A HIGH-FREQUENCY SOLUTION FOR THE PLANE WAVE DIFFRACTION FROM A 90° METALLIC WEDGE WITH A METAMATERIAL LAYER ON THE TOP SURFACE

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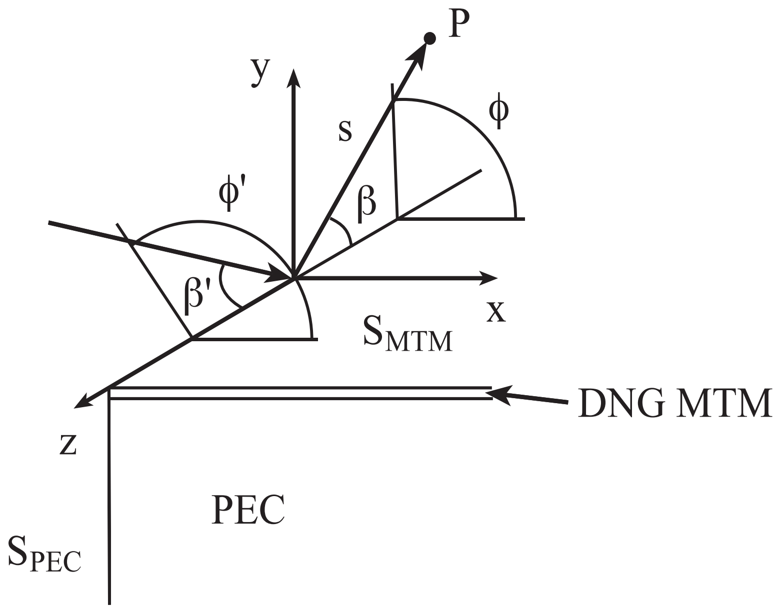
ABSTRACT

A uniform asymptotic solution is presented for evaluating the diffraction from a right-angled metallic wedge partially covered by a double negative metamaterial layer. A plane wave is assumed to impact the structure at skew incidence with respect to the edge. The problem is solved by means of an approach that is based on the Physical Optics approximation of surface currents radiating in the surrounding free space. The analytical procedure provides a closed form expression of the diffraction matrix in the framework of the Uniform Geometrical Theory of Diffraction. The related diffracted field is able to counterbalance the discontinuities of the Geometrical Optics field, thus producing a continuous total field also in correspondence of the shadow boundaries.

OUTLINE

- ✓ DIFFRACTION PROBLEM
- ✓ THE UNIFORM ASYMPTOTIC PHYSICAL OPTICS APPROACH
- ✓ THE UAPO DIFFRACTION COEFFICIENTS
- ✓ NUMERICAL EXAMPLES
- ✓ CONCLUSIONS

DIFFRACTION PROBLEM



$$\underline{E}^{d} = \begin{pmatrix} E_{\beta}^{d} \\ E_{\phi}^{d} \end{pmatrix} = \underline{\underline{D}} \frac{\exp(-jk_0 s)}{\sqrt{s}} \begin{pmatrix} E_{\beta'}^{i} \\ E_{\phi'}^{i} \end{pmatrix} = \underline{\underline{D}} \frac{\exp(-jk_0 s)}{\sqrt{s}} \underline{E}^{i}$$

Matrix of the diffraction coefficients

UNIFORM ASYMPTOTIC PO APPROACH

GO FIELD

↓

RADIATION INTEGRAL WITH A PO APPROXIMATION OF THE ELECTRIC AND MAGNETIC SURFACE CURRENTS

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USEFUL APPROXIMATION AND REPRESENTATION OF THE INTEGRALS

↓

STEEPEST DESCENT METHOD AND UNIFORM ASYMPTOTIC EVALUATION OF THE RESULTING INTEGRALS

↓

UAPO DIFFRACTED FIELD

UAPO DIFFRACTED FIELD (1)

RADIATION INTEGRAL WITH A PO APPROXIMATION OF THE ELECTRIC AND MAGNETIC SURFACE CURRENTS

$$\underline{E}^{S} \cong -jk_0 \iint_{S_{PEC}} \left(\underline{\underline{I}} - \hat{R}\hat{R}\right)\left(\zeta_0 \underline{J}_s^{PEC}\right) \frac{\exp\left(jk_0\left|\underline{r}-\underline{r}'\right|\right)}{4\pi\left|\underline{r}-\underline{r}'\right|} dS - jk_0 \iint_{S_{MTM}} \left[\left(\underline{\underline{I}} - \hat{R}\hat{R}\right)\left(\zeta_0 \underline{J}_s^{MTM}\right) + \underline{J}_{ms}^{MTM} \times \hat{R}\right] \frac{\exp\left(jk_0\left|\underline{r}-\underline{r}'\right|\right)}{4\pi\left|\underline{r}-\underline{r}'\right|} dS$$

$$\zeta_0 \underline{J}_s^{MTM} = \left[\left(1-\Gamma_{\perp}\right) E_{\perp}^{i} \sin\beta' \sin\phi' \, \hat{u}_{\perp} + \left(1+\Gamma_{\parallel}\right) E_{\parallel}^{i} (\hat{y} \times \hat{u}_{\perp})\right] \exp\left(j\varphi(\underline{r}')\right) U_{MTM} = \zeta_0 \underline{J}_s^{MTM}\Big|_{\varphi=0} \exp\left(j\varphi(\underline{r}')\right) U_{MTM}$$

$$\underline{J}_{ms}^{MTM} = \left[\left(1-\Gamma_{\parallel}\right) E_{\parallel}^{i} \sin\beta' \sin\phi' \, \hat{u}_{\perp} - \left(1+\Gamma_{\perp}\right) E_{\perp}^{i} (\hat{y} \times \hat{u}_{\perp})\right] \exp\left(j\varphi(\underline{r}')\right) U_{MTM} = \underline{J}_{ms}^{MTM}\Big|_{\varphi=0} \exp\left(j\varphi(\underline{r}')\right) U_{MTM}$$

U_{MTM} is equal to 1 or 0 accounting for the illumination of the MTM surface by the incident field. The reflection coefficients for the parallel and perpendicular polarizations are evaluated by means of the Equivalent Transmission Line circuit, i.,e.:

$$\Gamma_{\parallel,\perp} = \frac{Z_{\parallel,\perp}^{in} - Z_{\parallel,\perp}^{0}}{Z_{\parallel,\perp}^{in} + Z_{\parallel,\perp}^{0}}$$

UAPO DIFFRACTED FIELD (2)

USEFUL APPROXIMATION $\hat{R} \cong \hat{s}$

$$\underline{E}^S \cong -jk_0 \left[\left(\underline{\underline{I}} - \hat{s}\hat{s} \right) \zeta_0 \underline{J}_s^{MTM} \Big|_{\varphi=0} + \underline{J}_{ms}^{MTM} \Big|_{\varphi=0} \times \hat{s} \right] U_{MTM} \iint_{S_{MTM}} \exp\left(j\varphi(\underline{r}') \right) \frac{\exp\left(jk_0 \left| \underline{r} - \underline{r}' \right| \right)}{4\pi \left| \underline{r} - \underline{r}' \right|} dS \; +$$

$$-jk_0 \left[\left(\underline{\underline{I}} - \hat{s}\hat{s} \right) \zeta_0 \underline{J}_s^{PEC} \Big|_{\varphi=0} \right] U_{PEC} \iint_{S_{PEC}} \exp\left(j\varphi(\underline{r}') \right) \frac{\exp\left(jk_0 \left| \underline{r} - \underline{r}' \right| \right)}{4\pi \left| \underline{r} - \underline{r}' \right|} dS$$

$$\underline{E}^s = \begin{pmatrix} E_\beta^s \\ E_\phi^s \end{pmatrix} \cong \left[\underline{\underline{M}}\, I_{MTM}^s U_{MTM} + \underline{\underline{N}}\, I_{PEC}^s U_{PEC} \right] \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} = \underline{E}_{MTM}^s U_{MTM} + \underline{E}_{PEC}^s U_{PEC}$$

The matrices account for the expressions of the PO surface currents and

$$I_{MTM,PEC}^s = -jk_0 \iint_{S_{MTM,PEC}} \frac{\exp\left(j\varphi(\underline{r}') - jk_0 \left| \underline{r} - \underline{r}' \right| \right)}{4\pi \left| \underline{r} - \underline{r}' \right|} dS$$

UAPO DIFFRACTED FIELD (3)

The contribution of the DNG MTM layer with PEC backing…

$$\underline{\underline{M}} = \underline{\underline{M}}_1 \left[\underline{\underline{M}}_2 \underline{\underline{M}}_4 \underline{\underline{M}}_5 + \underline{\underline{M}}_3 \underline{\underline{M}}_4 \underline{\underline{M}}_6 \right] \underline{\underline{M}}_7$$

$$\underline{\underline{M}}_1 = \begin{pmatrix} \cos\beta'\cos\phi & \cos\beta'\sin\phi & -\sin\beta' \\ -\sin\phi & \cos\phi & 0 \end{pmatrix}$$

$$\underline{\underline{M}}_2 = \begin{pmatrix} 1-\sin^2\beta'\cos^2\phi & -\cos\beta'\sin\beta'\cos\phi \\ -\sin^2\beta'\sin\phi\cos\phi & -\cos\beta'\sin\beta'\sin\phi \\ -\cos\beta'\sin\beta'\cos\phi & \sin^2\beta' \end{pmatrix}$$

$$\underline{\underline{M}}_3 = \begin{pmatrix} 0 & -\sin\beta'\sin\phi \\ -\cos\beta' & \sin\beta'\cos\phi \\ \sin\beta'\sin\phi & 0 \end{pmatrix}$$

$$\underline{\underline{M}}_4 = \frac{1}{\sqrt{1-\sin^2\beta'\sin^2\phi'}} \begin{pmatrix} -\cos\beta' & -\sin\beta'\cos\phi' \\ -\sin\beta'\cos\phi' & \cos\beta' \end{pmatrix}$$

$$\underline{\underline{M}}_5 = \begin{pmatrix} 0 & (1-\Gamma_\perp)\sin\beta'\sin\phi' \\ 1+\Gamma_{||} & 0 \end{pmatrix}$$

$$\underline{\underline{M}}_6 = \begin{pmatrix} (1-\Gamma_{||})\sin\beta'\sin\phi' & 0 \\ 0 & -1-\Gamma_\perp \end{pmatrix}$$

$$\underline{\underline{M}}_7 = \frac{1}{\sqrt{1-\sin^2\beta'\sin^2\phi'}} \begin{pmatrix} \cos\beta'\sin\phi' & \cos\phi' \\ -\cos\phi' & \cos\beta'\sin\phi' \end{pmatrix}$$

UAPO DIFFRACTED FIELD (4)

$$I_{MTM}^{S} = \frac{-jk_0}{4\pi}\int_{0}^{\infty}\exp\left(jk_0\left(x'\sin\beta'\cos\phi'\right)\right)\int_{-\infty}^{\infty}\exp\left(-jk_0 z'\cos\beta'\right)\frac{\exp\left(-jk_0\sqrt{\left|\underline{\rho}-\underline{\rho}'\right|^2+\left(z-z'\right)^2}\right)}{\sqrt{\left|\underline{\rho}-\underline{\rho}'\right|^2+\left(z-z'\right)^2}}\,dz'\,dx'$$

where

$$\int_{-\infty}^{\infty}\exp\left(-jk_0 z'\cos\beta'\right)\frac{\exp\left(-jk_0\sqrt{\left|\underline{\rho}-\underline{\rho}'\right|^2+\left(z-z'\right)^2}\right)}{\sqrt{\left|\underline{\rho}-\underline{\rho}'\right|^2+\left(z-z'\right)^2}}\,dz' = -j\pi\exp\left(-jk_0 z\cos\beta'\right)H_0^{(2)}\left(k_0\left|\underline{\rho}-\underline{\rho}'\right|\sin\beta'\right)$$

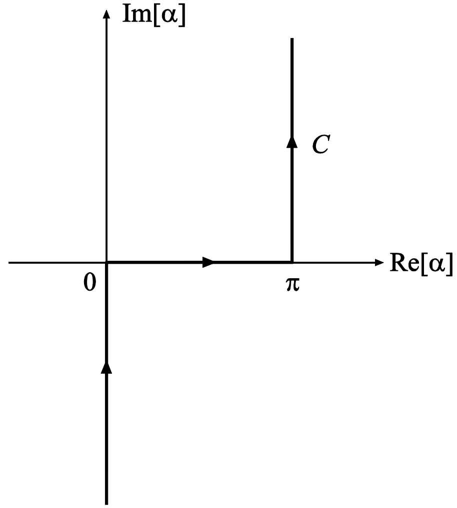
An integral representation of the zeroth order Hankel function of second kind and the application of the Sommerfeld-Maliuzhinets inversion formula provide:

$$I_{MTM}^{S} = \frac{\exp\left(-jk_0 z\cos\beta'\right)}{2\sin\beta'}\frac{1}{2\pi j}\int_{C}\frac{\exp\left(-jk_0\rho\sin\beta'\cos\left(\alpha\mp\phi\right)\right)}{\cos\alpha+\cos\phi'}\,d\alpha$$

UAPO DIFFRACTED FIELD (5)

Accordingly, the integral contribution is represented by

$$I^s(\Omega) = \frac{1}{2\pi j}\int_C g(\alpha)\exp(\Omega\, f(\alpha))\mathrm{d}\alpha \qquad \Omega = k_0\rho$$



STEEPEST DESCENT METHOD

$$I^s(\Omega) = \sum_i \mathrm{Res}_i - \frac{1}{2\pi j}\int_{SDP} g(\alpha)\exp(\Omega\, f(\alpha))\mathrm{d}\alpha = \sum_i \mathrm{Res}_i + I(\Omega)$$

… AND UNIFORM ASYMPTOTIC EVALUATION OF THE RESULTING INTEGRAL

$$I^d_{MTM} = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} \frac{F_t\left(2k_0 s \sin^2\beta' \cos^2\left(\frac{\phi \pm \phi'}{2}\right)\right)}{\sin^2\beta'(\cos\phi + \cos\phi')} \frac{\exp(-jk_0 s)}{\sqrt{s}}$$

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UAPO DIFFRACTED FIELD (6)

$$\underline{E}^{s}_{MTM} \;\Rightarrow\; \underline{E}^{d}_{MTM} = \underline{\underline{M}}\, I^{d}_{MTM} \begin{pmatrix} E^{i}_{\beta'} \\ E^{i}_{\phi'} \end{pmatrix}$$

… and the contribution of the PEC surface

$$\underline{E}^{s}_{PEC} \;\Rightarrow\; \underline{E}^{d}_{PEC} = \underline{\underline{N}}\, I^{d}_{PEC} \begin{pmatrix} E^{i}_{\beta'} \\ E^{i}_{\phi'} \end{pmatrix}$$

where

$$I^{d}_{PEC} = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} \frac{F_t\left(2k_0 s \sin^2\beta' \cos^2\left(\dfrac{(3\pi/2-\phi)\pm(3\pi/2-\phi')}{2}\right)\right)}{\sin^2\beta'\left[\cos(3\pi/2-\phi)+\cos(3\pi/2-\phi')\right]} \frac{\exp(-jk_0 s)}{\sqrt{s}}$$

UAPO DIFFRACTED FIELD (7)

$$\begin{pmatrix} E_\beta^d \\ E_\phi^d \end{pmatrix} = \underline{\underline{D}} \frac{\exp(-jk_0 s)}{\sqrt{s}} \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix}$$

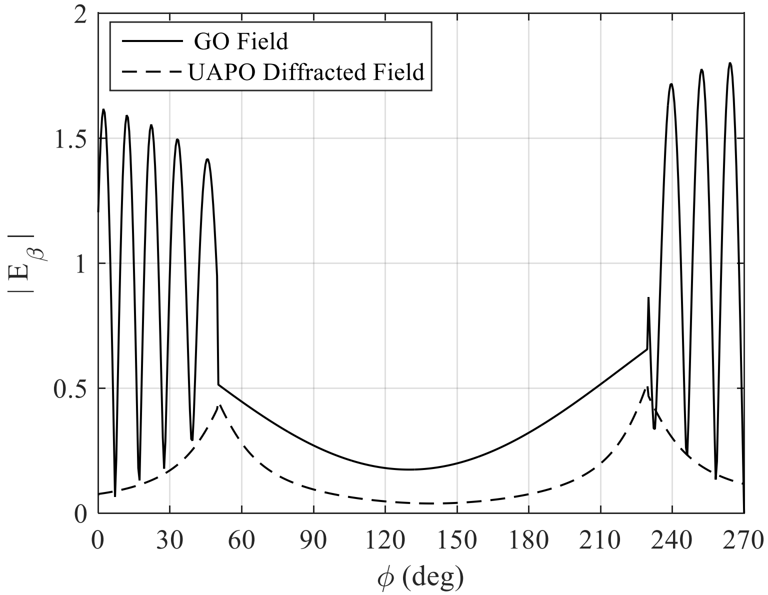
$$\begin{pmatrix} E_\beta^d \\ E_\phi^d \end{pmatrix} \cong \left[\underline{\underline{M}}\, I_{MTM}^d U_{MTM} + \underline{\underline{N}}\, I_{PEC}^d U_{PEC} \right] \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix}$$

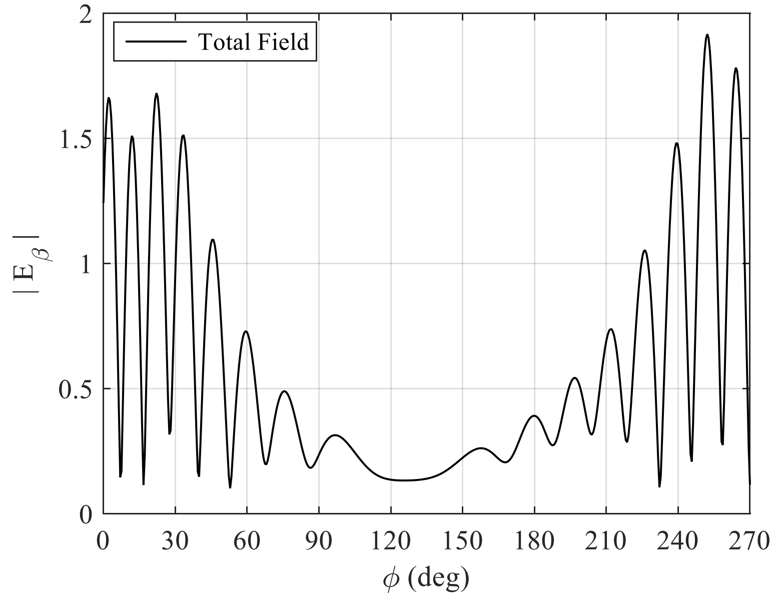
$$\underline{\underline{D}} = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} \frac{F_t\left(2k_0 s \sin^2\beta' \cos^2\left(\frac{\phi \pm \phi'}{2}\right)\right)}{\sin^2\beta'\left(\cos\phi + \cos\phi'\right)} U_{MTM}\, \underline{\underline{M}} + \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} \frac{F_t\left(2k_0 s \sin^2\beta' \cos^2\left(\frac{(3\pi/2-\phi) \pm (3\pi/2-\phi')}{2}\right)\right)}{\sin^2\beta'\left[\cos(3\pi/2-\phi) + \cos(3\pi/2-\phi')\right]} U_{PEC}\, \underline{\underline{N}}$$

UAPO DIFFRACTION MATRIX

$F_t(\cdot)$ is the UTD transition function

NUMERICAL EXAMPLE (1)



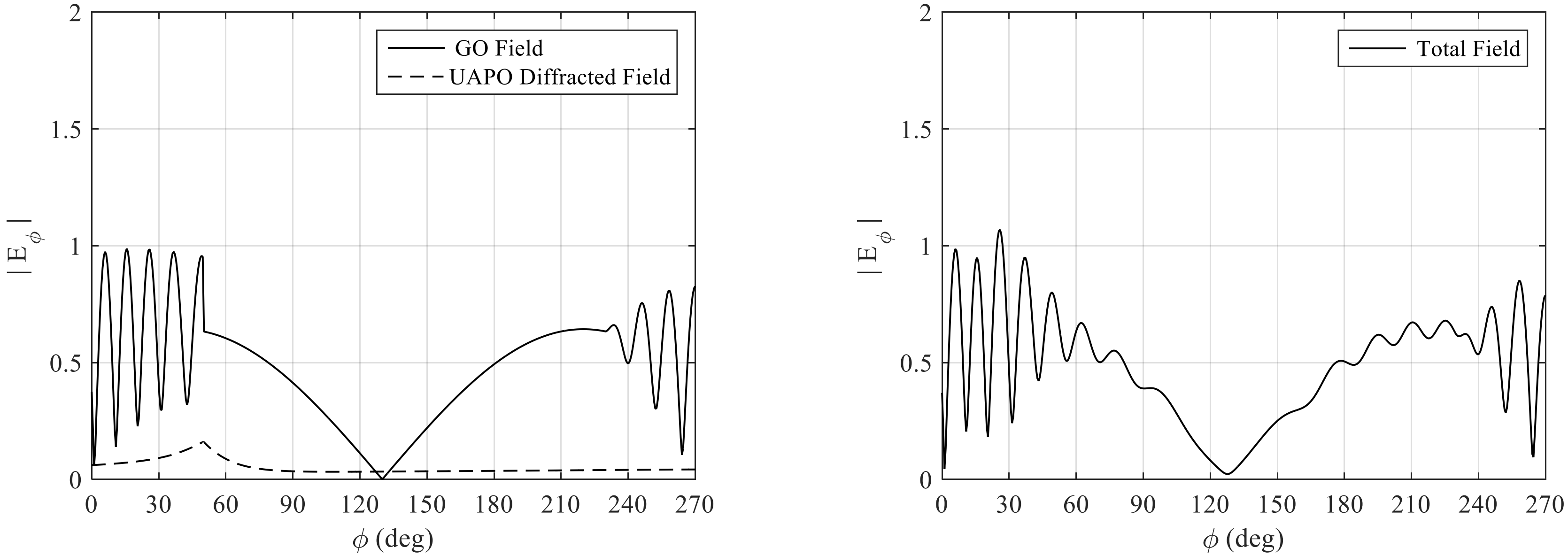


$$\beta' = 50°, \quad \phi' = 130°$$

$$E^i_{\beta'} = 1 \qquad E^i_{\phi'} = 0$$

$$\varepsilon_{r_{MTM}} = -3 - j0.01 \qquad \mu_{r_{MTM}} = -1 - j0.02 \qquad d = 0.2\lambda_0 \qquad \rho = 5\lambda_0$$

NUMERICAL EXAMPLE (2)



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$$\varepsilon_{r_{MTM}} = -3 - j0.01 \qquad \mu_{r_{MTM}} = -1 - j0.02 \qquad d = 0.2\lambda_0 \qquad \rho = 5\lambda_0$$

CONCLUSIONS

The UAPO solution has been presented for the evaluation of the field diffracted by a 90° PEC wedge with a lossy DNG MTM layer on the top surface.

Pros:

- Closed form analytic solution that is UTD-like, compact, user-friendly and computationally efficient.

Cons:

- Approximate solution that suffers from the PO limitations.

Future works will be devoted to comparisons with data obtained by using full-wave techniques.