Importance of modal analysis in vibratory microgyroscopes

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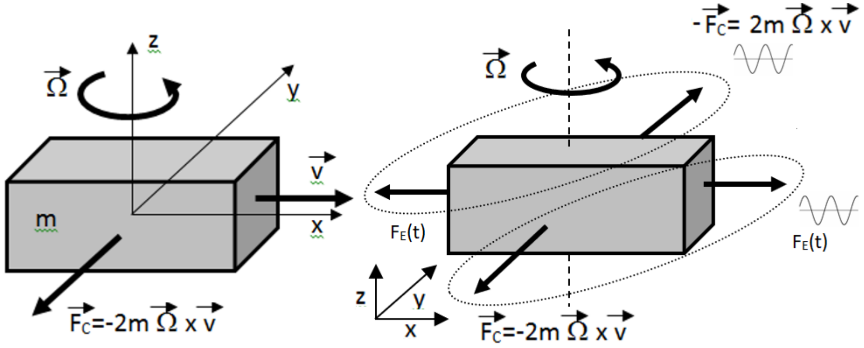
Introduction

- Microsystems are influenced by some negative factors, also by temperature,
- In case of small devices (particularly microsensors), temperature variation can be very destructive particularly there where accuracy is highly required,
- Natural frequencies should be taken into account in relation to geometry and temperature,
- In model one cycle of temperature was applied. We assumed also that temperatue value is constant over gyroscope surface. This assumption comes from the fact that in case of very small devices (on microlevel)
- The main purpose of the presentation is to show importance of modal analysis in MEMS devices performance assesement.

THEORETICAL BACKGROUND

Inertial sensor - gyroscope



- Coriolis effect,
- Displacement in y direction is proportional to angular velocity,
- Resonance effect,
- Amplitude influences on performance of device and range of measurement.

Motion equations

- Gyroscope principle of operation can be described with two 2nd order differental motion equations

Stiffness matrix

$$\begin{bmatrix} m_x & 0 \\ 0 & m_y \end{bmatrix} \begin{bmatrix} \frac{d^2x}{dt^2} \\ \frac{d^2y}{dt^2} \end{bmatrix} + \begin{bmatrix} c_x & 0 \\ 0 & c_y \end{bmatrix} \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} + \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2m_x\Omega\frac{dy}{dt} + F_x \\ -2m_y\Omega\frac{dx}{dt} + F_y \end{bmatrix}$$

Damping matrix

$$k_{yx} = \frac{3EI(l_1 - l_2)}{l_c l_1^3}$$

$$if \quad l_1 = l_2 \ then \ k_{yx} = 0$$

$$m_x\frac{d^2x}{dt^2} + c_x\frac{dx}{dt} + k_x x = F_D \sin(\omega t) + 2m_x\frac{dy}{dt}\Omega$$

$$m_y\frac{d^2y}{dt^2} + c_y\frac{dy}{dt} + k_y y = -2m_y\frac{dx}{dt}\Omega$$

- quantities temperature-dependent

Eigenfrequency and Q factor

$$\omega_x = \sqrt{\frac{k_x}{m_x}},\ \omega_y = \sqrt{\frac{k_y}{m_y}},$$

- Natural frequencies

$$Q_x = \frac{m_x \omega_x}{c_x},\ Q_y = \frac{m_y \omega_y}{c_y},\ \zeta_x = \frac{1}{2Q_x}\ \zeta_y = \frac{1}{2Q_y}$$

- Quality factors
- Damping ratios

$$\frac{d^2x}{dt^2} + \zeta_x \omega_x \frac{dx}{dt} + \omega_x^2 x = \frac{F_D}{m_x}$$

$$\frac{d^2y}{dt^2} + \zeta_y \omega_y \frac{dy}{dt} + \omega_y^2 y = -2\frac{dx}{dt}\Omega$$

- Modified Newton's motion equations

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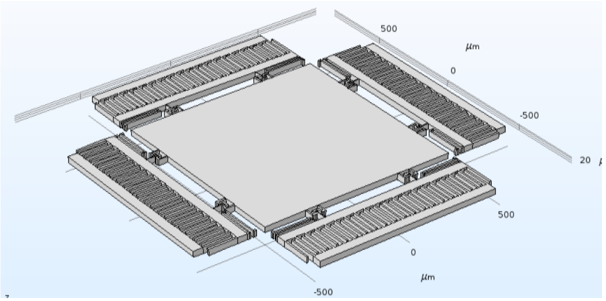
- Modified Newton's motion equations

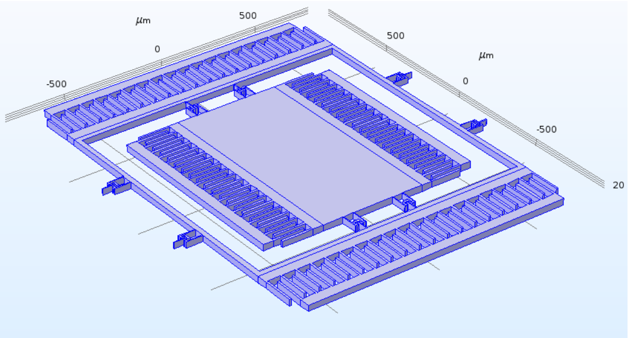
MODELS

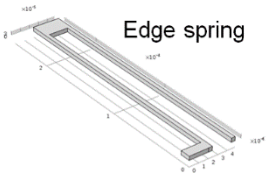
Models of inertial gyroscopes

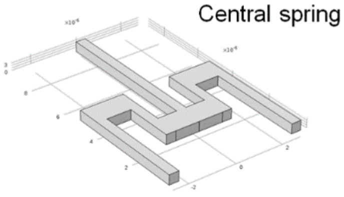
Two types of MEMS gyroscopes were considered:

- With one central inertial mass common for both drive and sense directions
- With two inertial masses: first, central mass for drive direction and second, with inertial frame.
  - **Note:** For sense directions mass is considered as a sum of both inertial frame and central mass.
  - Two different spring types were applied to device geometry structure.
  - The simplification – model does not assume fabrication imperfection and side wall effect



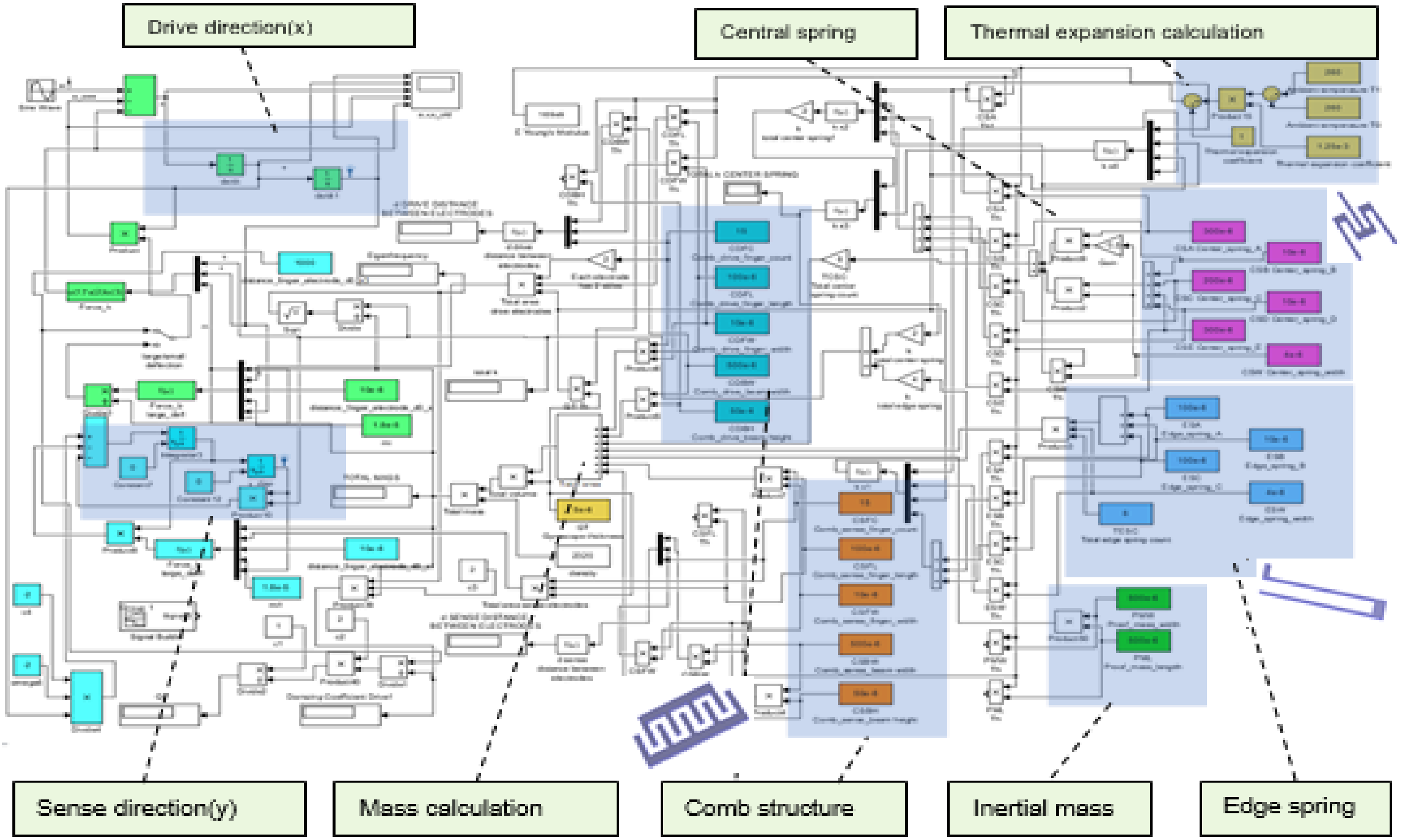



Edge spring

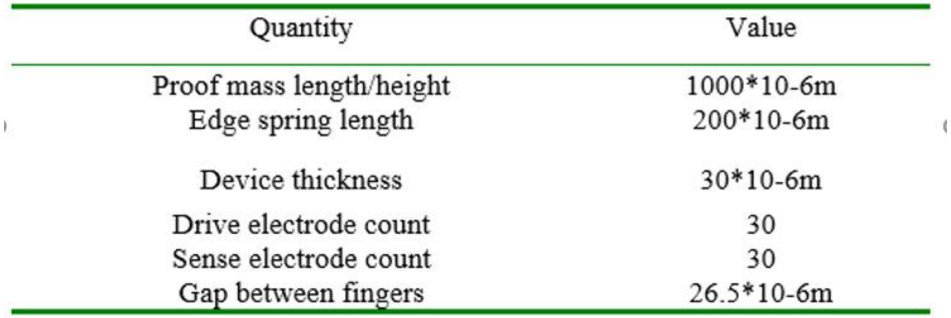

Central spring

MODEL in SIMULINK

Model in SIMULINK calculating two 2nd order Newton’s equations

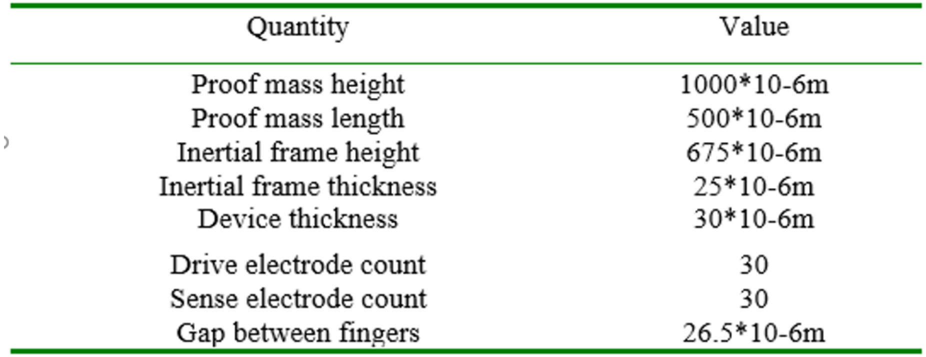


Geometry details of device



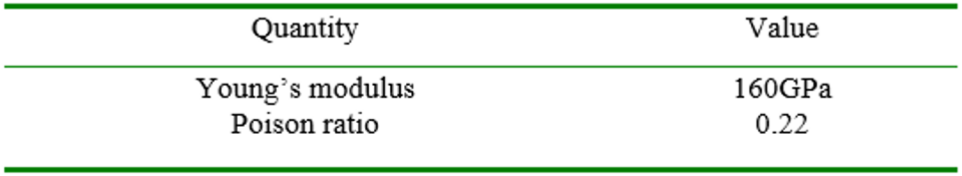
Quantity	Value
Proof mass length/height	1000*10-6m
Edge spring length	200*10-6m
Device thickness	30*10-6m
Drive electrode count	30
Sense electrode count	30
Gap between fingers	26.5*10-6m

Geometry details for device with one inertial mass



Quantity	Value
Proof mass height	1000*10-6m
Proof mass length	500*10-6m
Inertial frame height	675*10-6m
Inertial frame thickness	25*10-6m
Device thickness	30*10-6m
Drive electrode count	30
Sense electrode count	30
Gap between fingers	26.5*10-6m

Geometry details for device with inertial mass and frame

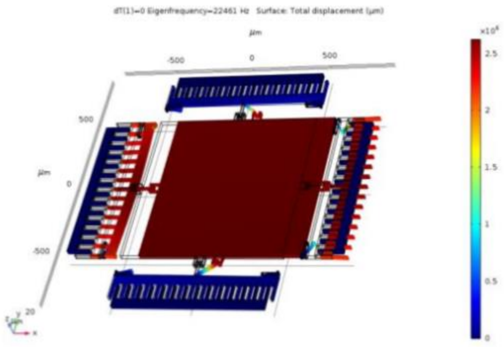


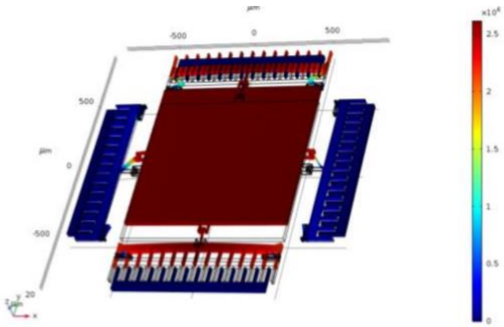
Quantity	Value
Young's modulus	160GPa
Poison ratio	0.22

Physical properties of polysilicon

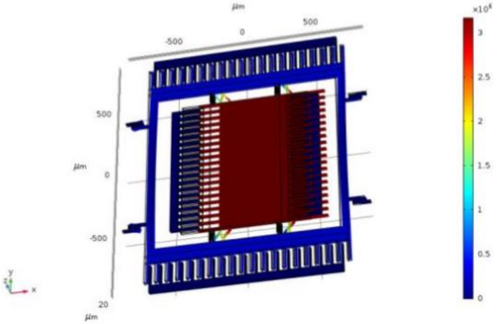
RESULTS

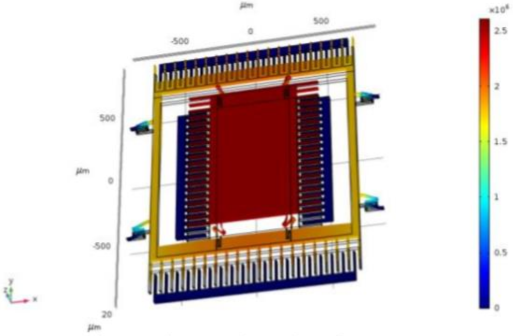
Eigenfrequency mode analysis





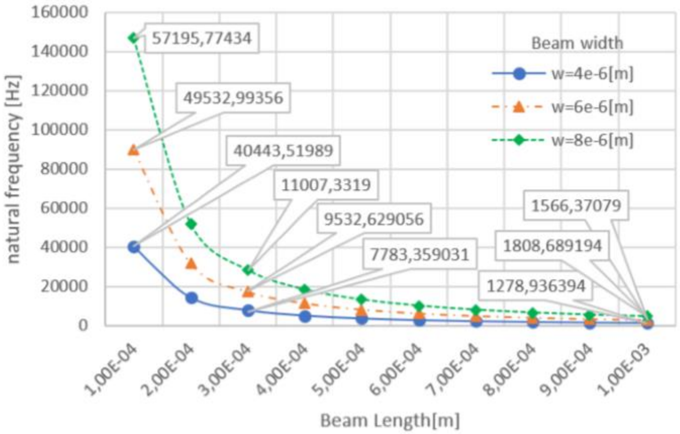
- 1st mode related to drive motion direction
- 2nd mode related to sense motion direction

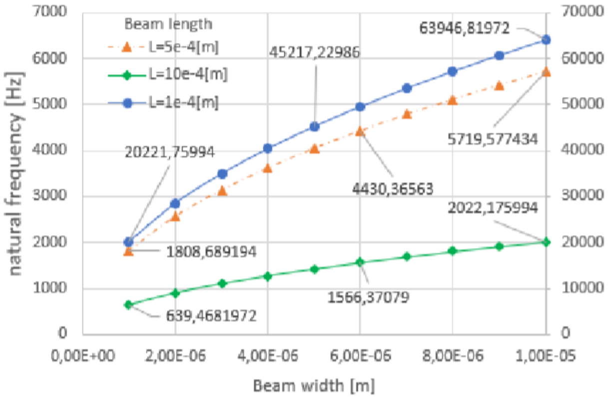


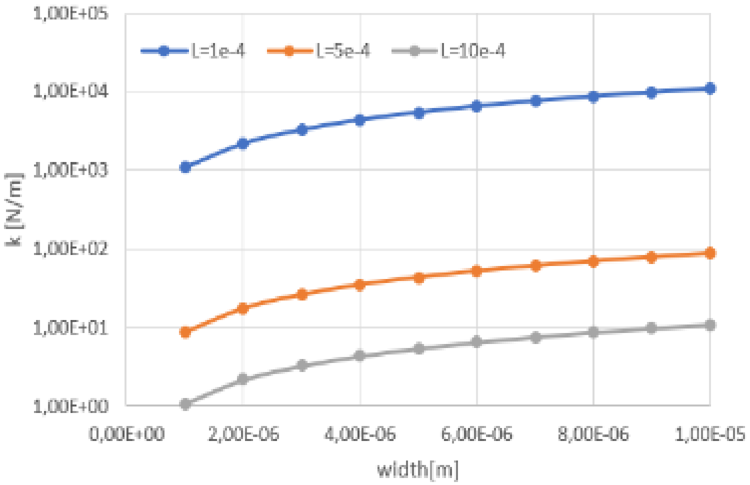


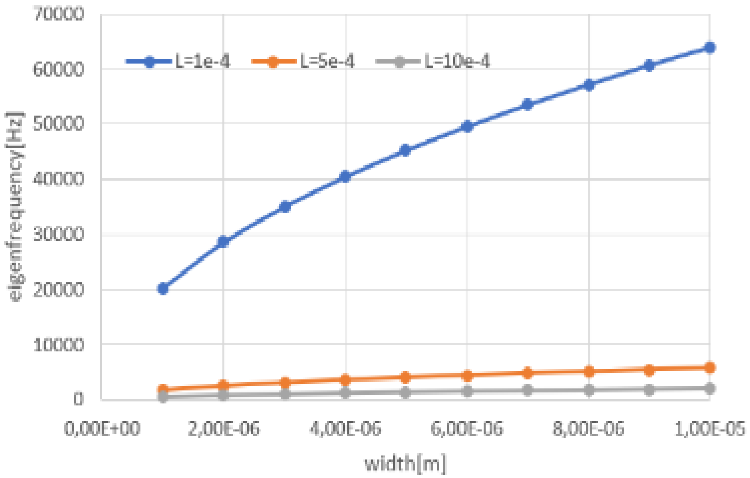
- Because of similar structures of both directions – eigenfrequencies are very similar - difference is about 15Hz

SIMULATION RESULTS

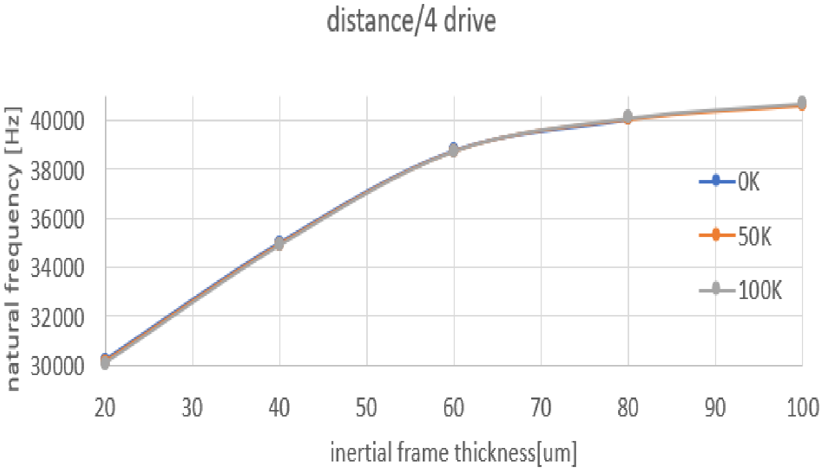






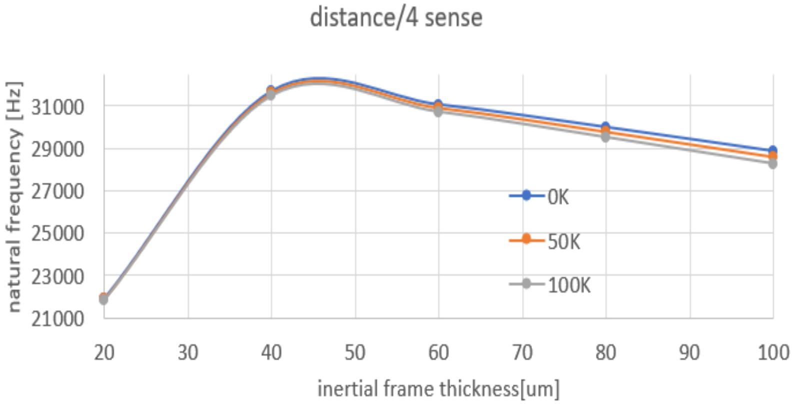


SIMULATION RESULTS



Natural frequency dependency on inertial frame thickness for different temperatures and for spring locations 1/4 distance from symmetry axis (drive direction)

Natural frequency dependency on inertial frame thickness for different temperatures and for spring locations 1/4 distance from symmetry axis (sense direction)



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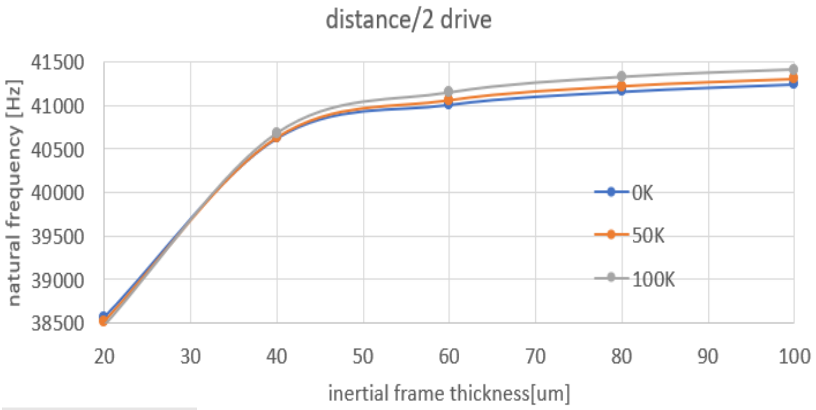
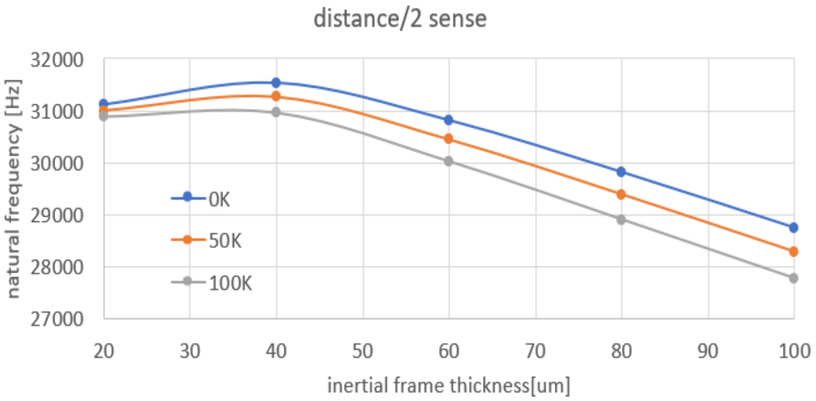
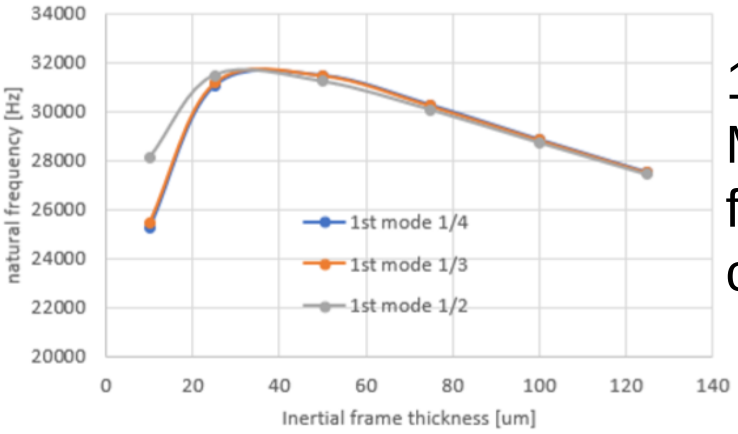


Natural frequency dependency on inertial frame thickness for different temperatures and for spring locations 1/2 distance from symmetry axis (drive direction) - gyroscope with two masses.

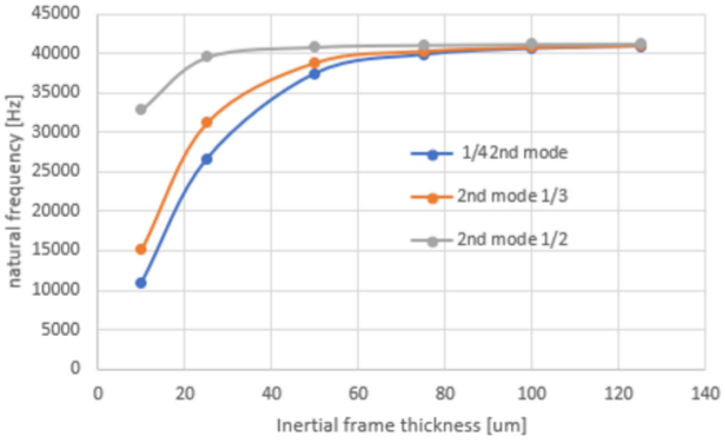
Natural frequency dependency on inertial frame thickness for different temperatures and for spring locations 1/2 distance from symmetry axis (sense direction) - gyroscope with two masses.



SIMULATION RESULTS

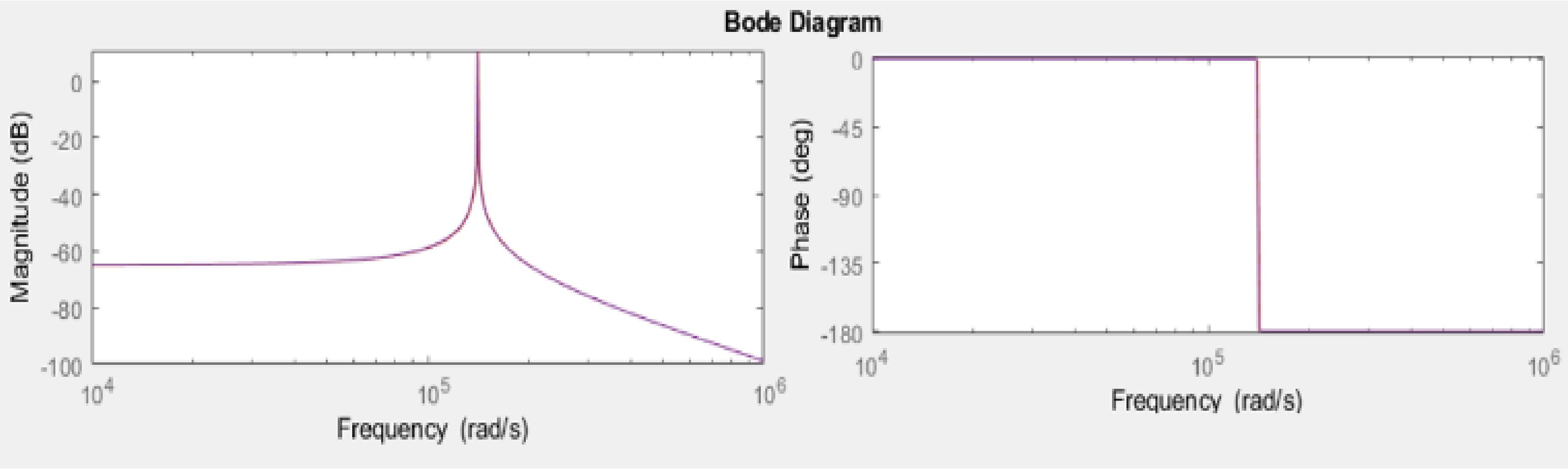


1^{nd} mode modal analysis of MEMS Gyroscope without inertial frame with inertial frame and central.

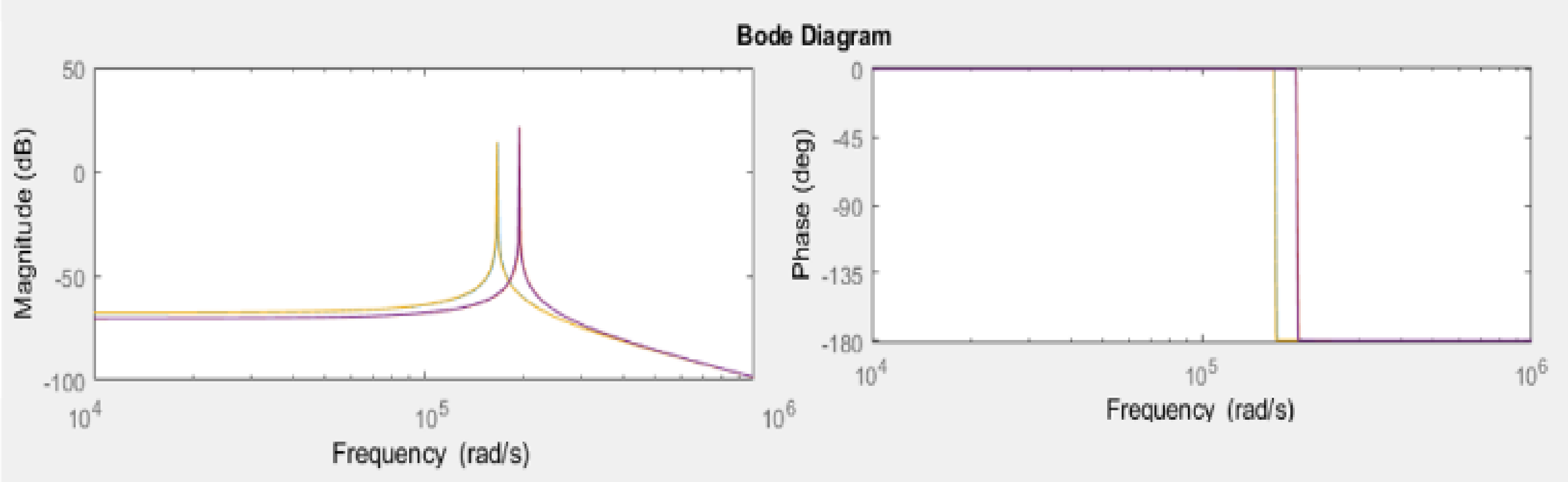


2^{nd} mode modal analysis of MEMS Gyroscope without inertial frame with inertial frame and central.

Magnitude and phase



Gyroscope with one inertial mass



Gyroscope with two masses.

CONCLUSIONS

Results presented here shows, how important is modal analysis in performance assesement for MEMS inertial devices. Some crucial conclusions extracted from these results are:

- Geometry details, dimensions and temperature influences on eigenfrequency response
- There are non-linear dependencies between dimensions and eigenfrequencies
- Optimal geometry is device with one inertial mass because of perfect mode-matching (it is seen in magnitude and phase plots)

CONCLUSIONS

A large numer of simulations were performed, however time presentation limit do not allow to show temperature dependencies for all geometry details.

- Results obtained from simulations clearly show that geometry and temperature variation have enormous influence on MEMS Gyroscope behavior and this factor cannot (!) be avoided during structure design
- In model we did not assumed fabrication imperfections and side wall angle, however both may empower (multiply) degradation of performance because of mode-matching lost.
- Each structure configuration should be considered separately, because the natural frequency dependency on temperature and dimensions gives different results.