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On the Feasibility of Using Inverse Scattering to Optimize the Design of EBG Devices

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OUTLINE

- Inverse Scattering as a design tool and the inherent issues

- Forward problem for a random set of (small) parallel cylinders: the scattering matrix method (SMM)

- Synthesis of EBG devices through the SMM

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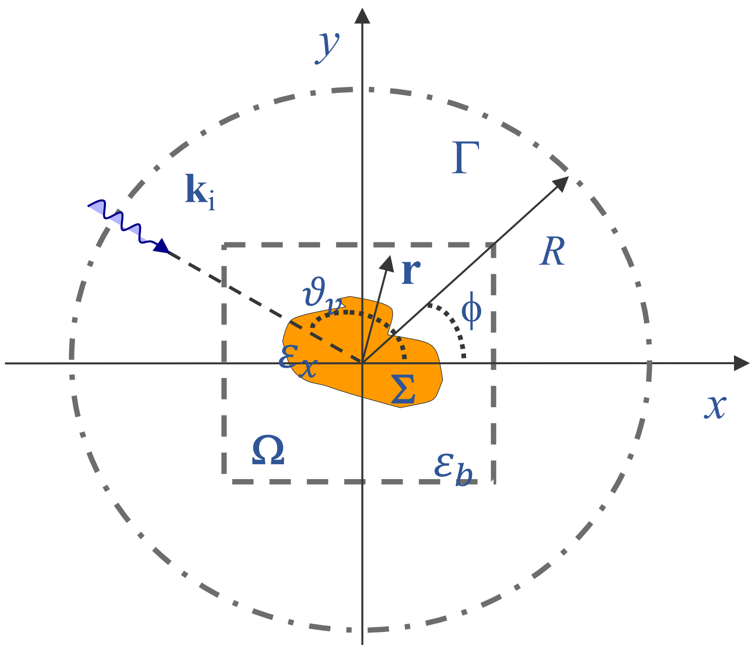
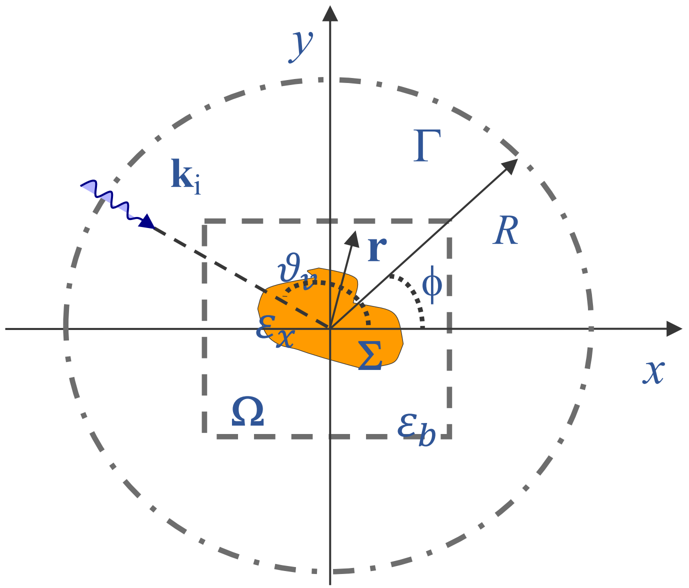
inverse scattering problem (ISP)



Let be :

- $\boldsymbol{\Omega}$ the region under investigation embedding the unknown target $\boldsymbol{\Sigma}$
- Γ the observation domain; it is usually a surface enclosing $\boldsymbol{\Omega}$
- $E_i\ (\boldsymbol{r}, \boldsymbol{r_t})$ the incident field illuminating $\boldsymbol{\Omega}$ from $\boldsymbol{r_t}$ impinging directions
- $E_s\ (\boldsymbol{r_m}, \boldsymbol{r_t})$ the scattered field measured at $\boldsymbol{r_m}$ observation directions in Γ, due to the induced contrast source $\boldsymbol{W}(\boldsymbol{r}, \boldsymbol{r_t})$ inside $\boldsymbol{\Omega}$

inverse scattering problem (ISP)



$E_i\,(\boldsymbol{r},\boldsymbol{r}_t), E_s\,(\boldsymbol{r}_m,\boldsymbol{r}_t)$

$\boldsymbol{r} \in \Omega, \quad \boldsymbol{r}_t, \boldsymbol{r}_m \in \Gamma$

$$\chi(\boldsymbol{r}) = \frac{\varepsilon_x(\boldsymbol{r})}{\varepsilon_b(\boldsymbol{r})} - 1$$

CONTRAST FUNCTION

encodes target properties (e.m. parameters, shape)

The inverse scattering problem is described by two main equations :

CONTRAST SOURCE $W = \chi E$

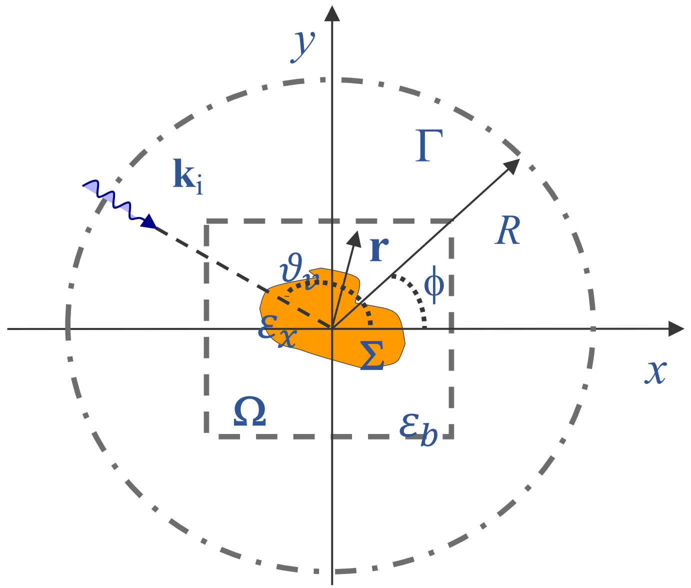
$$E_s(\boldsymbol{r}_m,\boldsymbol{r}_t) = \mathcal{A}_e[W(\boldsymbol{r},\boldsymbol{r}_t)]$$ ***'data equation'***

$\boldsymbol{r} \in \boldsymbol{\Omega}, \boldsymbol{r}_m, \boldsymbol{r}_t \in \boldsymbol{\Gamma}$

$$W(\boldsymbol{r},\boldsymbol{r}_t) = \chi E_i\,(\boldsymbol{r},\boldsymbol{r}_t) + \chi \mathcal{A}_i[W(\boldsymbol{r},\boldsymbol{r}_t)]$$ ***'state equation'***

NOTE: χ and W are unknowns ! The only available data are E_i and E_s.

inverse scattering problem (ISP)



$E_i\,(\boldsymbol{r},\boldsymbol{r}_t), E_s\,(\boldsymbol{r}_m,\boldsymbol{r}_t)$

$\boldsymbol{r} \in \Omega, \quad \boldsymbol{r}_t, \boldsymbol{r}_m \in \Gamma$

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non-linear and ill-posed inverse problem

Inverse Scattering as a design tool

THE USUAL AIM:

Given a set of incident fields, find the e.m. characteristics of the region under test (i.e., $\chi(\mathbf{r})$) in such a way that the scattered (total) field *obeys the collected measurements*

Inverse Scattering as a design tool

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A DIFFERENT AIM:

Given a set of incident fields, find the e.m. characteristics of the region under test (i.e., $\chi(\boldsymbol{r})$) in such a way that the scattered (total) field *obeys to given specifications*

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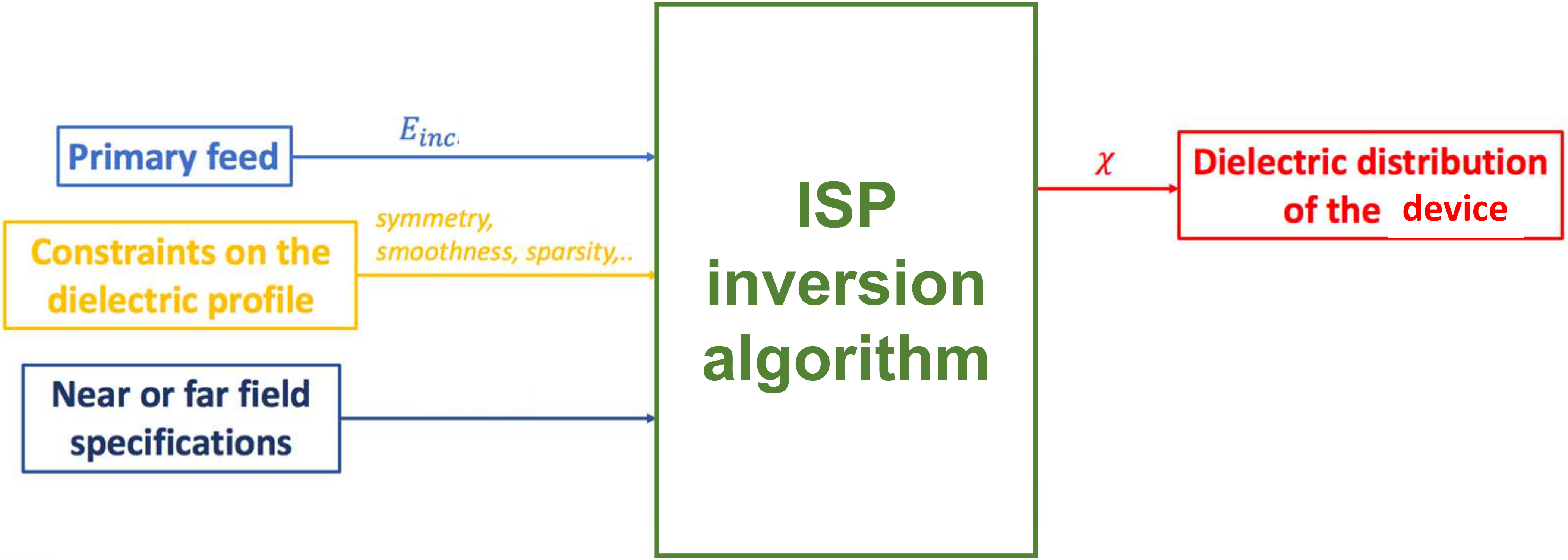
A DIFFERENT AIM:
Given a set of incident fields, find the e.m. characteristics of the region under test (i.e., $\chi(\boldsymbol{r})$) in such a way that the scattered (total) field *obeys to given specifications*

Inverse scattering theory and solution procedures can be seen as a **design tool rather than as a mean for recovery/imaging**

Innovative devices can be hopefully designed

ISP-based design procedure



Manufacturing a GRIN (GRADED refractive index) device is not a trivial task

Sometimes, homogenization techniques do not work

[*] R. Palmeri, et al., “Design of Artificial-Materials-Based Antennas Using Inverse Scattering Techniques”, IEEE TAP 2018.

A design example: direct synthesis of a Graded Artificial Material (GAM)-based device

A novel expansion for the contrast function allowing the direct synthesis of GAM-based device

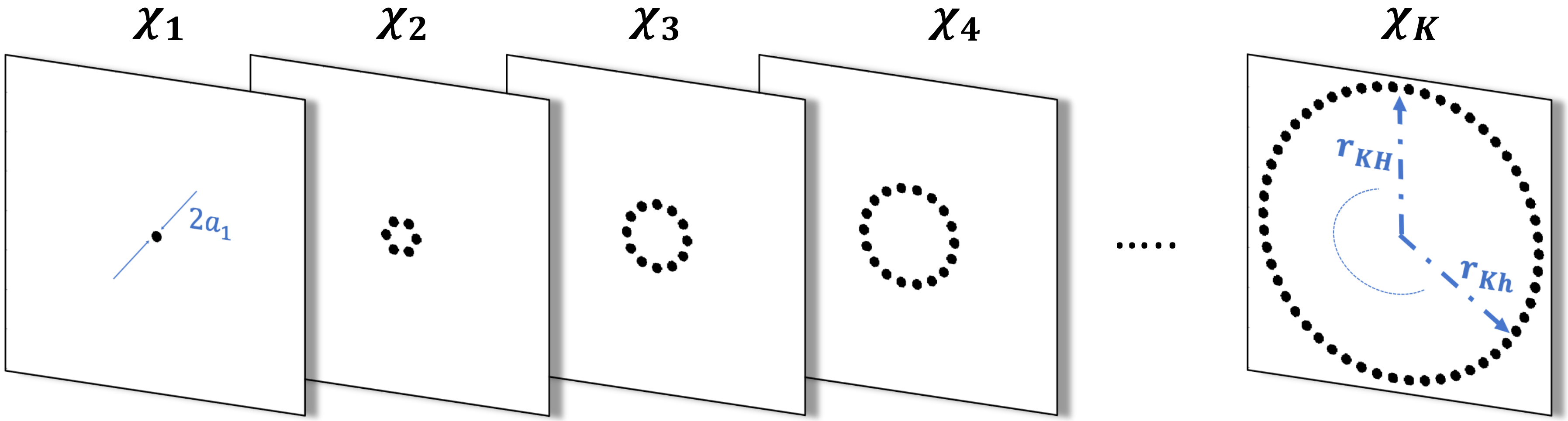
$$\chi(\boldsymbol{r}) = \sum_{k=1}^{K} \chi_k \, \Pi(\boldsymbol{r})$$

$\Pi(\boldsymbol{r})$ is the representation basis projecting χ into the space of 'inclusions'

χ_k coefficients are the new unknowns of the problem

GAM with a gradient of the refractive index (GAM_R)

$$\chi(\boldsymbol{r}) = \sum_{k=1}^{K} \chi_k \sum_{h=1}^{H_k} \Pi_{kh}\left(\frac{\boldsymbol{r} - \boldsymbol{r}_{kh}}{a_k}\right)$$



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Pro and Cos of the ISP-based GAM design

- **Dielectric profiles obeying non-canonical solutions**
- **Dielectric profiles satisfying desired spatial distributions constraints**
- ***Multi-view* Inverse Scattering Problems turn into *multi-purpose* device**

- **Most of inversion algorithms are based on discretization of the investigation domain in subdomains/cells and are not suitable for GAM design.**

  Smaller and smaller mesh elements → higher and higher number of unknowns

  Staircasing errors

- **Less flexibility with respect to the 'kind' of unknowns** (radius, position of the inclusions)

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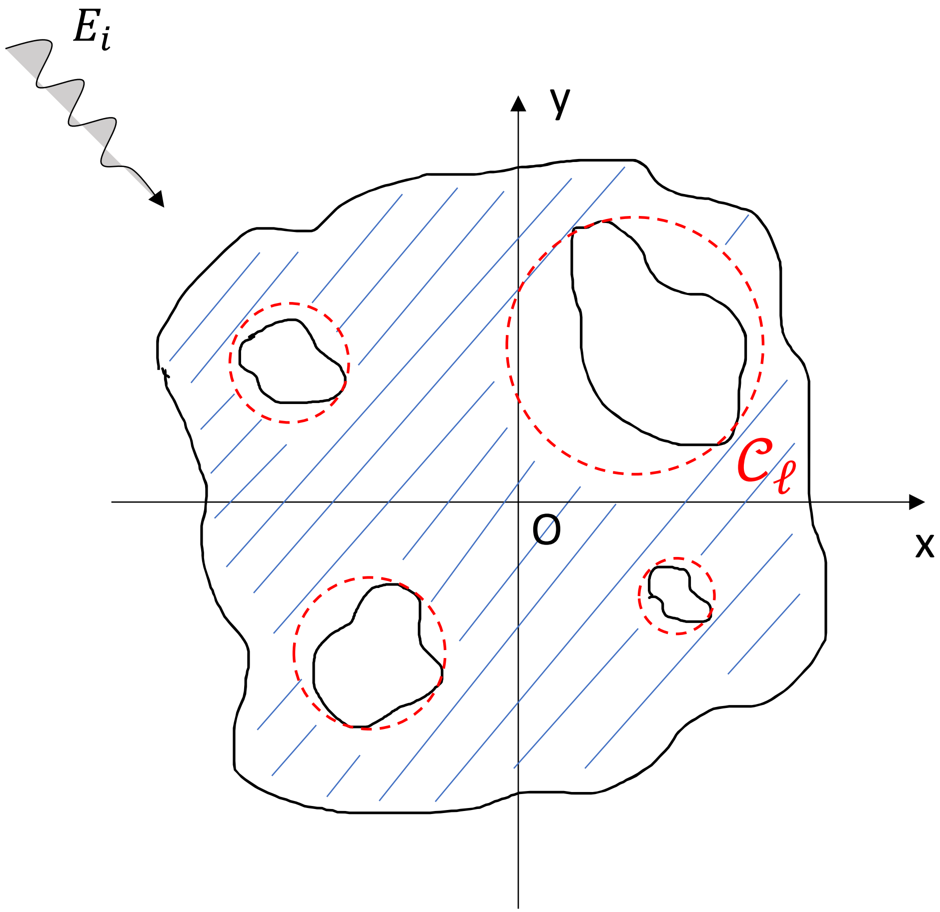
Multiple scatterers oriented tool:
Scattering Matrix Method (SMM)



Modal expansion of the total field outside cylinders

$$E_{tot}(\underline{r}) = \sum_{m=-\infty}^{+\infty} a_{\ell,m} J_m(kr_\ell) e^{jm\theta_\ell} + \sum_{m=-\infty}^{+\infty} b_{\ell,m} H_m^{(2)}(kr_\ell) e^{jm\theta_\ell}$$

local incident field on cylinder $\mathcal{C}_\ell$
(i.e., primary + secondary incident field)

scattered field by cylinder $\mathcal{C}_\ell$

In a matrix form the interactions amongst the different cylinders are described by:

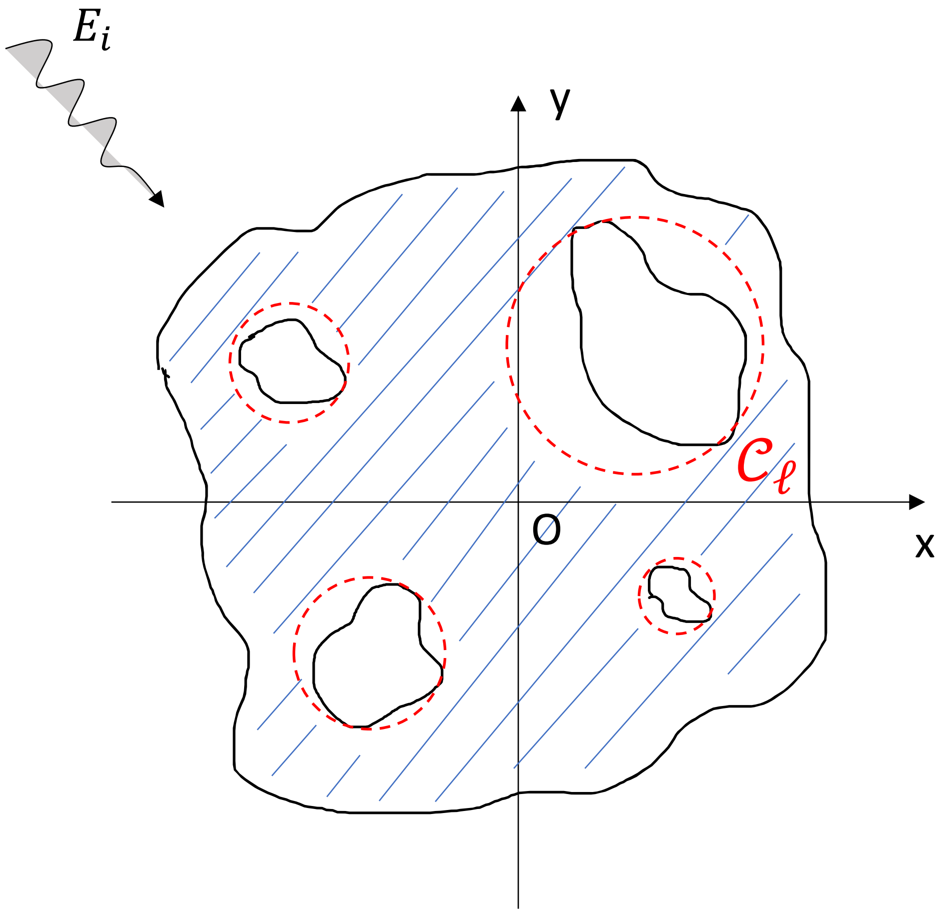
$$\mathbf{a}_\ell = \mathbf{Q}_\ell + \sum_{i=1, i\neq\ell}^{N} \mathbf{T}_{i,\ell} \mathbf{b}_i$$

LINEAR RELATIONSHIP FOR EACH CYLINDER

N being the number of inclusions

[*] D. Felbacq, et al., “Scattering by a random set of parallel cylinders”, JOSA A 1994.

Multiple scatterers oriented tool: Scattering Matrix Method (SMM)



As one can write

$$\mathbf{b}_\ell = \boldsymbol{S}_\ell \mathbf{a}_\ell$$

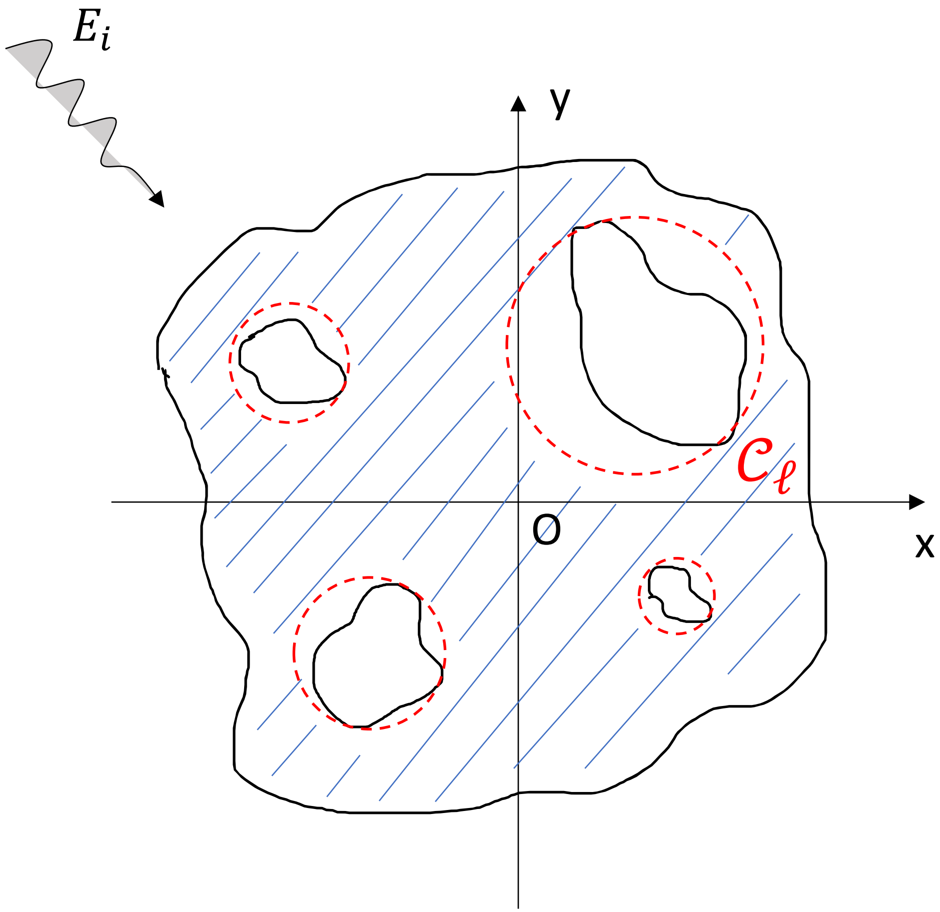
where $\boldsymbol{S}_\ell$ is the '<u>scattering matrix</u>' of the inclusion, and depends on its characteristics

$$\mathbf{b}_\ell - \sum_{i=1, i\neq\ell}^{N} \boldsymbol{S}_\ell \boldsymbol{T}_{\ell,i}\, \mathbf{b}_i = \boldsymbol{S}_\ell \mathbf{Q}_\ell \qquad \ell = 1, .., N$$

LINEAR SYSTEM IN THE UNKNOWN SCATTERING COEFFICIENTS $\mathbf{b}_\ell$

[*] D. Felbacq, et al., "Scattering by a random set of parallel cylinders", JOSA A 1994.

Multiple scatterers oriented tool: Scattering Matrix Method (SMM)



$$\mathbf{b}_\ell - \sum_{i=1, i\neq\ell}^{N} \boldsymbol{S}_\ell \boldsymbol{T}_{\ell,i}\, \mathbf{b}_i = \boldsymbol{S}_\ell \mathbf{Q}_\ell \qquad \ell = 1, .., N$$

- **Specific scattering properties of each object are considered**
- **Coupling phenomena between objects are taken into account**
- **Computational complexity grows with the perimeters of the different inclusions (rather than with volume of the region under test)**
- **No staircasing errors**

[*] D. Felbacq, et al., "Scattering by a random set of parallel cylinders", JOSA A 1994.

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formulation of the SMM

Let consider the expanded arrayal form of the linear system:

$$\begin{pmatrix} \mathbf{I} & -\mathbf{S_1 T_{1,2}} & \dots. & -\mathbf{S_1 T_{1,N}} \\ -\mathbf{S_2 T_{2,1}} & \mathbf{I} & \dots. & -\mathbf{S_2 T_{2,N}} \\ \dots & \dots & & \dots \\ -\mathbf{S_N T_{N,1}} & -\mathbf{S_N T_{N,2}} & \dots. & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{b_1} \\ \mathbf{b_2} \\ \dots \\ \mathbf{b_N} \end{pmatrix} = \begin{pmatrix} \mathbf{S_1 Q_1} \\ \mathbf{S_2 Q_2} \\ \dots \\ \mathbf{S_N Q_N} \end{pmatrix}$$

and remember that [*] :

- the square matrix $\mathbf{T}_{l,i}$ of the (*m*,*q*)-th element $\mathrm{T}_{l,i,m,q}$ takes into account the coupling mutual interactions
- the column matrix $\mathbf{Q}_l$ of *m*-th element $\mathrm{Q}_{l,m}$ represents the coefficients of the Fourier-Bessel expansion of primary incident fields
- the square matrix $\mathbf{S}_l$ is the scattering matrix and depends on the parameters of the *l*-th cylinder
- the column matrix $\boldsymbol{b}_l$ of *m*-th element $b_{l,m}$ represents the coefficients of the rigorous modal expansion of the scattered field

[*] D. Felbacq, et al., “Scattering by a random set of parallel cylinders”, JOSA A 1994.

Inverse formulation of the SMM (I-SMM)

Let consider the expanded arrayal form of the linear system:

$$\begin{pmatrix} \mathbf{I} & -\mathbf{S}_1\mathbf{T}_{1,2} & \dots. & -\mathbf{S}_1\mathbf{T}_{1,N} \\ -\mathbf{S}_2\mathbf{T}_{2,1} & \mathbf{I} & \dots. & -\mathbf{S}_2\mathbf{T}_{2,N} \\ \dots & \dots & & \dots \\ -\mathbf{S}_N\mathbf{T}_{N,1} & -\mathbf{S}_N\mathbf{T}_{N,2} & \dots. & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \dots \\ \mathbf{b}_N \end{pmatrix} = \begin{pmatrix} \mathbf{S}_1\mathbf{Q}_1 \\ \mathbf{S}_2\mathbf{Q}_2 \\ \dots \\ \mathbf{S}_N\mathbf{Q}_N \end{pmatrix}$$

CONCEPTUAL DESIGN PROBLEM

Determine cylinders' parameters able to behave like a desired device.

MATHEMATICAL DESIGN PROBLEM

Determine $\boldsymbol{S}_l$ and $\boldsymbol{b}_l$ able to scatter a given field on a given surface.

Inverse formulation of the SMM (I-SMM)

The (new) inverse scattering equations

$$\begin{pmatrix} \mathbf{I} & -\mathbf{S}_1\mathbf{T}_{1,2} & \dots & -\mathbf{S}_1\mathbf{T}_{1,N} \\ -\mathbf{S}_2\mathbf{T}_{2,1} & \mathbf{I} & \dots & -\mathbf{S}_2\mathbf{T}_{2,N} \\ \dots & \dots & & \dots \\ -\mathbf{S}_N\mathbf{T}_{N,1} & -\mathbf{S}_N\mathbf{T}_{N,2} & \dots & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \dots \\ \mathbf{b}_N \end{pmatrix} = \begin{pmatrix} \mathbf{S}_1\mathbf{Q}_1 \\ \mathbf{S}_2\mathbf{Q}_2 \\ \dots \\ \mathbf{S}_N\mathbf{Q}_N \end{pmatrix}$$

state equation

$$E_s(R,\theta) = \sum_{i=1}^{N} \sum_{m=-M}^{+M} b_{i,m} H_m^{(2)}(\beta R) e^{jm\theta}\,, \qquad R \notin cylinders$$

data equation

wherein $$\boldsymbol{S_n} = \begin{pmatrix} s_{-M} & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & s_m & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & s_M \end{pmatrix}$$

and s_m depends on the kind of inclusion (dielectric, metallic, magnetics,..) and its dimension

Design procedure

Determine inclusions D_i such to

$$\Phi = \min_{D_i,\tau} \frac{\left\| E_{tot}(\underline{r}) - E_{tot}^{specified}(\underline{r}) \right\|_\Gamma^2}{\left\| E_{tot}^{specified}(\underline{r}) \right\|_\Gamma^2}$$

subject to desired constraints on D_i.

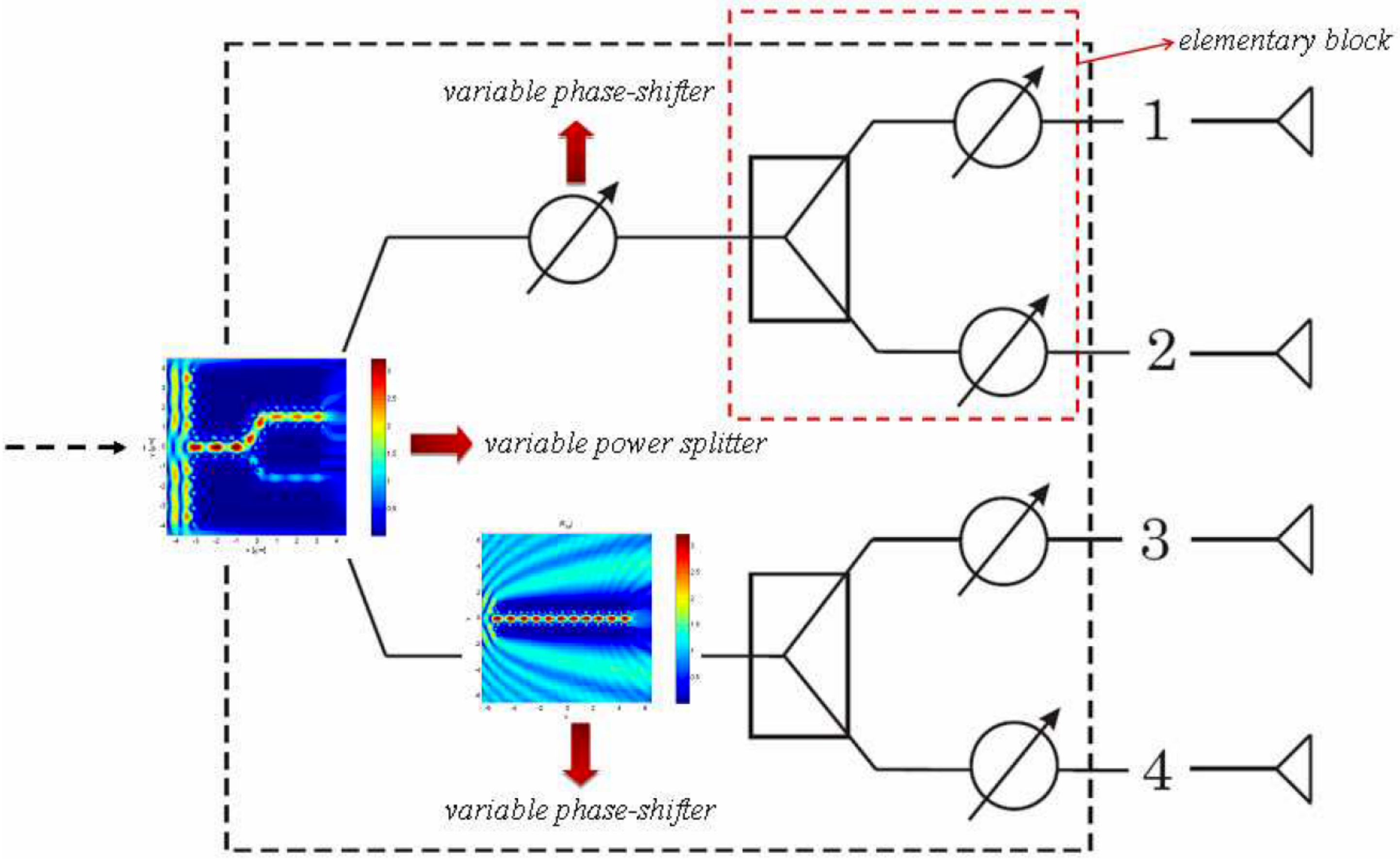
N.B.1. D_i could mean permittivity, radius, both ones, …

N.B.2. Favourable starting points are needed to find a satisfying solution

Numerical Assessment

Design of a beam-forming network for antennas array

Design of a beam-forming network for antennas array

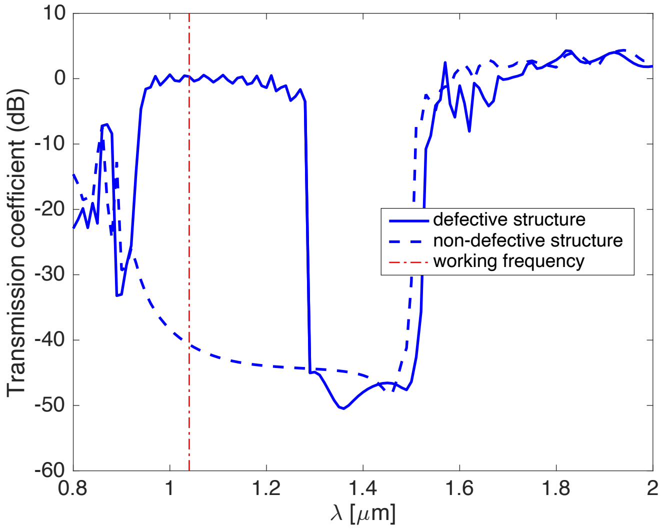


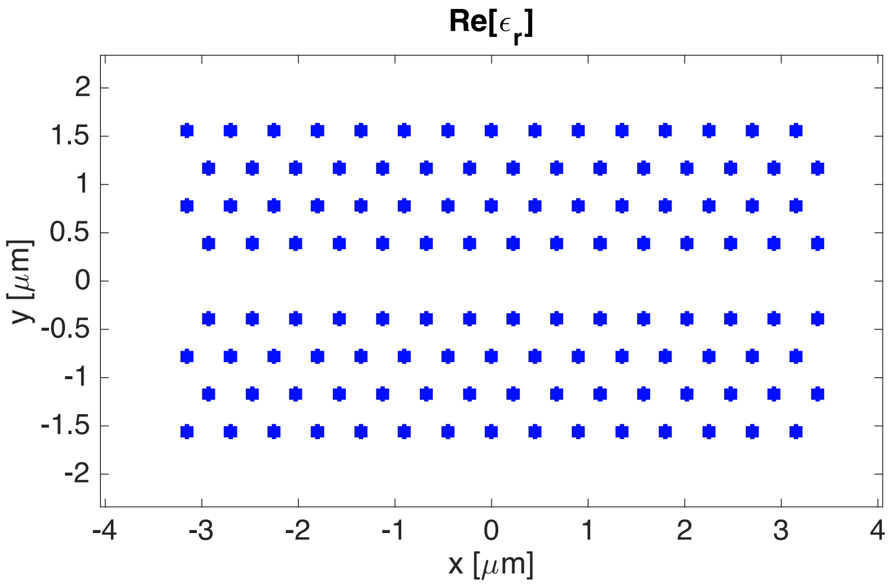
Numerical Assessment

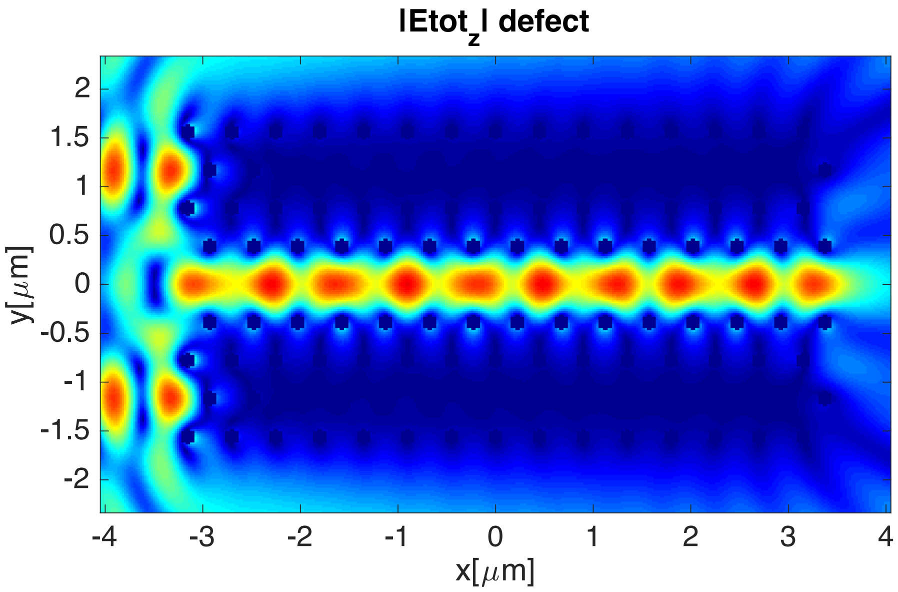
Design of a beam-forming network for antennas array

Optimization of basic elements

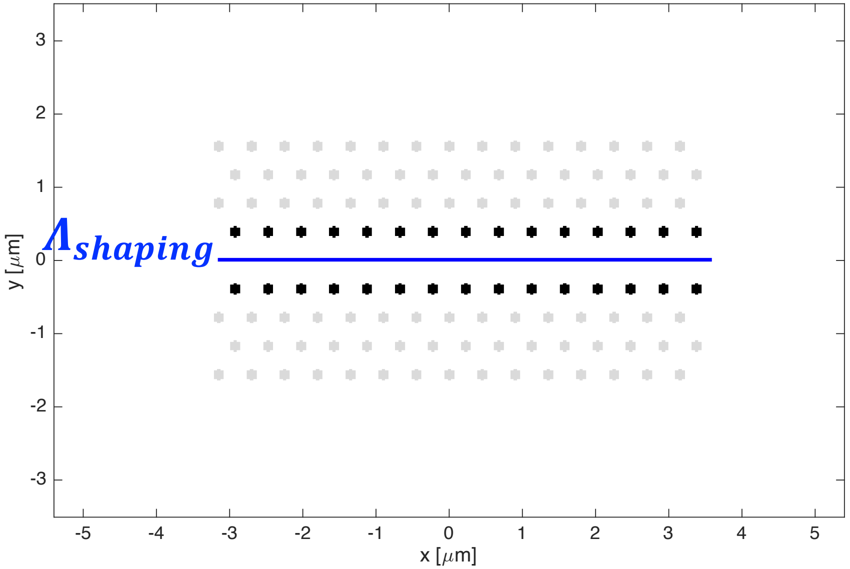
Basic element #1: EBG straight waveguide

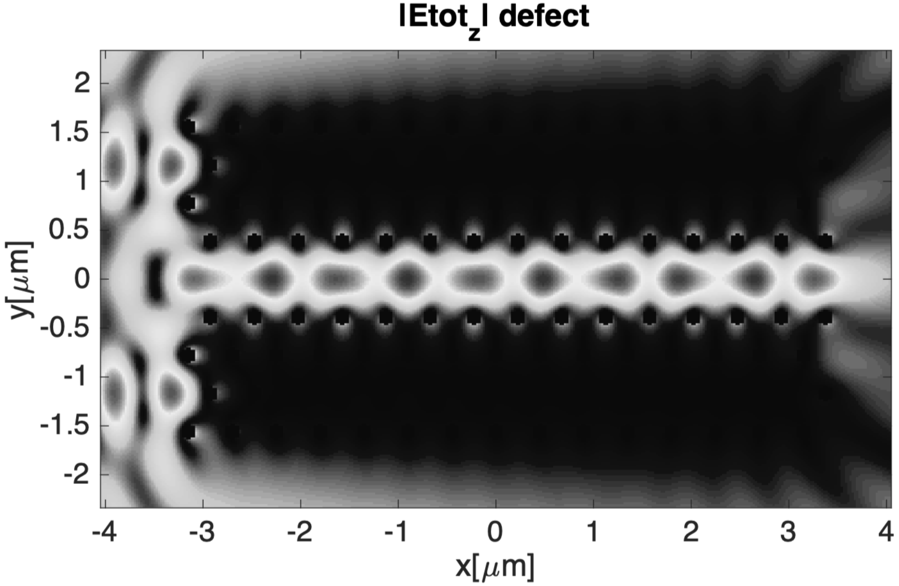






Basic element #1: EBG straight waveguide





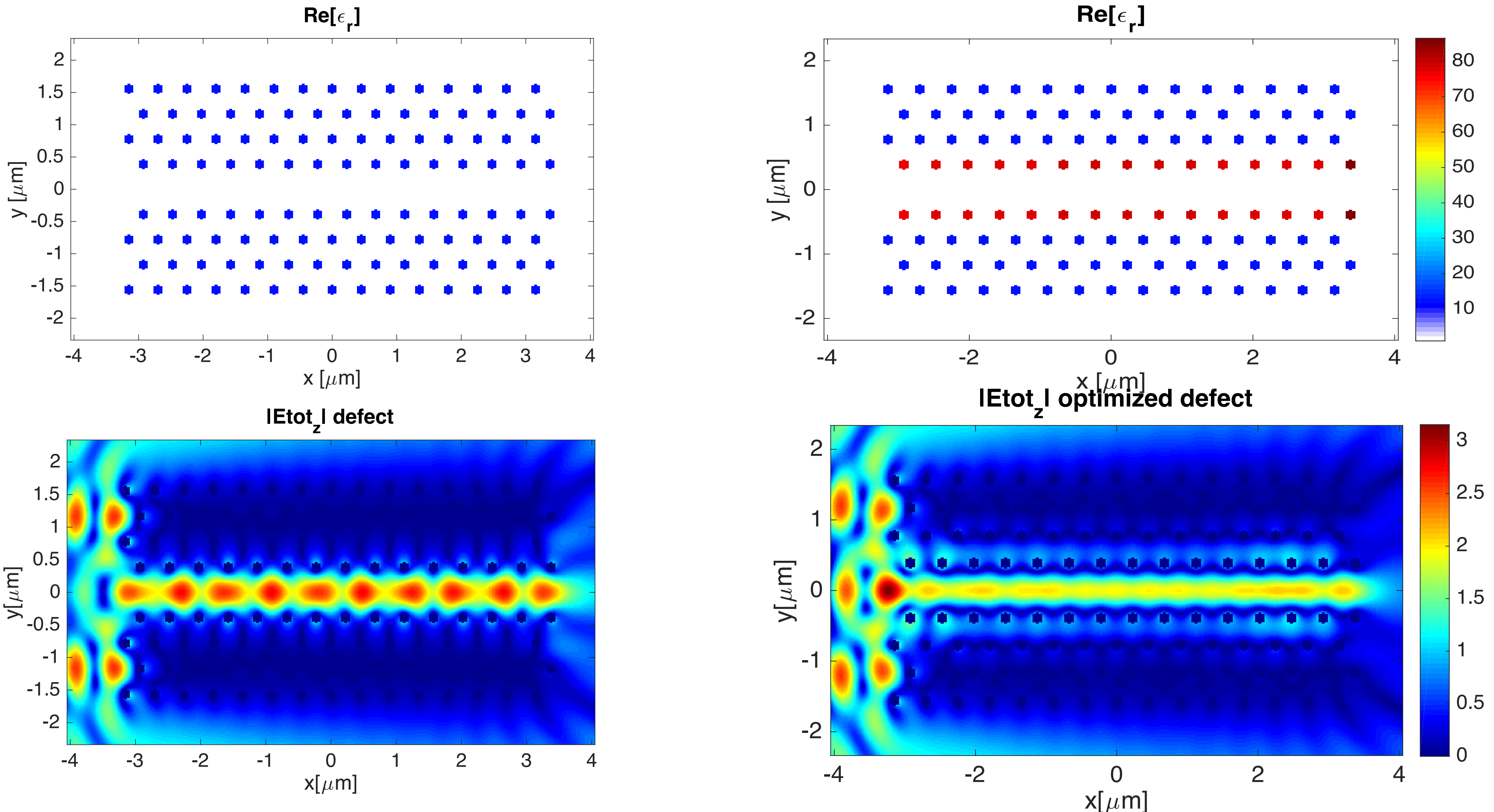
OPTIMIZATION PROBLEM

$$\min_{\varepsilon_r} \left\{ \frac{\max|E_{tot}| - \min|E_{tot}|}{2} \right\}_{\Lambda_{shaping}}$$

subject to

$$\varepsilon_r^{up} = \varepsilon_r^{down}$$

Basic element #1: EBG straight waveguide



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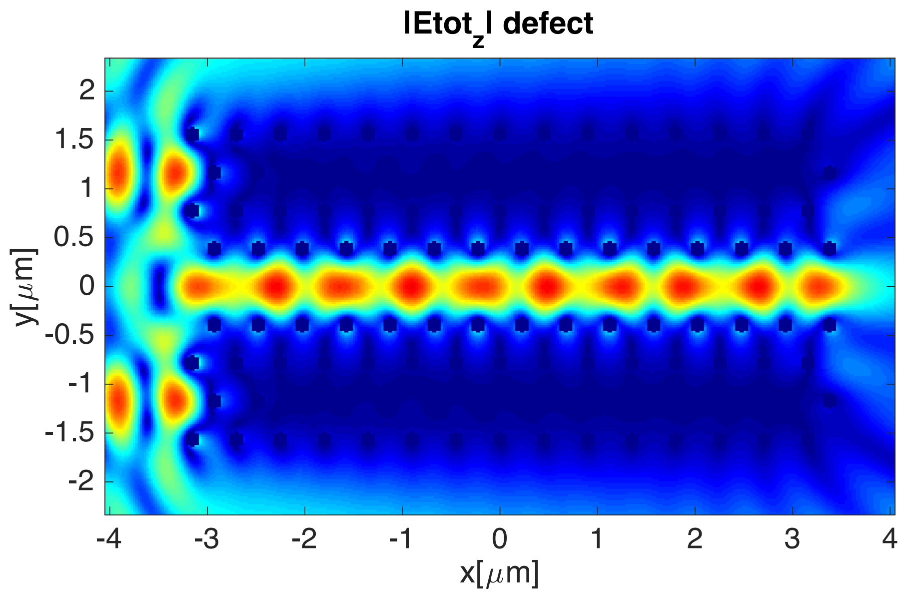
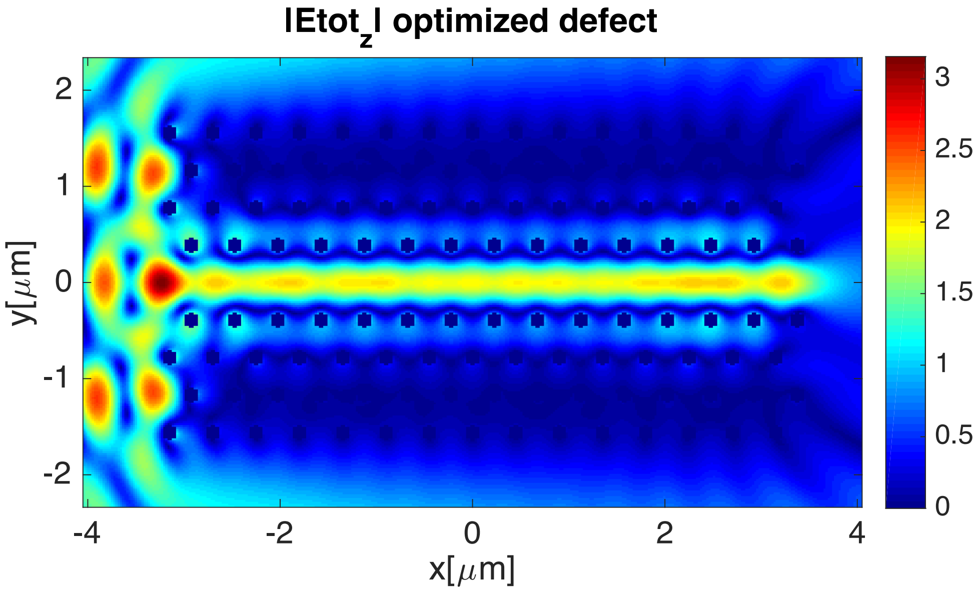
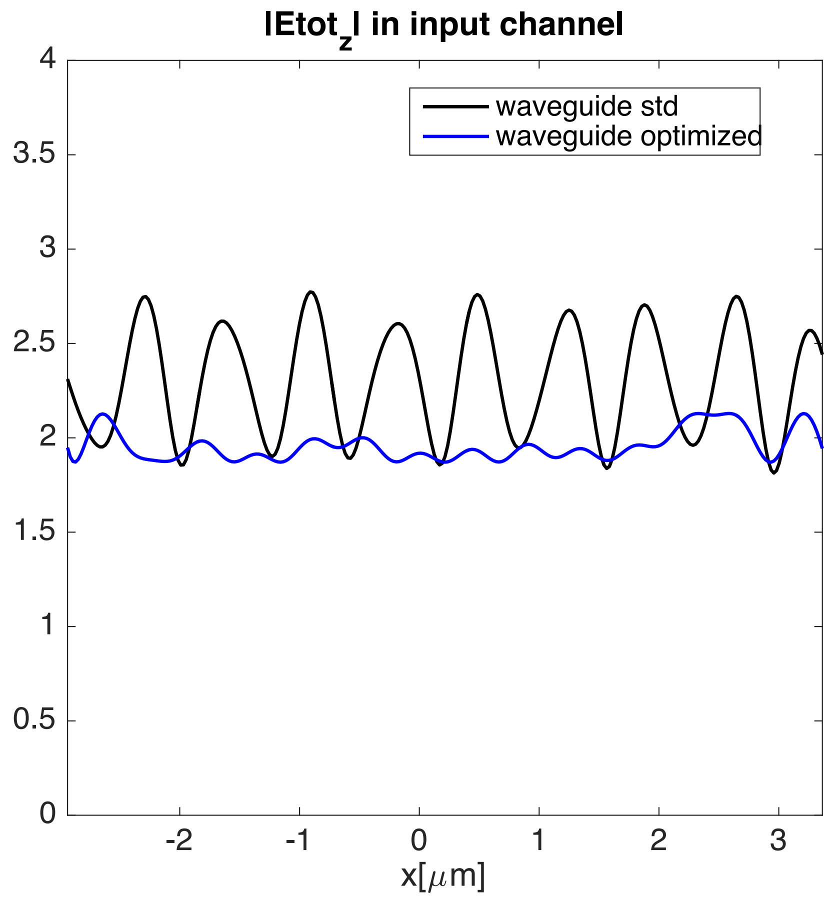


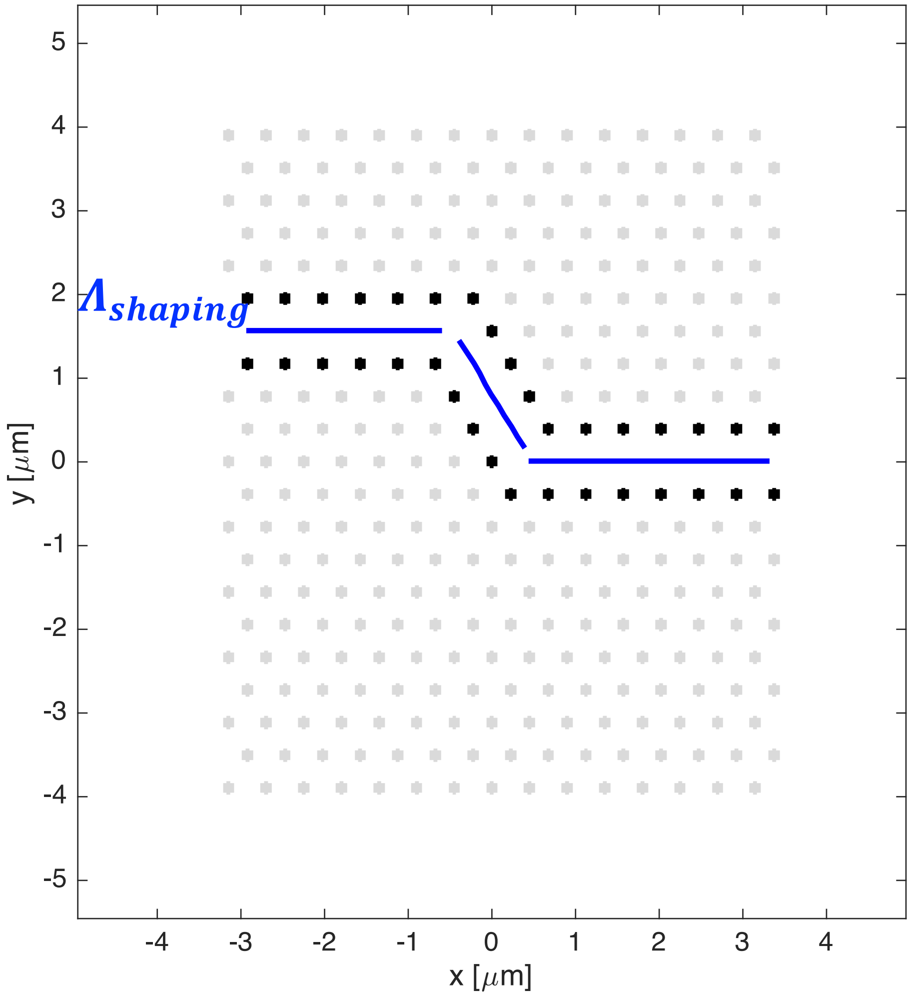
ripple = 0.4801



ripple = 0.1289



Basic element #2: 60° bend EBG waveguide



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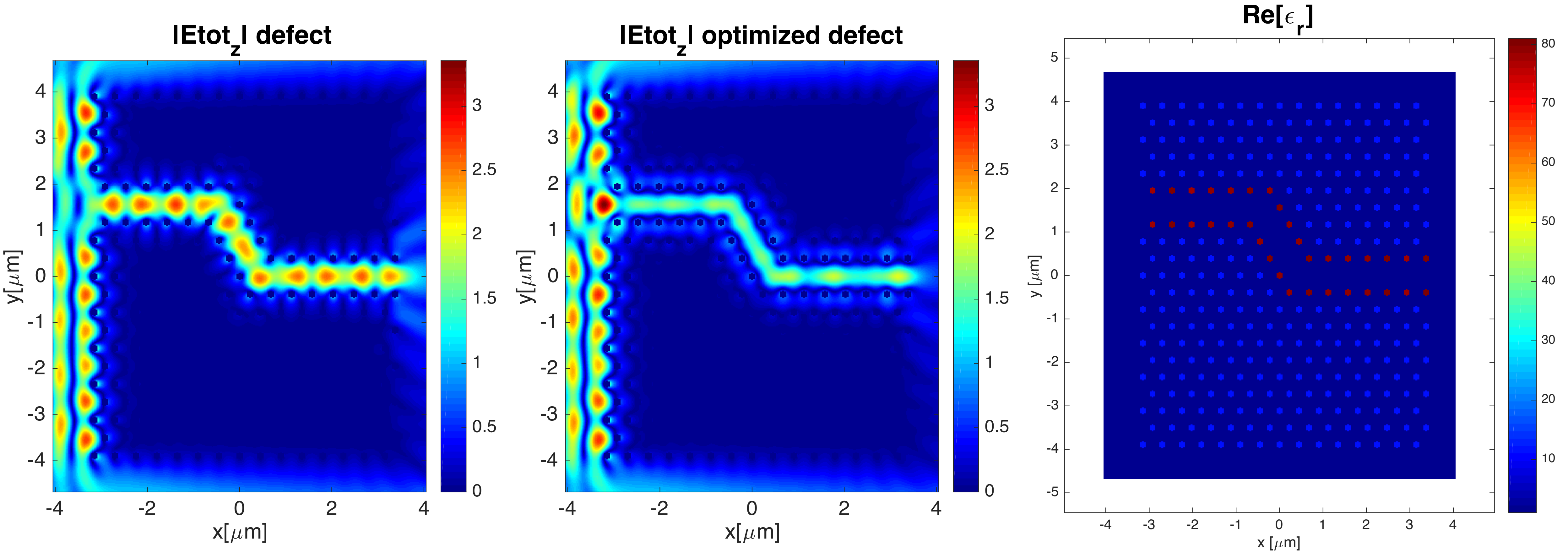
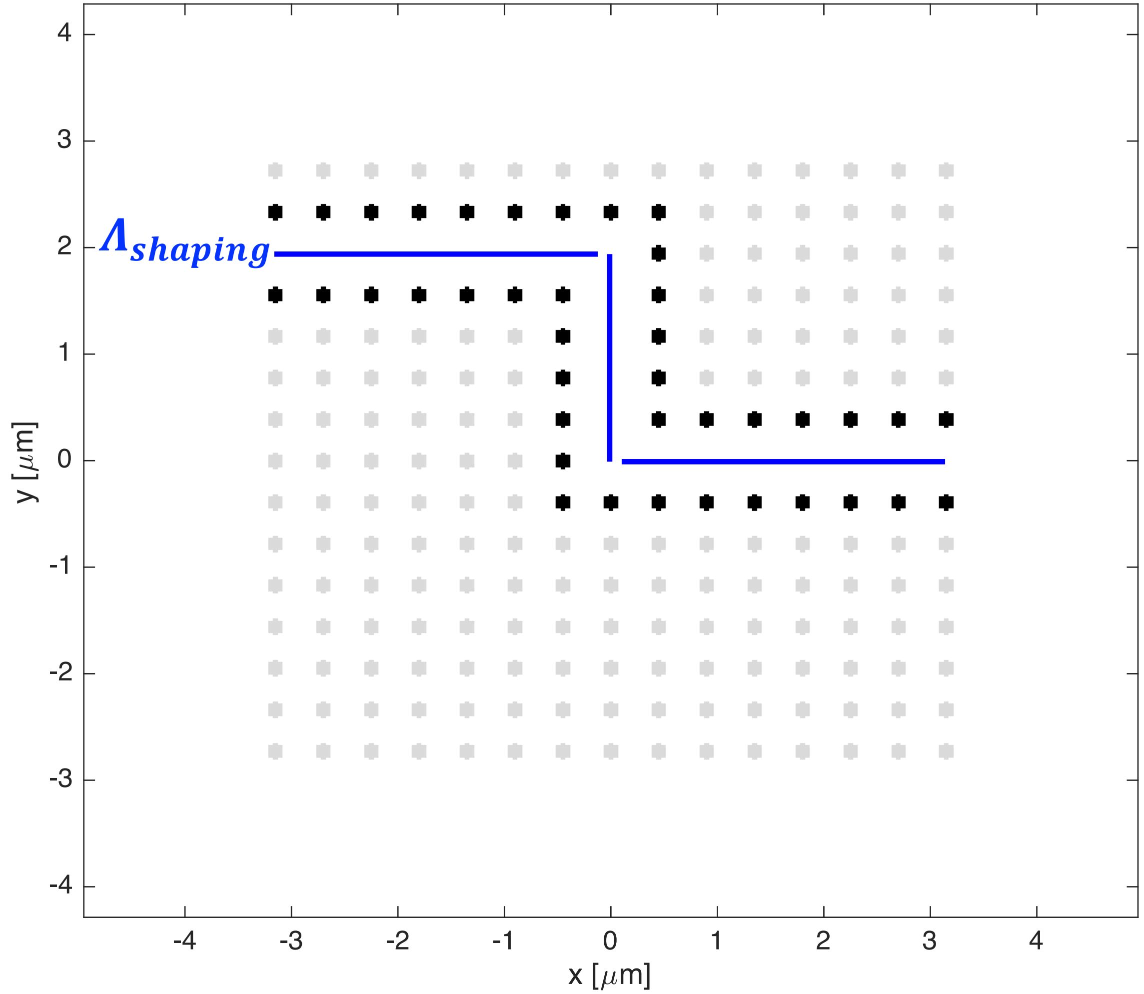


ripple_INPUT_channel = 0.4455
ripple_OBLIQUE_channel = 0.4459
ripple_OUTPUT_channel = 0.4380

ripple_INPUT_channel = 0.2459
ripple_OBLIQUE_channel = 0.1090
ripple_OUTPUT_channel = 0.3573

Basic element #3: 90° bend EBG waveguide



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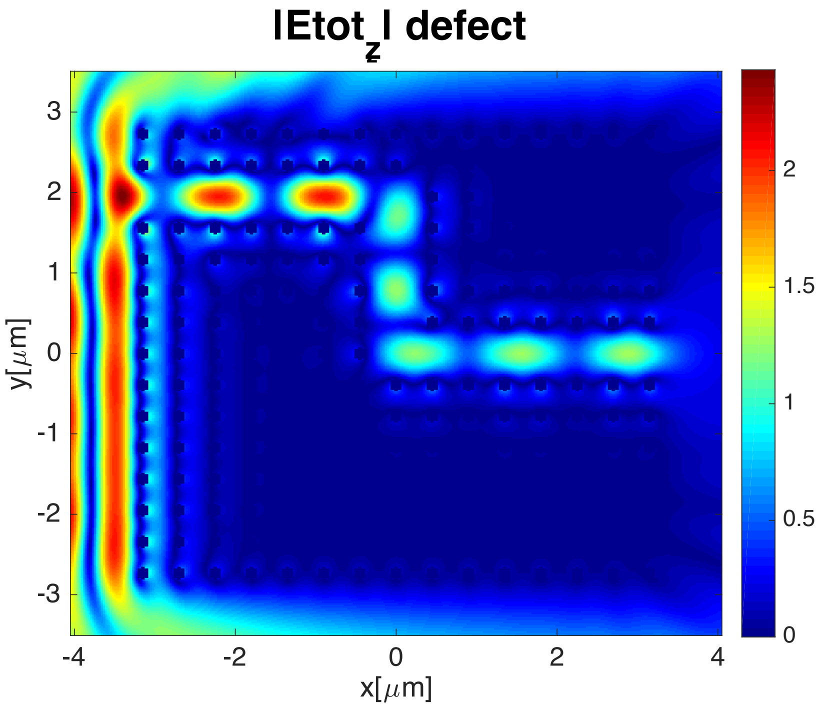
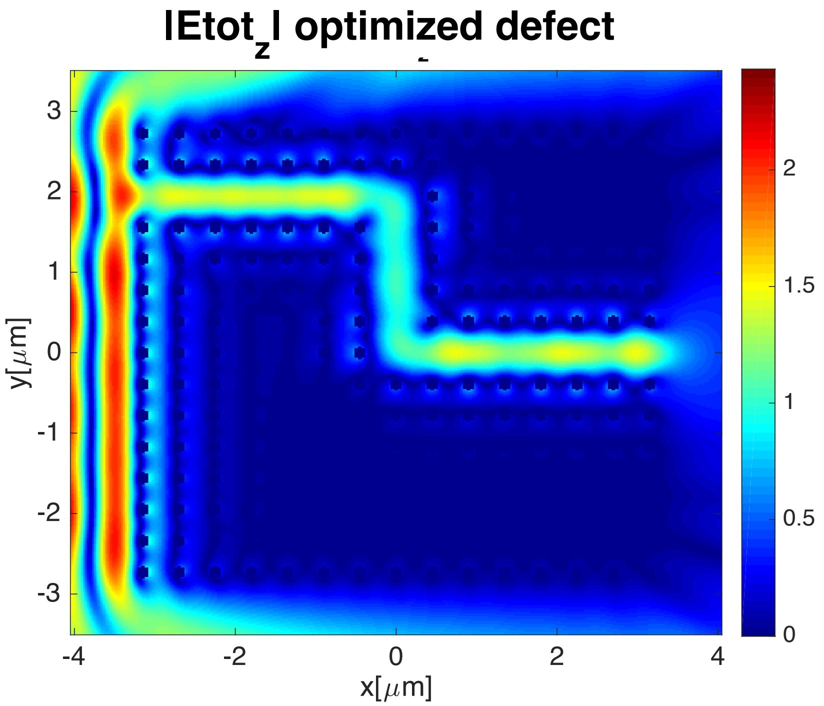
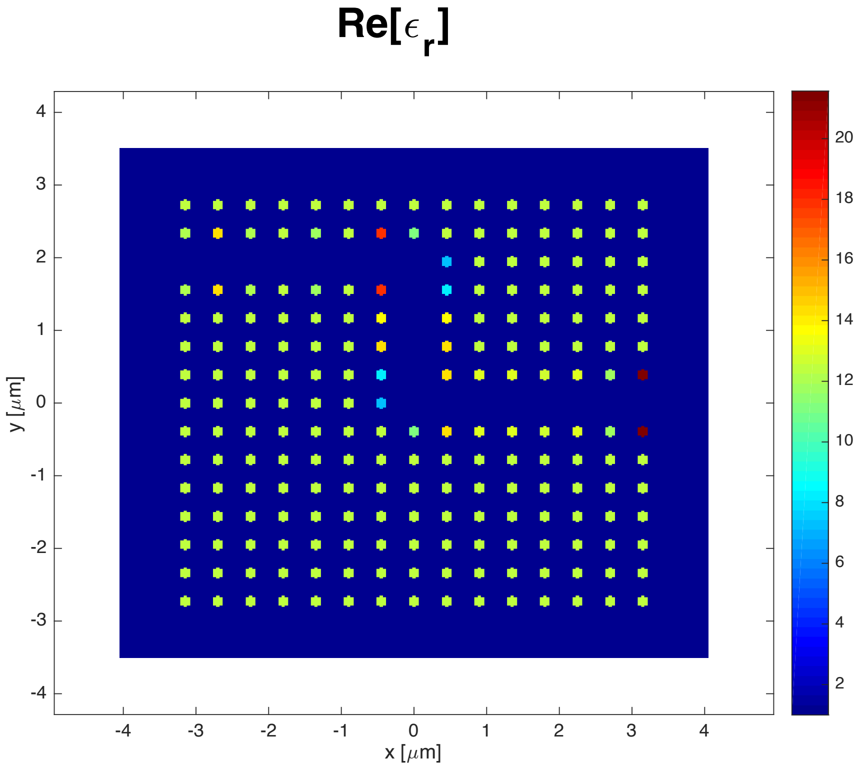


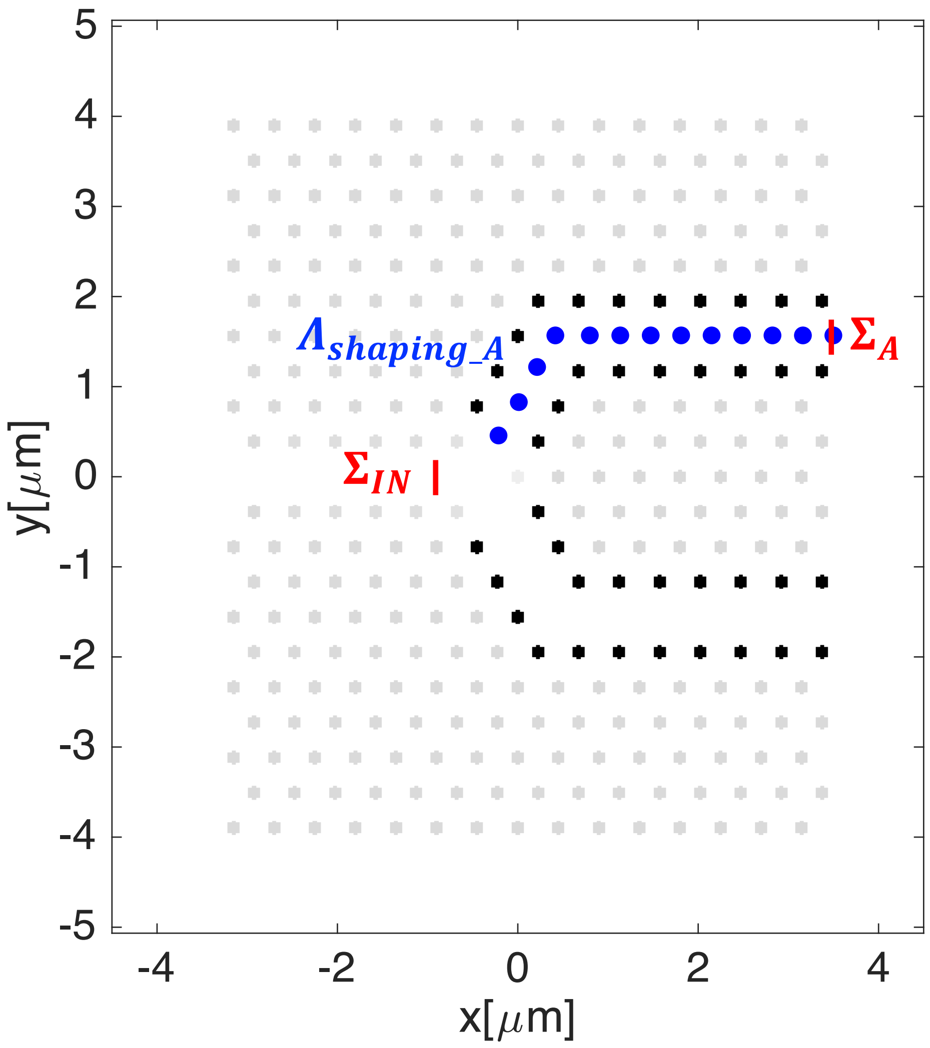




ripple_INPUT_channel = 0.8641
ripple_VERTICAL_channel = 0.4762
ripple_OUTPUT_channel = 0.3829

ripple_INPUT_channel = 0.2349
ripple_VERTICAL_channel = 0.0556
ripple_OUTPUT_channel = 0.1985

Basic element #4: 50-50 EBG power splitter



OPTIMIZATION PROBLEM

$$\min_{\varepsilon_r} \left\{ Ripple_{\Lambda_{shaping\_A}} \right\}$$

subject to

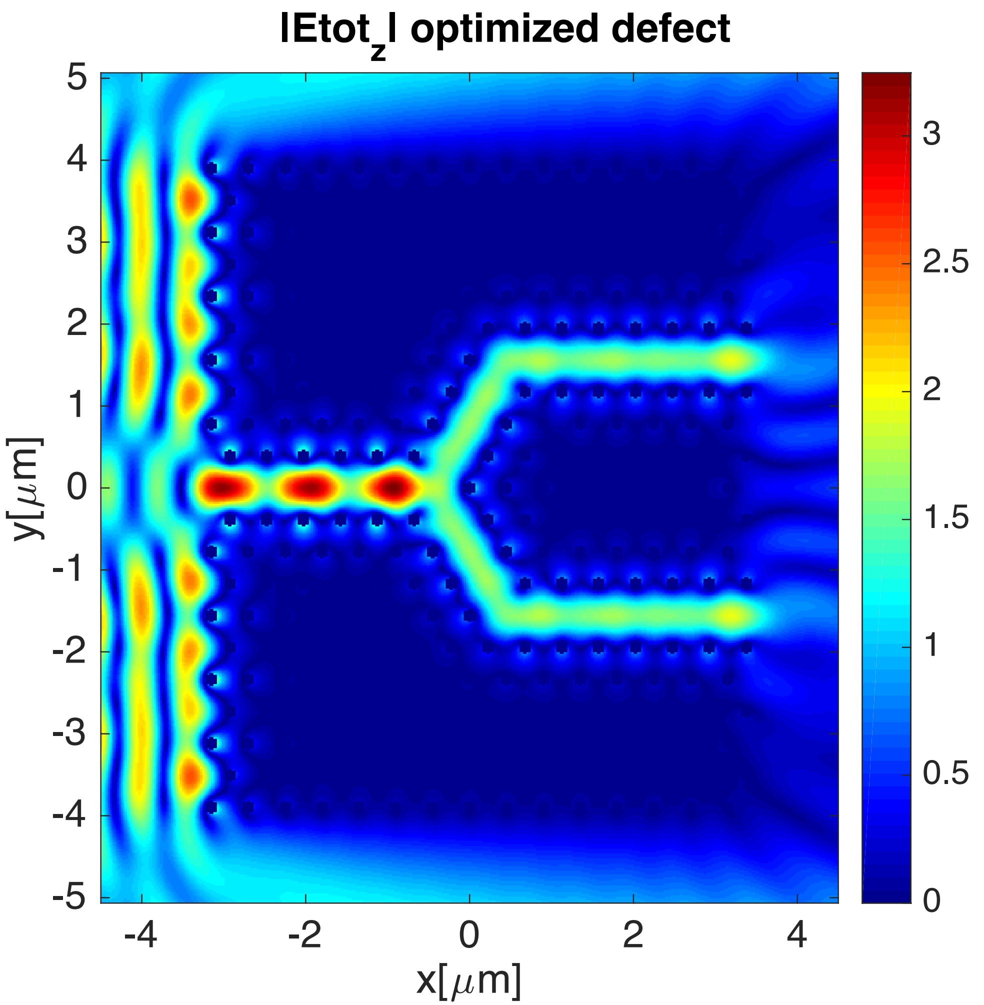
$$\varepsilon_r^{up} = \varepsilon_r^{down}$$

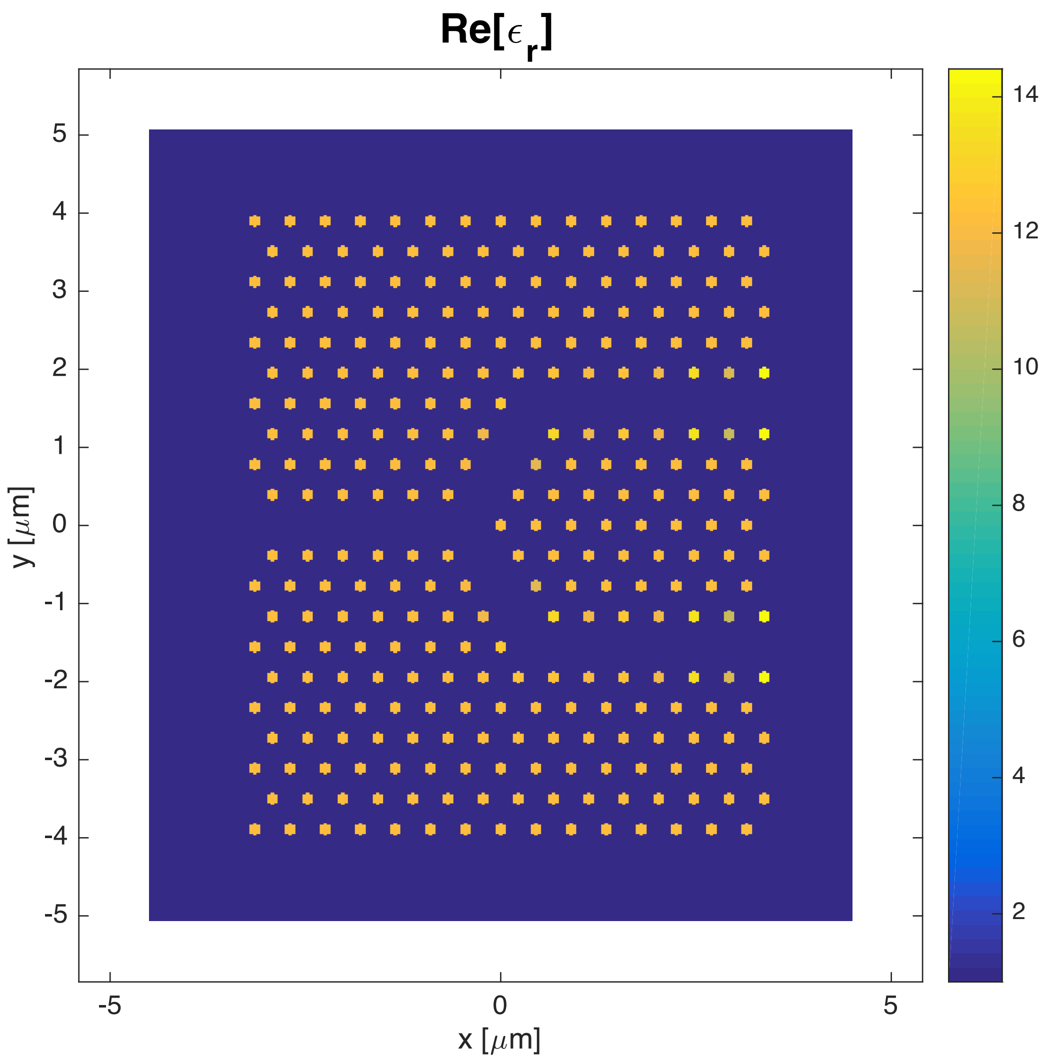
$$\varepsilon_r \geq 1$$

$$\left\| \phi_{\Sigma_A} - 0.5\, \phi_{\Sigma_{IN}} \right\|_2 \leq 0.001$$

Basic element #4: 50-50 EBG power splitter

ripple = 0.2545





Conclusions

- **Inverse scattering theory and solution procedures can be used as convenient and flexible design tool.**

- **A new suitable and efficient design tool based on the Scattering Matrix Method has been proposed and preliminary assessed**
    - *The method is roughly two orders of magnitude faster than the previous full wave method;*
    - *Classes of possible inclusions and objective functions can be exploited.*

- **Future works: other devices** *(for instance, EBG phase shifter, 75-25 power splitter, …)*, **other inclusion shapes** *(for instance elliptical)*, **unknown arrangements.**

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