First- and Second Order Characterization of Temporal Moments of Stochastic Multipath Channels

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Introduction

- Wideband radio channels are characterized using the received power, mean delay, and rms delay spread calculated from the first three raw temporal moments of the received signal y defined as:

$$m_i = \int_0^\infty |y(\tau)|^2 \tau^i d\tau, \quad i = 0, 1, 2 \ldots$$

- Recently, temporal moments have been used to calibrate stochastic multipath models, avoiding the need for multipath extraction.

  [Bharti and Pedersen, 2020, Bharti et al., 2019c, Bharti et al., 2019a, Bharti et al., 2019b, Bharti et al., 2020]

- Temporal moments are widely used to sumarize data but analysis of their proporties has been largely ignored in the literature.
- For stochastic multipath models, the raw temporal moments are jointly random variables.
  - Mean: $\mu_i = \mathbb{E}[m_i] = \int P(\tau)\tau^i d\tau$, where $P(\tau)$ is the power delay spectrum.
    Welknown — see any propagation textbook!
  - Covariance: $\sigma_{ij} = \mathrm{Cov}(m_i, m_j) = \mathbb{E}[m_i m_j] - \mu_i \mu_j =?$
    To the author's knowledge not been published. (Remarkably!)

Are Expressions for the Covariance Needed?

Having access to formulas connecting the temporal moments to model parameters help designers to understand the connections between system and channel parameters.

- The mean alone does not carry information on the dispersion of the temporal moments.
- Without such formulas, it is necessary to run simulations to understanding the effects of
  - channel model parameters, (e.g. path arrival rate)
  - system parameters (e.g. antenna directivity)
- It is in general difficult to relate the often reported ECDFs of rms delay spread to model parameters.
- Recently, temporal moments have been recently modeled as a jointly log normal with mean and covariance as parameters. [Ayush Bharti, 2020]

In this contribution we derive a *general expression for covariance of all temporal moments* of any Uncorrelated Scattering (US) stochastic multipath model. We apply this general expression to Turin's multipath model.

Stochastic Multipath Models as a Point Process

Multipath models for the radio channel yield a received signal of the form:

$$y(t) = \sum_{x \in \mathcal{X}} \alpha_x s(t - \tau_x).$$

where x denotes the pair (τ_x, α_x) of delay τ_x and complex gain α_x.

The collection of pairs $\mathcal{X} = \{x_1, x_2, x_3, \dots\}$ is a marked point process with points $\{\tau_x\}$ and associated marks $\{\alpha_x\}$.

The intensity function, or arrival rate, is denoted by $\lambda(t)$.

The conditional mean square of the marks is denoted by $\sigma_\alpha^2(\tau) = \mathbb{E}[|\alpha_x|^2 | \tau_x = \tau]$

The power delay spectrum: $P(\tau) = \sigma_\alpha^2(\tau)\lambda(\tau)$

Mean of Temporal Moments

For simplicity, assume high signal bandwidths, such that the temporal moment read

$$m_i = \sum_{x \in \mathcal{X}} |\alpha_x|^2 \tau_x^i, \quad i = 0, 1, 2, \ldots$$

Law of total expectation and Campbell's theorem gives the wellknown,

$$\mu_i = \mathbb{E}[m_i] = \int \sigma_\alpha^2(\tau)\lambda(\tau)\tau^i d\tau = \int P(\tau)\tau^i d\tau, \quad i = 0, 1, 2, \ldots$$

Notice that the power delay spectrum $P(\tau)$ completely specifies the means of all the temporal moments.

First order Campbell Theorem:

$$\mathbb{E}[\sum_{x \in \mathcal{X}} f(x)] = \int f(x)\lambda(x)dx$$

where $f(x)$ is some function of a single point and $\lambda(x)$ is the intensity function of $\mathcal{X}$.

Interpretation:
$\lambda(x)dx \approx$ *Prob. of a point in* dx.

Covariance of Temporal Moments

To obtain the covariance $\sigma_{ij} = \mathrm{Cov}(m_i, m_j) = \mathbb{E}[m_i m_j] - \mu_i \mu_j$, it suffices to compute $\mathbb{E}[m_i m_j]$. By the law of total expectation,

$$\mathbb{E}[m_i m_j] = \iint \mathbb{E}\left[\sum_{x,x'} A(\tau_x, \tau_{x'}) \tau_x^i \cdot \tau_{x'}^j\right] d\tau_x d\tau_{x'}$$

with $A(\tau_x, \tau_{x'}) = \mathbb{E}[|\alpha_x|^2 |\alpha_{x'}|^2 | \tau_x, \tau_{x'}]$. First and second order Campbell theorems lead to

$$\mathbb{E}[m_i m_j] = \int A(\tau, \tau)\lambda(\tau)\tau^{i+j} d\tau + \iint A(\tau, \tau')\lambda^{(2)}(\tau, \tau')\tau^i \tau^{j'} d\tau$$

This result gives (convergence provided) the covariance structure for the temporal moments for any model where $A(\tau, \tau')$, $\lambda(\tau)$ and $\lambda^{(2)}(\tau, \tau')$ are known.

Second order Campbell Thm:

$$\mathbb{E}[\sum_{x \neq x'} g(x, x')] = \iint g(x, x')\lambda^{(2)}(x, x') dx dx'$$

where $g(x, x')$ is some function of pairs of points and $\lambda^{(2)}(x, x')$ is the "second order factorial intensity function" of $\mathcal{X}$.

Interpretation:
$\lambda^{(2)}(x, x') dx dx' \approx$ Prob. of a point in each of dx and dx'

Example: Application to Turin's Model

For Turins model [Turin et al., 1972], the mean of the temporal moments are wellknown, but the covariance does not appear in the literature.

- Here, $\mathcal{X}$ is an independently marked Poisson process specified by the arrival rate $\lambda(\tau)$ and the conditional mark density $p(\alpha|\tau)$.
- For a Poisson point proceses, $\lambda^{(2)}(\tau,\tau') = \lambda(\tau)\lambda(\tau')$.
- Since the marks are independent, we have

$$A(\tau,\tau') = \begin{cases} \kappa_\alpha(\tau), & \tau = \tau' \\ \sigma_\alpha^2(\tau)\sigma_\alpha^2(\tau'), & \tau \neq \tau' \end{cases}$$

  with second and fourth moments of $p(\alpha|\tau)$ are denoted as $\sigma_\alpha^2(\tau)$ and $\kappa_\alpha(\tau)$, respectively.
- Then our equation for the covariance, gives after cancelling terms

$$\sigma_{ij} = \int \kappa_\alpha(\tau)\lambda(\tau)\tau^{i+j}d\tau.$$

Example: Application to Turin's Model — Remarks

- For specific $\lambda(\tau)$, and $\kappa_\alpha(\tau)$, the resulting integral

$$\sigma_{ij} = \int \kappa_\alpha(\tau)\lambda(\tau)\tau^{i+j} d\tau$$

  can be computed analytically or numerically.
- For some i, j settings, covariances are undefined and the integral diverges.
- Distinct settings with the same power delay spectra, and thus mean temporal moments, may lead to very different covariance structures.
- This confirms observations from [Pedersen, 2018, Pedersen, 2019] that models with the same power delay spectrum, but different higher moment spectra, produced different distribution of temporal moments.

Simulation Example: Room Electromagnetics Turin Model

We simulate a special case of the Turin model was studied for the room electromagnetic setting in [Pedersen, 2019] specified as:

$$P(t) = G_0 \exp(-t/T), \quad t > 0, \qquad \lambda(t) = at^b, \quad t > 0$$

The reverberation gain G_0, reverberation time T and the arrival rate parameters a and b control the model.

Mark distribution $p(\alpha|\tau)$: circular complex Gaussian with conditional second moment $\sigma_\alpha^2(\tau) = P(\tau)/\lambda(\tau)$.

For this model, we obtain analytical results:

$$\mu_i = G_0 T^{i+1} i! \qquad \text{and} \qquad \sigma_{ij} = \frac{2G_o^2}{a}\left(\frac{T}{2}\right)^{i+j-b}\Gamma(i+j-b+1), \qquad i+j-b+1>0.$$

Simulation Example (Contd.)

We compare the theoretical to Monte Carlo simulations with the settings according to room electromagnetics:

$$G_0 = \frac{4\pi c}{V}, \quad T = -\frac{4V}{cS \ln g}, \quad a = \frac{4\pi c^3 \omega^2}{V}, \quad b = 2.$$

- V: room volume
- S: room surface area
- c: speed of light

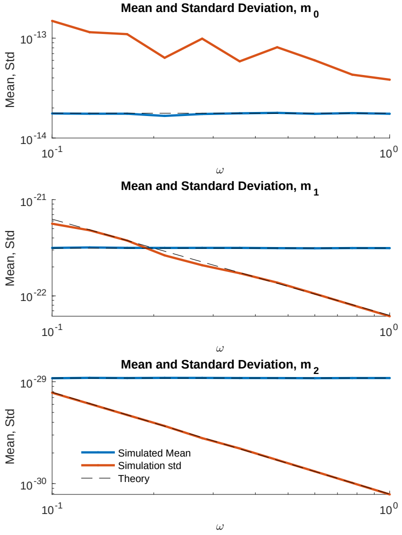
The beam coverage fraction ω accounts for the directivity of the transmitter and receiver antennas. Isotropic antennas: $\omega = 1$. Hemisphere antennas $\omega = 0.5$.

The power delay spectrum is not affected by ω.

Simulation Settings

Parameter	Value
Room dim.	$5 \times 5 \times 3\,\mathrm{m}^3$
g	0.6
c	$3 \cdot 10^8\,\mathrm{m/s}$
Max sim. time $t_{\max}$	$120\,\mathrm{ns}$
No. Monte Carlo runs	10^4

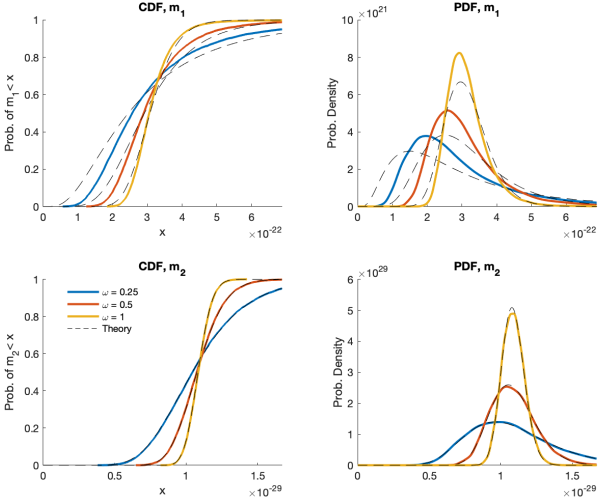
Simulation: Mean and Standard Deviation of Temporal moments



Simulated and theoretical mean and standard deviation of temporal moments as functions of ω.

- The expression for the variance of m_0 diverges.
- The simulation results follow closely the theoretical values.
- Mean values of the temporal moments are unaffected by ω.
- Variance (standard deviations) depend on ω.
- Higher antenna directivity (lower ω) gives higher variance of temporal moments.

Simulation: Distribution of Temporal Moments



- Empirical distribution of temporal moments with ω as parameter.
- Log-normal distributions are included according to [Ayush Bharti, 2020] with theoretical mean and variance.
- Mostly, the temporal moments are well modeled by a log-normal distribution.
- The effect of ω on the distributions of moments is well captured, especially for m_2.

Conclusion

- The expression enable us to compute and analyse the impact of model parameters on the covariance structure of temporal moments:
  - The mean depends the arrival rate λ.
  - The covariance depends on the second-order factorial intensity $\lambda^{(2)}$
- This highlights the importance of $\lambda^{(2)}$ in a stochastic channel model, an entity which is commonly ignored.
- The results are applicable to models for which $\lambda^{(2)}$ can be obtained.
  - This entity is known for the Poisson process, but also for many others such as binomial and Cox process.
  - For stochastic mutipath models which can be identified as a wellknown point process, our result is straight-forward to apply.
- Currently work is ongoing to obtain the mean and covariance of received power, mean delay and rms delay spread also.

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