) = \frac{1}{2\pi j}\int_C g(\alpha)\exp(\Omega\, f(\alpha))\,\mathrm{d}\alpha \qquad \Omega = k_0\rho$$



STEEPEST DESCENT METHOD

$$I^s(\Omega) = \sum_i \mathrm{Res}_i - \frac{1}{2\pi j}\int_{SDP} g(\alpha)\exp(\Omega\, f(\alpha))\,\mathrm{d}\alpha = \sum_i \mathrm{Res}_i + I(\Omega)$$

… AND UNIFORM ASYMPTOTIC EVALUATION OF THE RESULTING INTEGRAL

$$I^d_{DNG} = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}}\,\frac{F_t\left(2k_0 s\sin^2\beta'\cos^2\left(\frac{\phi\pm\phi'}{2}\right)\right)}{\sin^2\beta'(\cos\phi+\cos\phi')}\,\frac{\exp(-jk_0 s)}{\sqrt{s}}$$

UAPO DIFFRACTED FIELD (6)

$$\underline{E}^{d}_{DNG} = \underline{\underline{M}}\, I^{d}_{DNG} \begin{pmatrix} E^{i}_{\beta'} \\ E^{i}_{\phi'} \end{pmatrix}$$

… and the DPS sheet contribution

$$\underline{E}^{d}_{DPS} = \underline{\underline{N}}\, I^{d}_{DPS} \begin{pmatrix} E^{i}_{\beta'} \\ E^{i}_{\phi'} \end{pmatrix}$$

where

$$I^{d}_{DPS} = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} \frac{F_t\left(2k_0 s \sin^2\beta' \cos^2\left(\dfrac{(\pi-\phi)\pm(\pi-\phi')}{2}\right)\right)}{\sin^2\beta'\left[\cos(\pi-\phi)+\cos(\pi-\phi')\right]} \frac{\exp(-jk_0 s)}{\sqrt{s}}$$

UAPO DIFFRACTED FIELD (7)

$$\begin{pmatrix} E_\beta^d \\ E_\phi^d \end{pmatrix} = \underline{\underline{D}} \frac{\exp(-jk_0 s)}{\sqrt{s}} \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix}$$

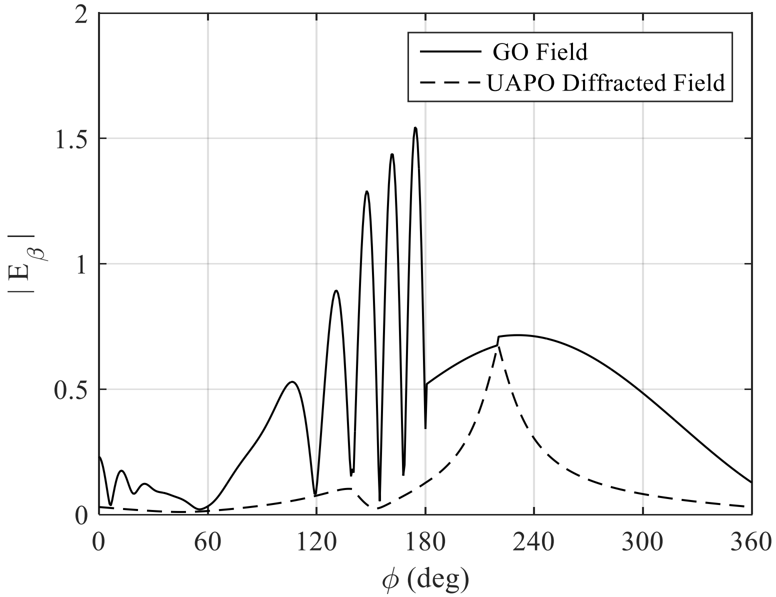
$$\begin{pmatrix} E_\beta^d \\ E_\phi^d \end{pmatrix} \cong \left[\underline{\underline{M}}\, I_{DNG}^d + \underline{\underline{N}}\, I_{DPS}^d \right] \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix}$$

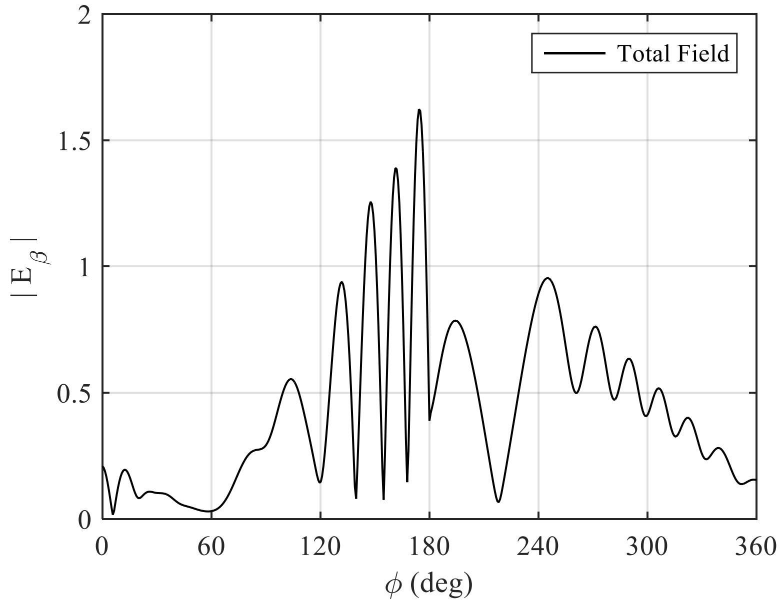
$$\underline{\underline{D}} = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} \frac{F_t\left(2k_0 s \sin^2\beta' \cos^2\left(\frac{\phi \pm \phi'}{2}\right)\right)}{\sin^2\beta'\left(\cos\phi + \cos\phi'\right)} \underline{\underline{M}} + \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} \frac{F_t\left(2k_0 s \sin^2\beta' \cos^2\left(\frac{(\pi-\phi) \pm (\pi-\phi')}{2}\right)\right)}{\sin^2\beta'\left[\cos(\pi-\phi) + \cos(\pi-\phi')\right]} \underline{\underline{N}}$$

UAPO DIFFRACTION MATRIX

$F_t(\cdot)$ is the UTD transition function

NUMERICAL EXAMPLE (1)





$$\beta' = 45°, \quad \phi' = 40°$$

$$E^i_{\beta'} = 1 \qquad E^i_{\phi'} = 0$$

$$\varepsilon_{r_{DNG}} = -2 - j0.01 \qquad \mu_{r_{DNG}} = -1 - j0.1 \qquad \varepsilon_{r_{DPS}} = 4 - j0.001 \qquad \mu_{r_{DPS}} = 1$$

$$d = 0.2\lambda_0 \qquad \rho = 5\lambda_0$$

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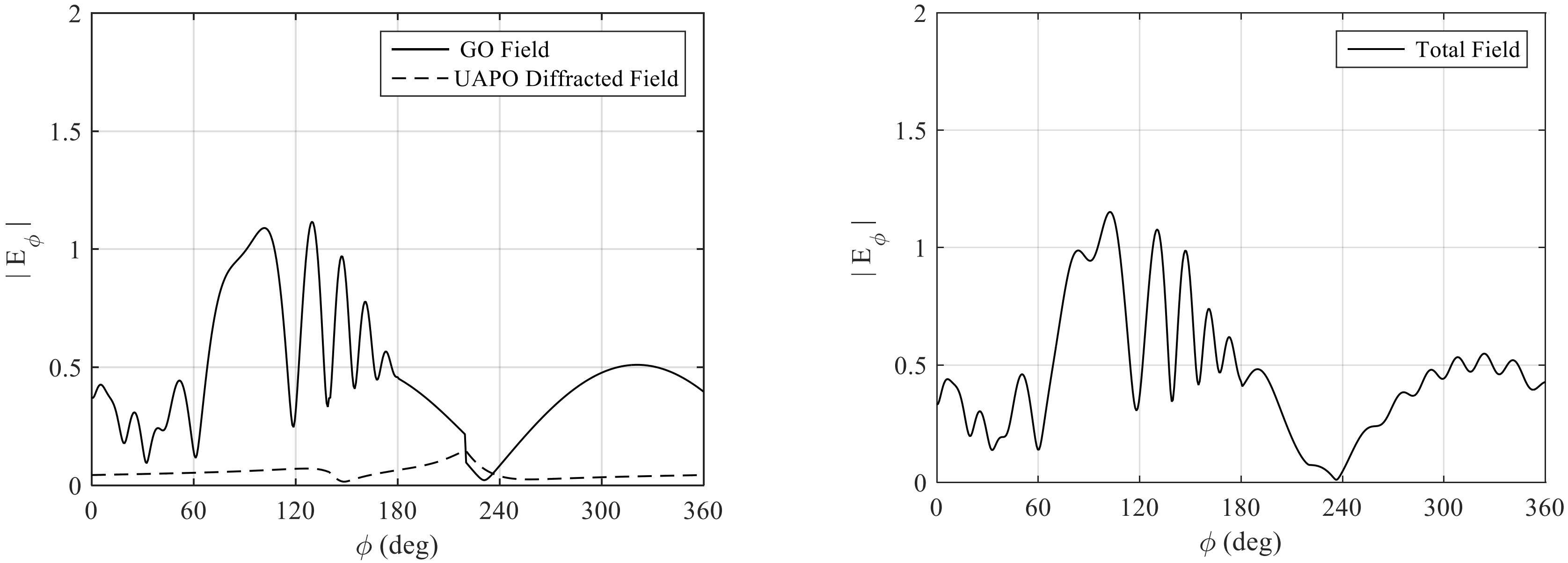
NUMERICAL EXAMPLE (2)



$$\beta' = 45°, \quad \phi' = 40°$$

$E^i_{\beta'} = 1 \qquad E^i_{\phi'} = 0$

$\varepsilon_{r_{DNG}} = -2 - j0.01 \qquad \mu_{r_{DNG}} = -1 - j0.1 \qquad \varepsilon_{r_{DPS}} = 4 - j0.001 \qquad \mu_{r_{DPS}} = 1$

$d = 0.2\lambda_0 \qquad \rho = 5\lambda_0$

CONCLUSIONS

The UAPO solution has been presented for the evaluation of the field diffracted by a planar junction that is formed by a lossy DNG MTM slab and a lossy DPS slab.

Pros:

- Closed form analytic solution that is UTD-like, compact, user-friendly and computationally efficient.

Cons:

- Approximate solution that suffers from the PO limitations.

Future works will be devoted to comparisons with data obtained by using full-wave techniques.