Retrieval and Synthesis of Sources having a Circular Support and Generating Shaped Patterns

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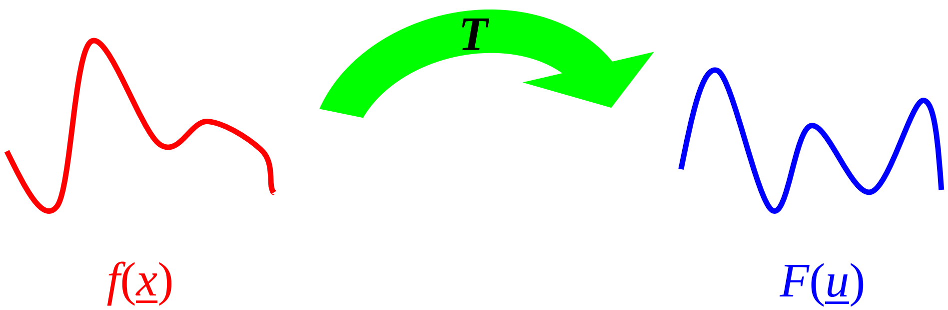
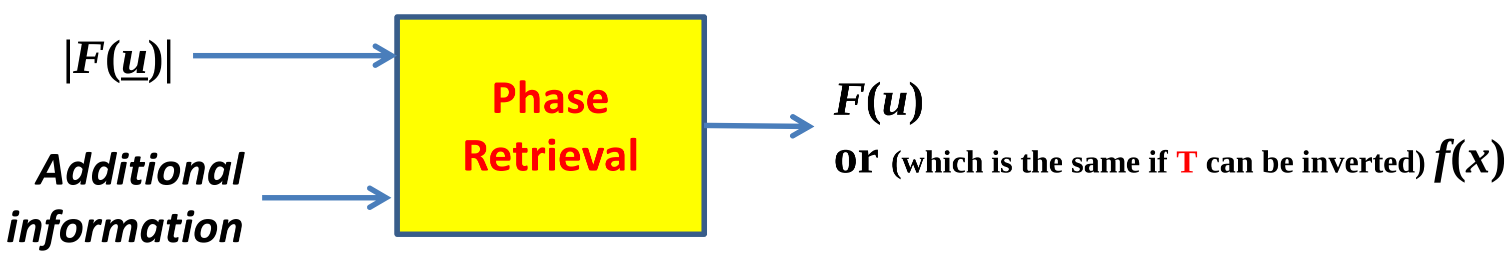
2) Consorzio Nazionale Interuniversitario per le Telecomunicazioni (CNIT), viale G. Usberti, Parma, Italy

The Phase Retrieval Problem

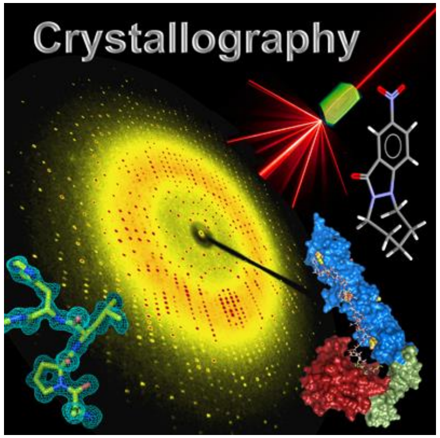


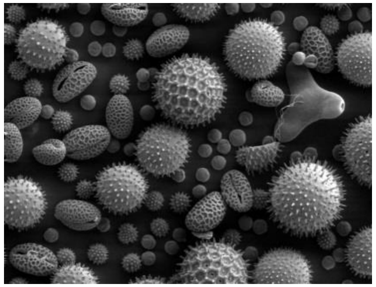
$$F(\underline{u}) = T\left[f(\underline{x})\right] = \left|F(\underline{u})\right| e^{j\varphi(\underline{u})}$$

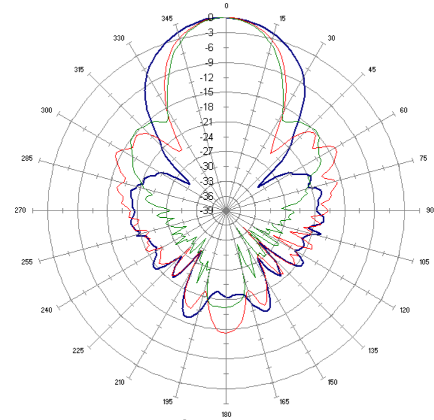
$f(\underline{x})$ is an unknown signal
T is a known operator
$\underline{x}$ and $\underline{u}$ are the vectors spanning the corresponding multidimensional domains

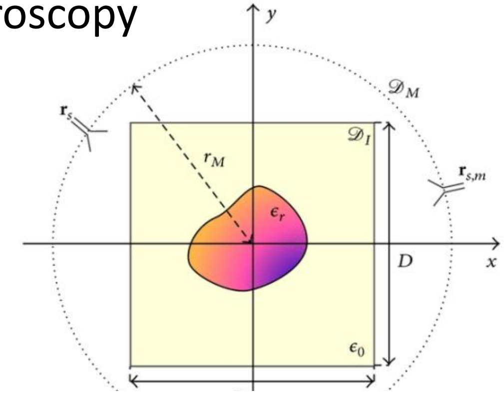


Interest



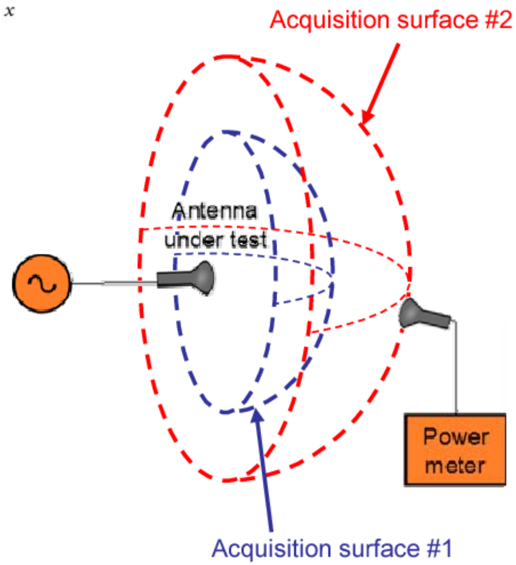

electron microscopy


antenna characterization



inverse scattering


antenna testing through UAV

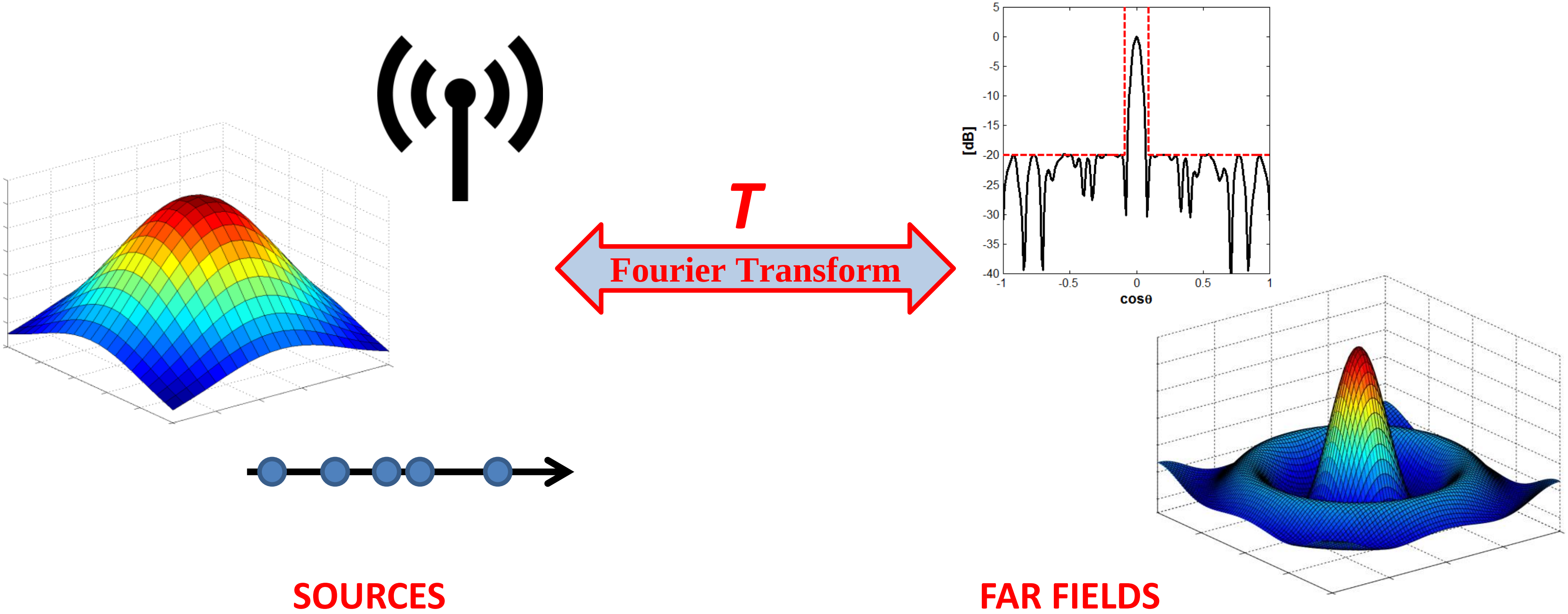


near-field and far-field diagnostics

Terahertz


astronomy

Our specific problem



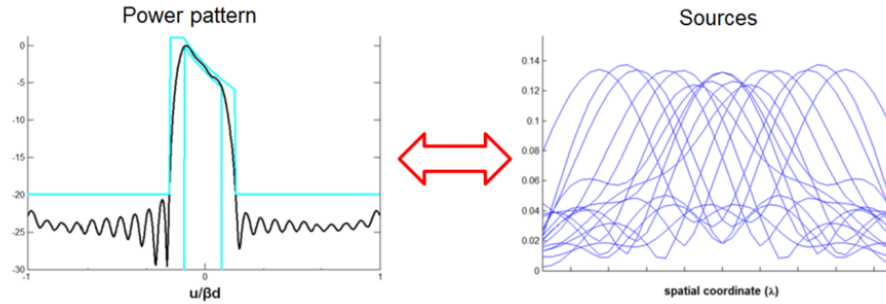
Assumption (*): We will deal in the following with Fourier operator and discrete sources,
i.e., array antennas and (by the sake of simplicity) far field phaseless data

(*) Since the far field of any non-superdirective source can be processed, in the visible part of the spectrum, as it is radiated by a 'virtual' equispaced array , the following results have a range of validity which is not restricted to discrete sources.

Bad and good news

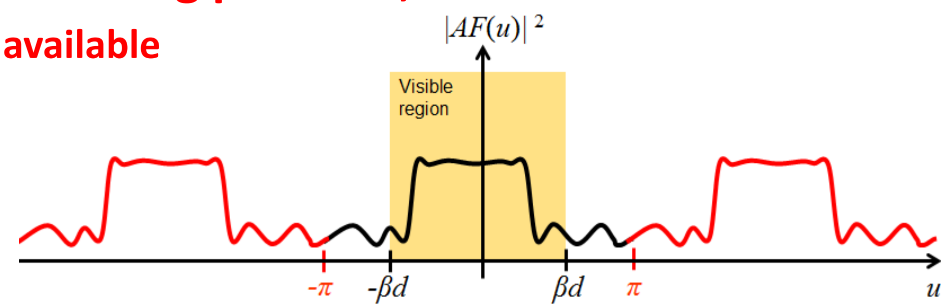
- ‘Trivial ambiguities’ (in all cases)
  (constant phase, linear phase, conjugation, combinations of the above)
  They can be fixed somehow by simple additional a-priori information

- Additional non uniqueness in the 1-D case
  (but all the different solutions can be found)



- Uniqueness (but for a zero-measure set of cases) in the 2-D case,
  BUT the problem is still ill-posed (a solution may not exist at all): ‘False solutions’ may occur

- Harder than the corresponding Signal-Processing problem,
  as only data in the visible part of the spectrum is available



The proposed procedure: aim, and the basic idea

Solve the 2-D PR problem avoiding false solutions by using a single set of measurements plus some minimal additional information

Exploit all the knowledge and procedures available in 1-D PR problems plus coherence relationships amongst columns and rows

		1 B	2 T				3 A		
	4 H	E	L	5 P		6 C	R	7 E	8 W
9			C	H	10 E	S	T	E	R
11 I	O	T		12 I	E	T			A
			13 H				14 I	O	P
15 I	E	E	E						

Suitable words:

- ✓ Chichester
- ✓ Manchester
- ✓ Winchester

HORIZONTAL

1. British Telecom. **4.** It is needed if you're in trouble! **6.** Group of people who work together on a ship. **9. UK's city.** **11.** Internet Of Things. **12.** The Institution of Engineering and Technology. **14.** Publishing company to which 'Inverse Problems' belongs. **15.** The Institute of Electrical and Electronics Engineers.

VERTICAL

1. [Missing] **2.** Italian shortening for telecommunications. **3.** Range of human activities expressing the author's technical skills. **4.** [Missing] **5.** A spherical coordinate. **6.** Simulation software providing 3-D numerical solutions to Maxwell's Equations. **7.** Electrical Engineer. **8.** Constrain the phase to either the interval $(-\pi, \pi]$ or $[0, 2\pi)$. **9.** [Missing] **10.** Same as 'vertical 7'. **13.** Co-Editor-in-Chief of PIER Journal.

		1 B	2 T				3 A		
	4 H	E	L	5 P		6 C	R	7 E	8 W
9		N	C	H	10 E	S	T	E	R
11 I	O	T		12 I	E	T			A
			13 H				14 I	O	P
15 I	E	E	E						

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		1 B	2 T				3 A		
	4 H	E	L	5 P		6 C	R	7 E	8 W
9 W	I	N	C	H	10 E	S	T	E	R
11 I	O	T		12 I	E	T			A
F			13 H				14 I	O	P
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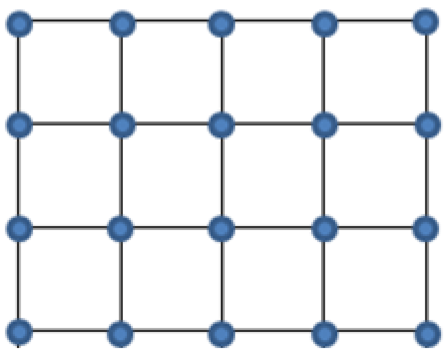
VERTICAL

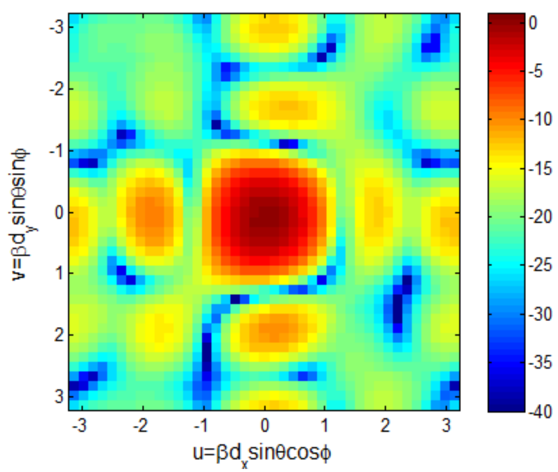
1. Sometimes a waveguide is… **2.** Italian shortening for telecommunications. **3.** Range of human activities expressing the author's technical skills. **4.** Fienup's algorithm for phase retrieval. **5.** A spherical coordinate. **6.** Simulation software providing 3-D numerical solutions to Maxwell's Equations. **7.** Electrical Engineer. **8.** Constrain the phase to either the interval $(-\pi, \pi]$ or $[0, 2\pi)$. **9.** Technology for radio wireless local area networking. **10.** Same as 'vertical 7'. **13.** Co-Editor-in-Chief of PIER Journal.

The proposed procedure: assumptions

- ✓ ***NxM*** **equispaced array on a rectangular grid**
- ✓ **knowledge of the source support**
- ✓ **measurements available in all the *T* domain**



NxM equispaced array



$$F(\underline{u}) = T[f(\underline{x})] = |F(\underline{u})| e^{j\varphi(\underline{u})}$$

Array Factor

Array Excitations

$$F(u,v) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} I_{nm}\, e^{jnu} e^{jmv}$$

- Data: $|F(u,v)|$
- Unknwons: I_{nm}

The proposed procedure: flowchart

Step 1

Choose a row in the data space.

By solving the corresponding 1-D PR problem, identify all the admissible fields/words on such a row

Step 1

Determining the 'dictionary' of all complex-field admissible behaviors

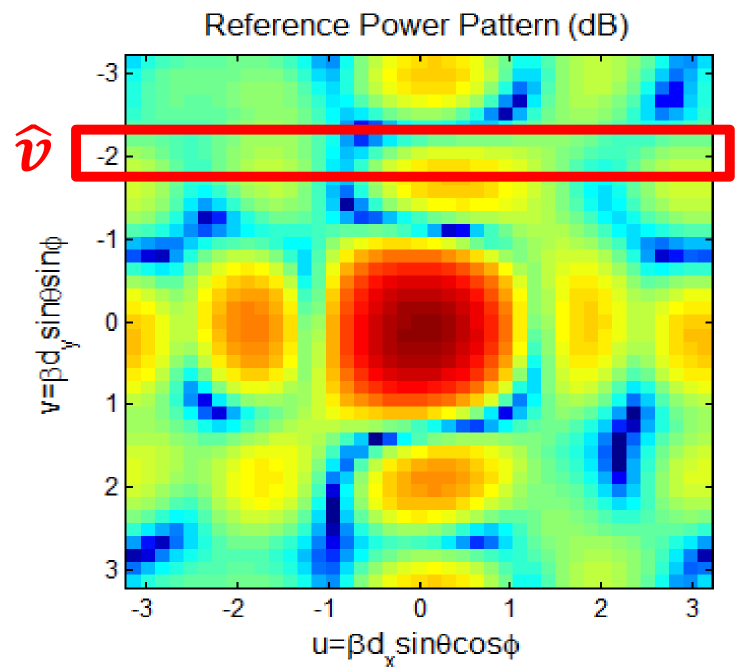
$$|F(u,v)|^2 = P(u,v) = \sum_{p=-N+1}^{N-1} \sum_{q=-M+1}^{M-1} D_{pq}\, e^{jpu} e^{jqv}$$

$$|F(u,\hat{v})|^2 = P(u,\hat{v}) = \sum_{p=-N+1}^{N-1} \widehat{D}_p\,(\hat{v}) e^{jpu}$$

$$with\ \ \widehat{D}_p(\hat{v}) = \sum_{q=-M+1}^{M-1} D_{pq}\, e^{jq\hat{v}}$$

$$F(u,\hat{v}) = \sum_{n=0}^{N-1} \hat{I}_n(\hat{v})\, e^{jnu}$$

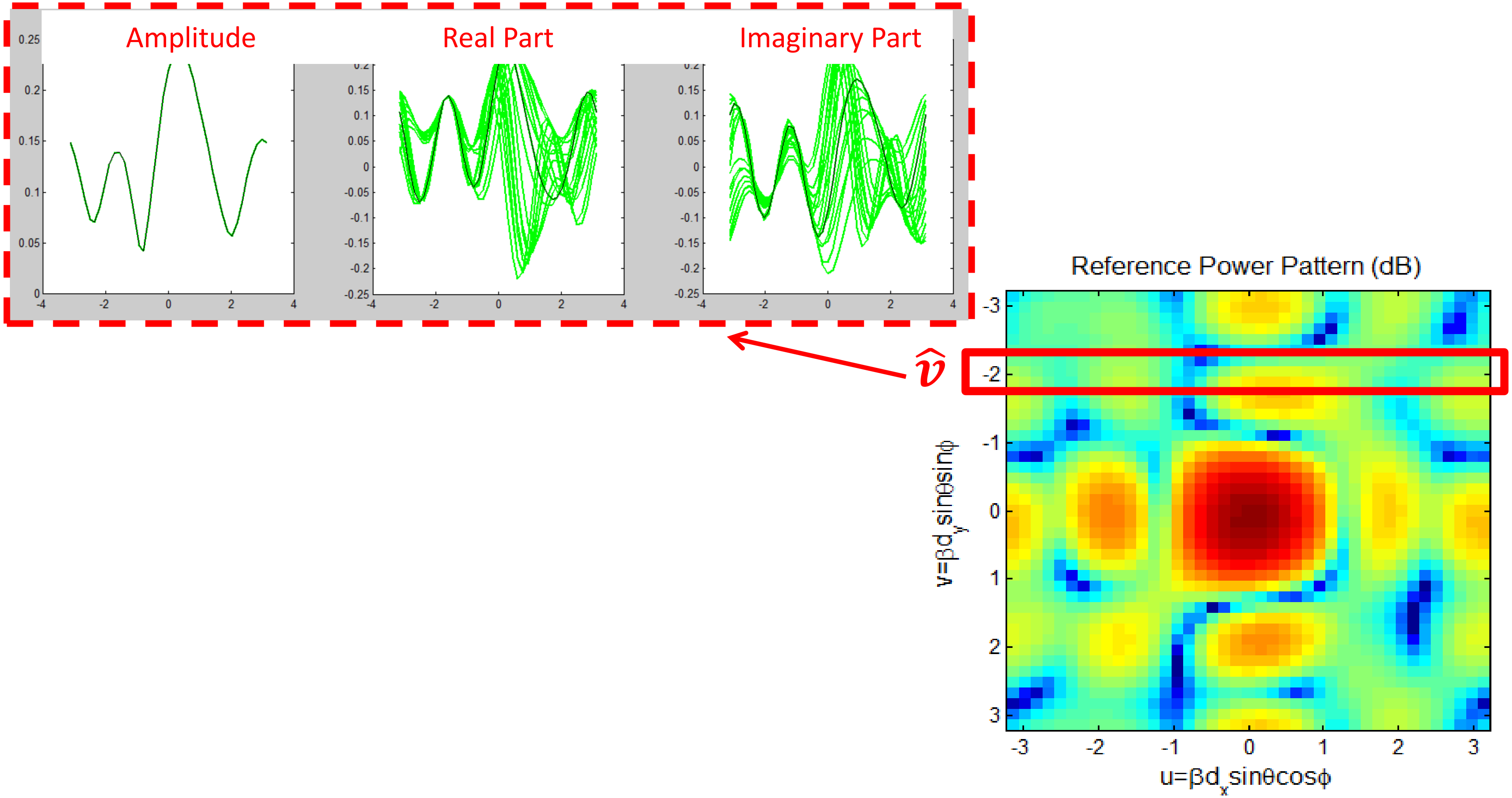
$$with\ \ \hat{I}_n(\hat{v}) = \sum_{m=0}^{M-1} I_{nm}\, e^{jm\hat{v}}$$



T. Isernia, O. M. Bucci, and N. Fiorentino, "Shaped beam antenna synthesis problem: Feasibility criteria and new strategies," *JEMWA*, 1998.

Step 1

Determine the 'dictionary' of all complex-field admissible behaviors



Along a fixed line of the spectral plane, the Spectral Factorization technique is able to provide all available 1-D solutions of the problem

T. Isernia, O. M. Bucci, and N. Fiorentino, "Shaped beam antenna synthesis problem: Feasibility criteria and new strategies," *JEMWA*, 1998.

The proposed procedure: flowchart

Step 1

Choose a row in the data space.

By solving the corresponding 1-D PR problem, identify all the admissible fields/words on such a row

↓

Step 2

Choose a column in the data space.

By solving the corresponding 1-D PR problem, identify all the admissible fields/words on such a column

Step 2

Determine the dictionary of all complex-field admissible behaviors

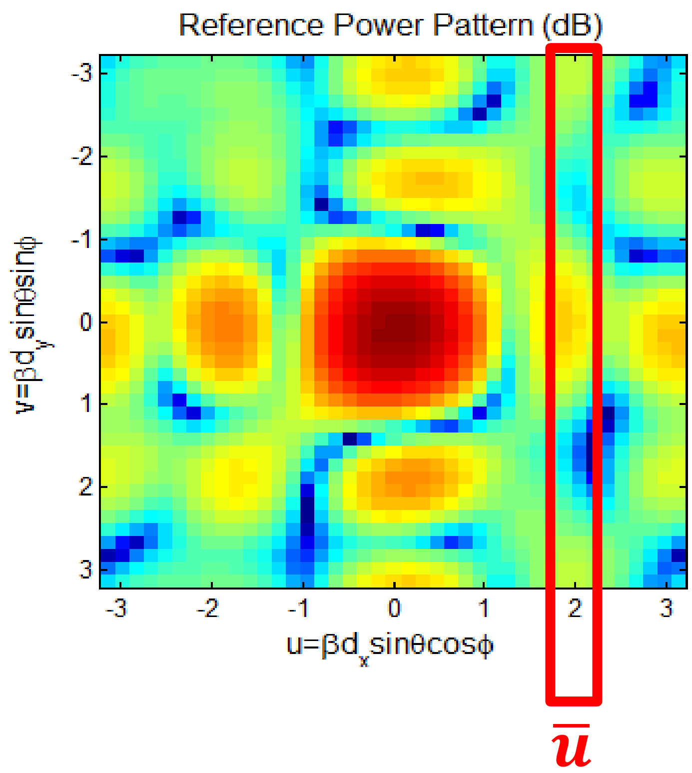
$$|F(u,v)|^2 = P(u,v) = \sum_{p=-N+1}^{N-1} \sum_{q=-M+1}^{M-1} D_{pq}\, e^{jpu} e^{jqv}$$

$$|F(\bar{u},v)|^2 = P(\bar{u},v) = \sum_{q=-M+1}^{M-1} \bar{D}_q\,(\bar{u}) e^{jqv}$$

$$with\ \ \bar{D}_q(\bar{u}) = \sum_{p=-N+1}^{N-1} D_{pq}\, e^{jp\bar{u}}$$

$$F(\bar{u},v) = \sum_{m=0}^{M-1} \bar{I}_m(\bar{u})\, e^{jmv}$$

$$with\ \bar{I}_m(\bar{u}) = \sum_{n=0}^{N-1} I_{nm}\, e^{jn\bar{u}}$$



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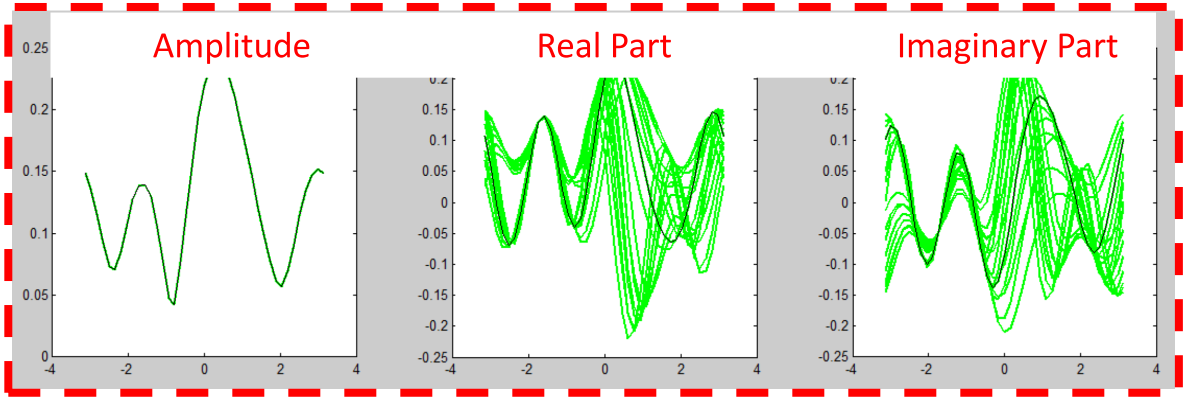
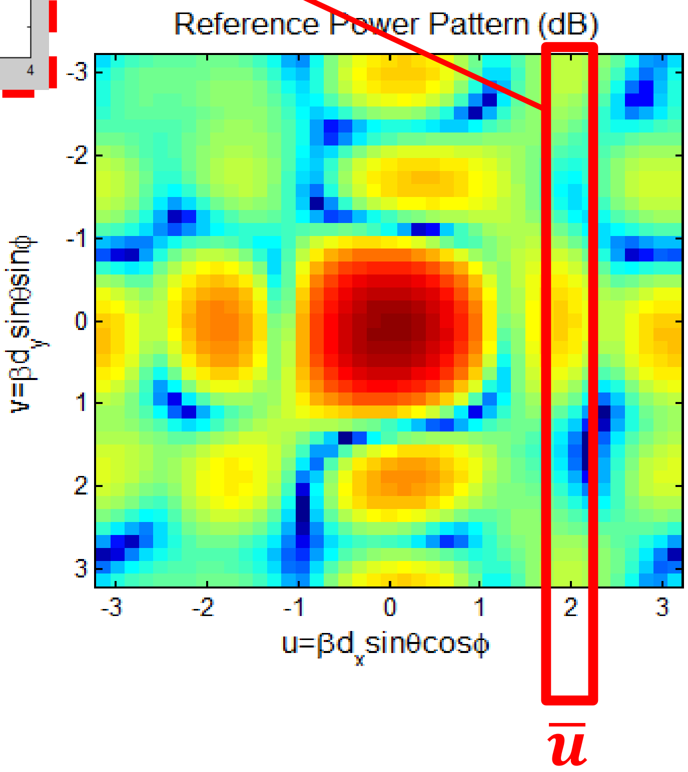
Step 1

Determine the 'dictionary' of all complex-field admissible behaviors





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The proposed procedure: flowchart

Step 1

Choose a row in the data space.
By solving the corresponding 1-D PR problem, identify all the admissible fields/words on such a row.

Step 2

Choose a column in the data space.
By solving the corresponding 1-D PR problem, identify all the admissible fields/words on such a column.

Step 3

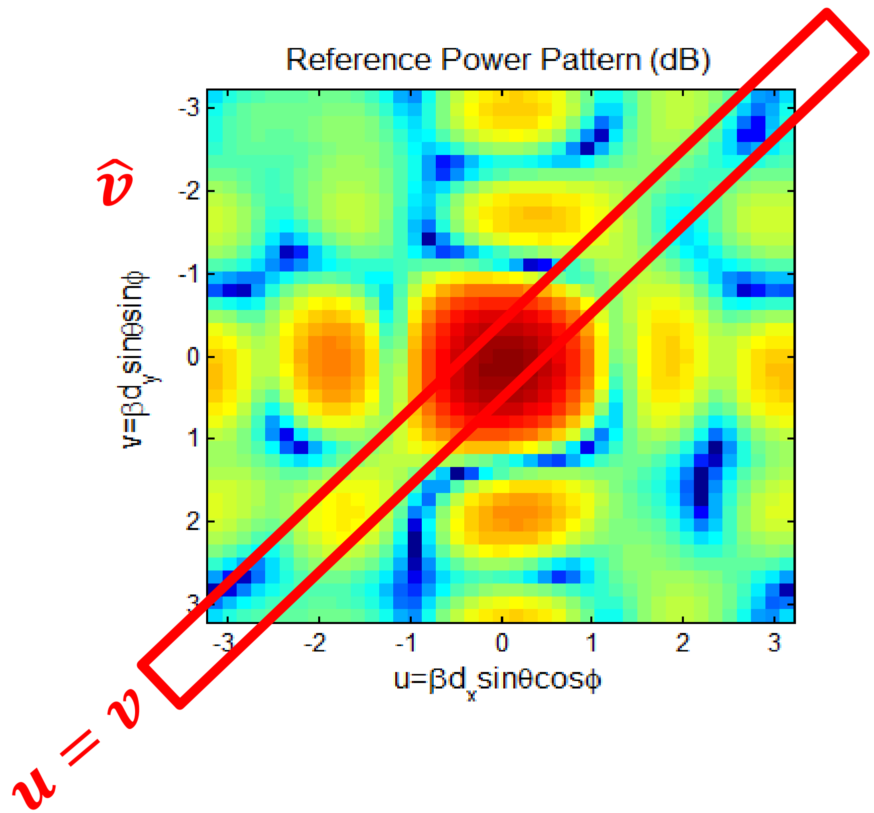
Choose a diagonal of the data space.
By solving the corresponding 1-D phase retrieval problem find all the admissible fields/words on such a diagonal.

Step 3

Determining the dictionary of all complex-field admissible behaviors

$$|F(w)|^2 = P(w) = \sum_{h=-2(N+M)}^{2(N+M)} \widetilde{D}_h \, e^{jhw}$$

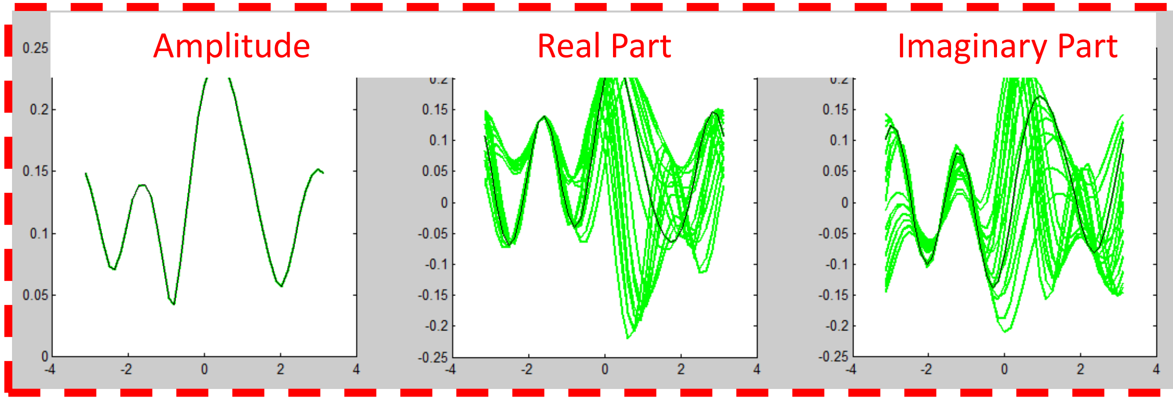
$$F(w) = \sum_{h=-N-M}^{N+M} \tilde{I}_h \, e^{jhw}$$

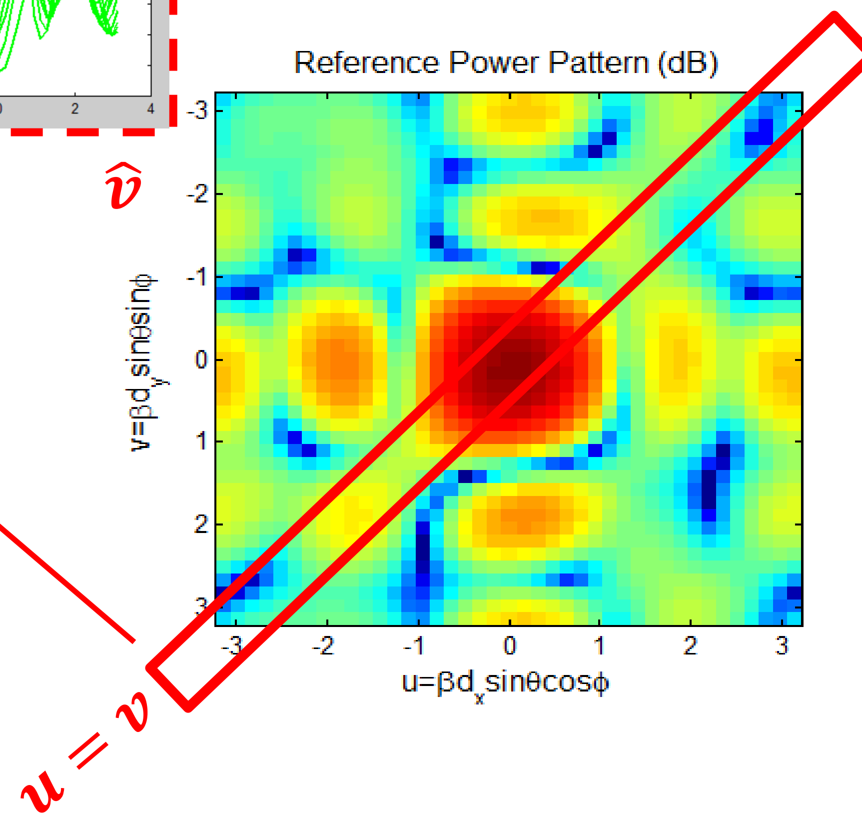


T. Isernia, O. M. Bucci, and N. Fiorentino, "Shaped beam antenna synthesis problem: Feasibility criteria and new strategies," *JEMWA*, 1998.

Step 3

Determining the dictionary of all complex-field admissible behaviors

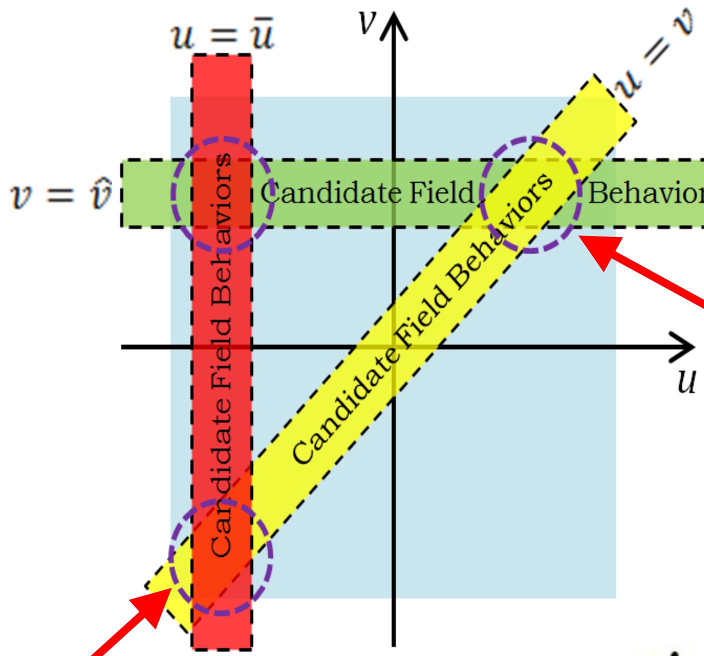




T. Isernia, O. M. Bucci, and N. Fiorentino,
"Shaped beam antenna synthesis problem:
Feasibility criteria and new strategies," *JEMWA*, 1998.

Step 4

Identify the correct field behavior(s) through the usual strategy when solving "Crosswords" approach



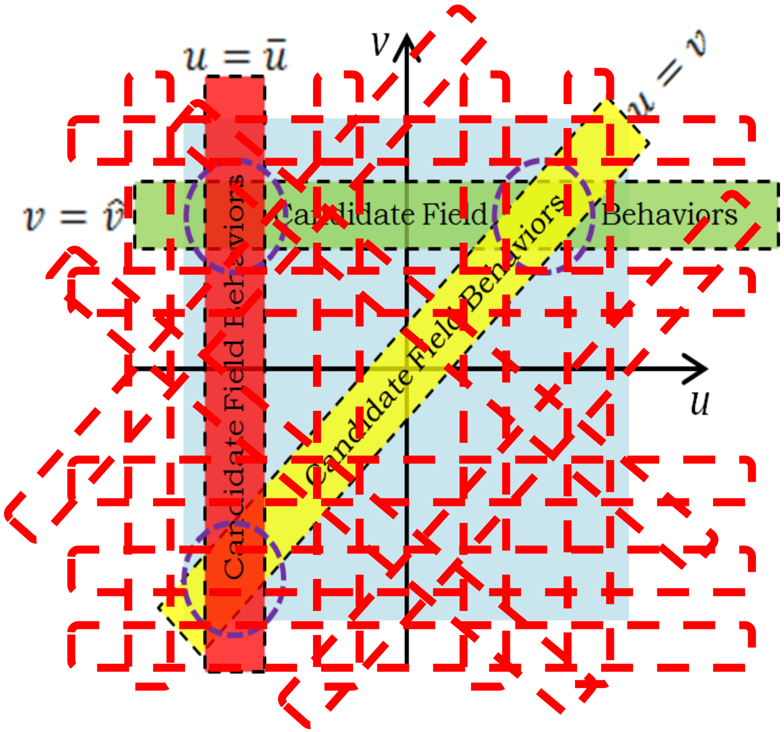
…the phase of the oblique field will generally be different from the one of the horizontal field.

A proper choice of the corresponding phase constant can allow any oblique candidate field to correctly intersect the candidate vertical field, BUT at the other intersection…

One will be able to discard a number of possibilities and hopefully identify the correct triplet of words (or at least to considerably reduce the number of possibilities).

Further Steps…

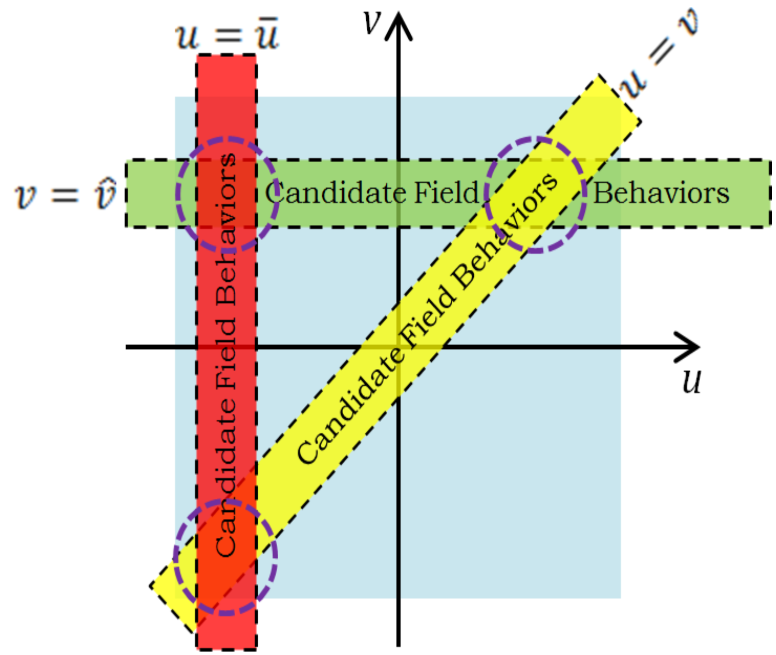
Find the correct field behavior(s) through a "Crosswords" approach



Completion of the scheme is rather intuitive for 'crosswords' solvers:

Consider additional horizontal, vertical, and oblique lines in order to identify the correct field behavior (i.e., the correct 'words') amongst the very many possible ones.

…and tricks (mimicking crosswords solution !)

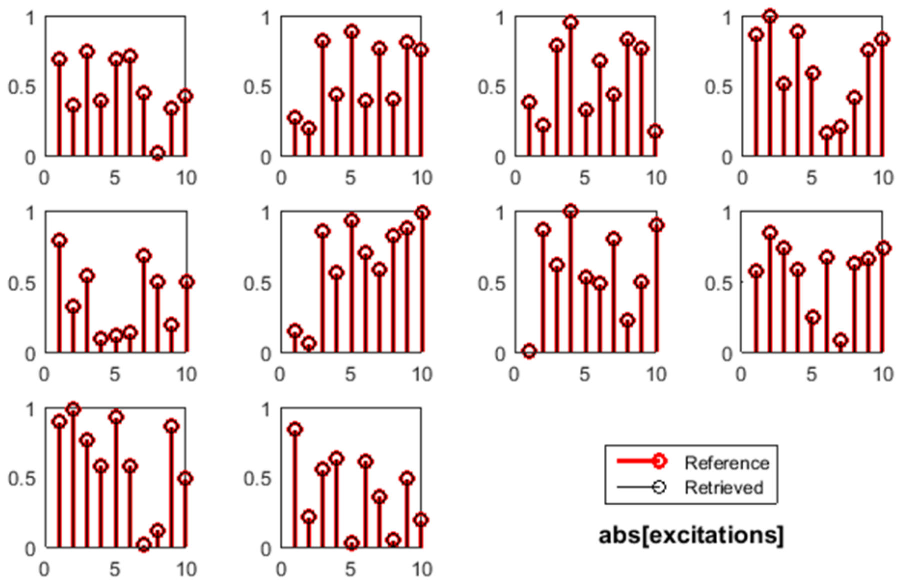


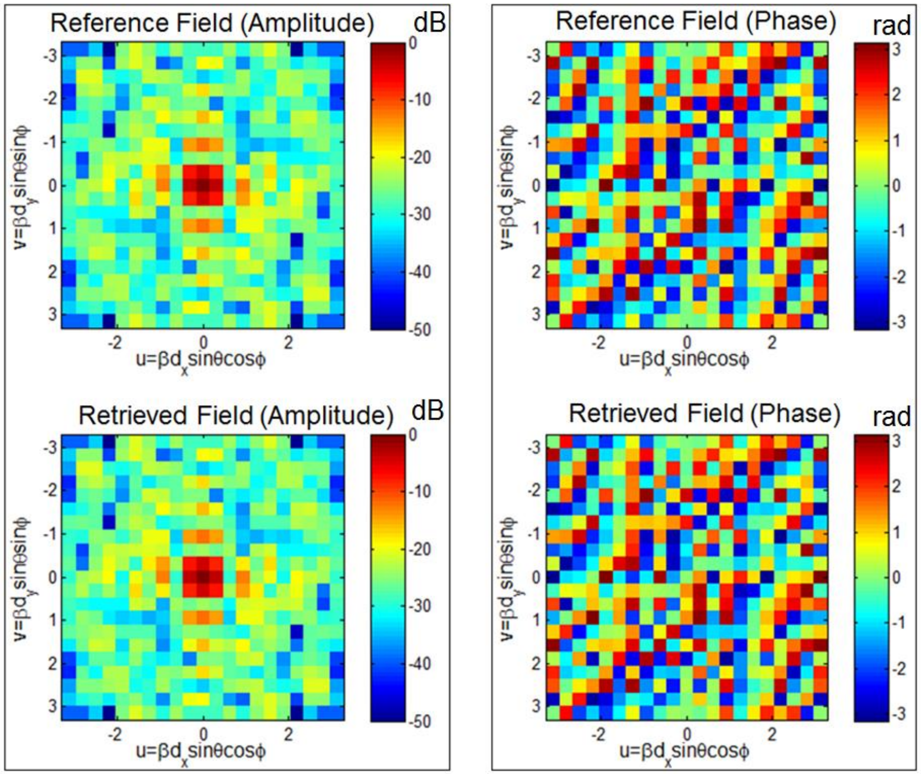
Computational burden can be reduced by starting from the simpler lines (which are those where many zeroes are present).

In fact, such a strategy allow to reduce the number of combinations to be checked (every zero reduces the number of ambiguities)

Numerical Examples (1/2)

✓ **100** elements **λ/2** spaced square array with (**real and positive**) **random** excitations (Knowing source support plus **3** field's complex samples along the *u*=*v* diagonal)

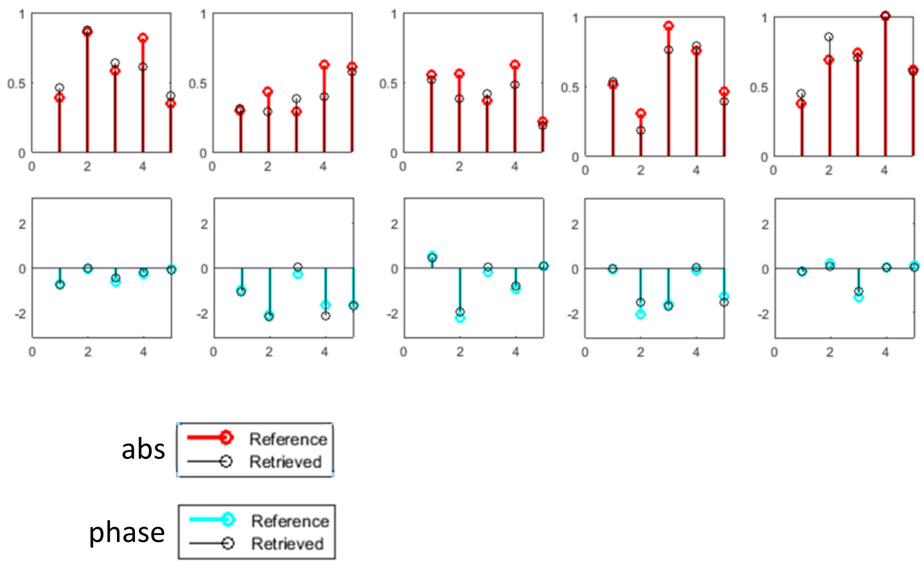




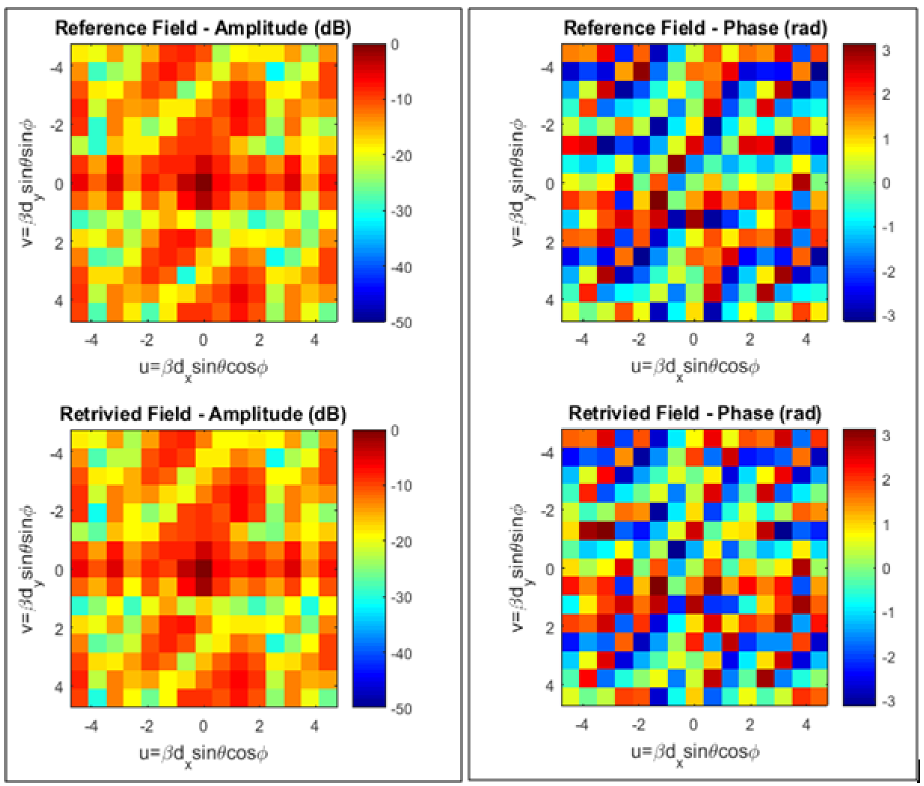
$$NMSE = \frac{\left\| I_{ref} - I_{retrieved} \right\|^2}{\left\| I_{ref} \right\|^2} = 3.7\text{e-}12$$

Numerical Examples (2/2)

- ✓ **25** elements **0.707λ** spaced square array with (**complex**) **random** excitations
  (Knowing source support plus **3** field's complex samples along the *u=v* diagonal)
- ✓ **SNR=30dB**



$$NMSE = \frac{\left\| I_{ref} - I_{retrieved} \right\|^2}{\left\| I_{ref} \right\|^2} = 0.06$$

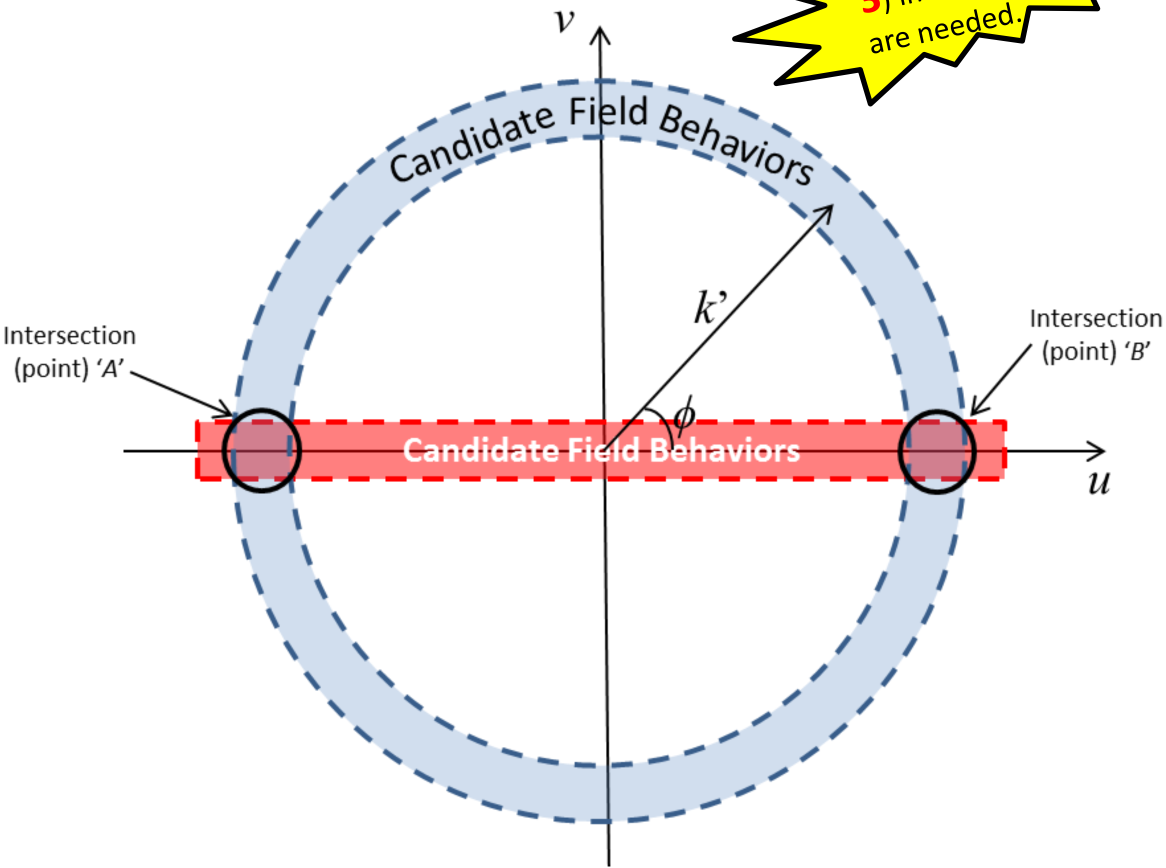


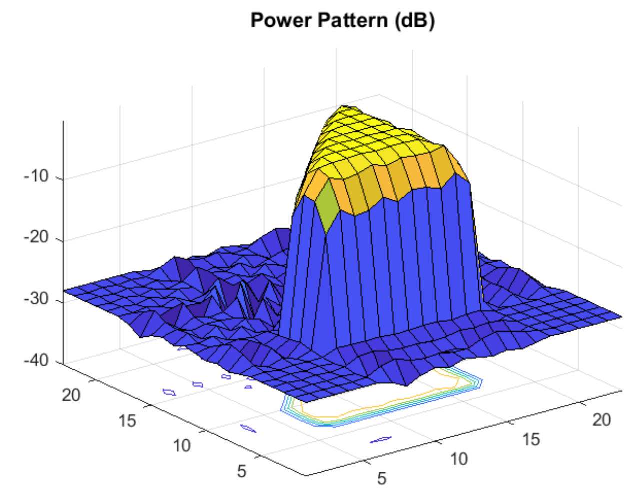
An even more powerful approach: exploit concentric crosswords!

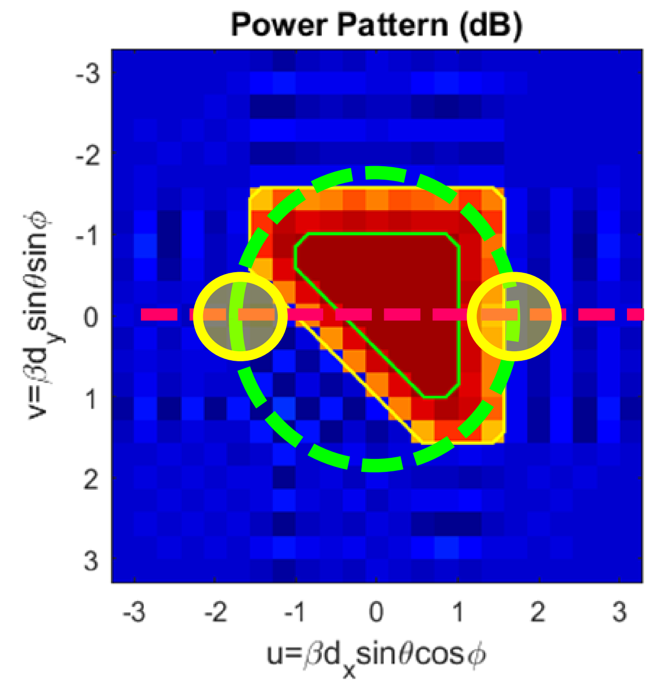
$$F(k',\phi) = \sum_{\ell=-\infty}^{\infty} F_\ell(k') e^{j\ell\phi}$$

$$k' = \sqrt{u^2 + v^2} = \beta \sin\theta$$

2 (rather than 3) intersections are needed.

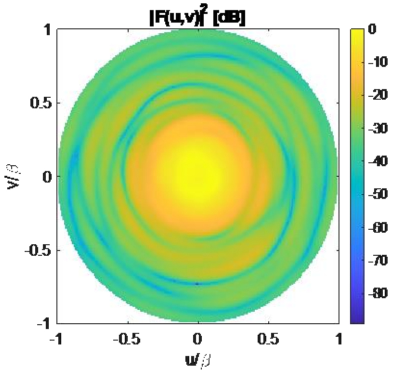


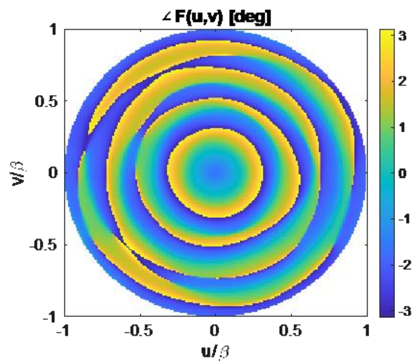




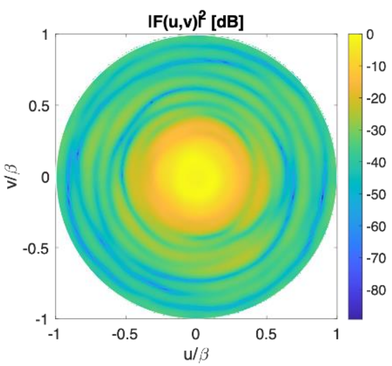
Numerical Examples

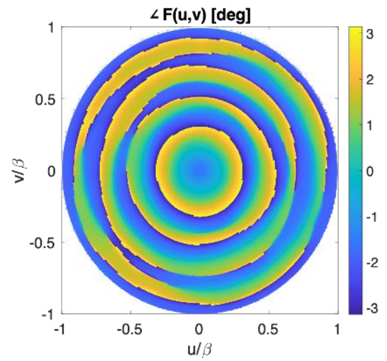
- ✓ Deformation Detection of Reflector Antennas: **Continuous aperture field with a circular support of radius 5λ**
- ✓ **Phase deformation:** random in the range $\left[-\frac{\lambda}{30}, \frac{\lambda}{30}\right]$.





Square amplitude and phase of the reference far field





Retrieved distributions through the proposed strategy

$$NMSE = \frac{\left\|F^{actual}(k',\phi) - F^{recovered}(k',\phi)\right\|^2}{\left\|F^{actual}(k',\phi)\right\|^2} = 2.2302 \cdot 10^{-4}$$

Jin, H., Huang, J., Ye, Q., Meng, G., Xiang, B., and Wang, N., "Surface Shape Detection with a Single Far-Field Intensity by Combined Amplitude and Phase Retrieval," International Journal of Antennas and Propagation, vol. 2019, 2019.

Remarks and Conclusions

- ✓ A completely new point of view has been presented by exploiting a kind of reasoning commonly adopted in very far away problems.

- ✓ It exploits the specific reason which renders 2-D problems 'simpler' than 1-D problems [i.e., the need of bandlimitedness (and congruence) along all the different directions].

- ✓ It finds in a deterministic fashion all the different solutions to the PR problem in case the solution is not unique (i.e., in the case of factorable patterns).

- ✓ Single set (surface) of measurements; NO global opt.; Extensible to near-field case.

Thank you!