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Resonant Inductive WPT Link with Multiple Transmitters

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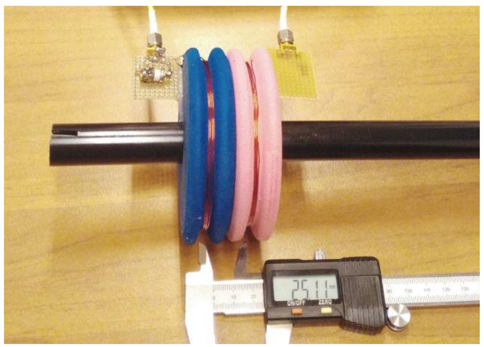
Outline

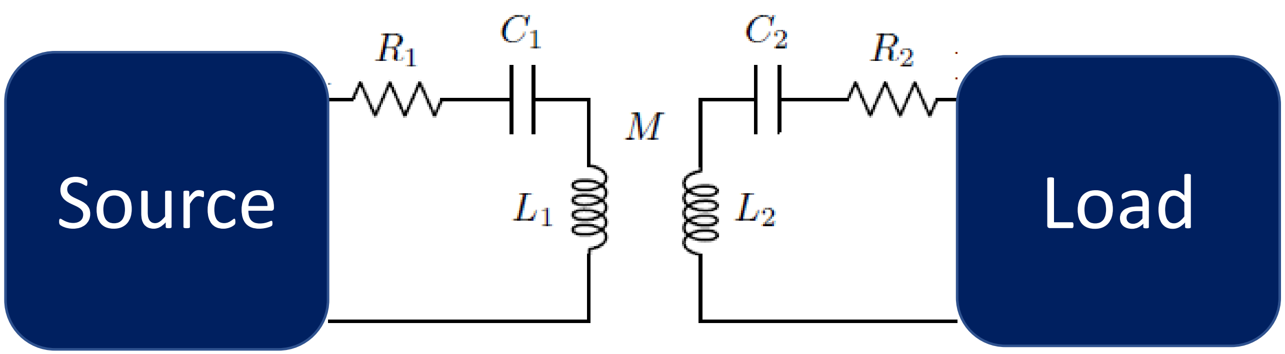
- Introduction
- Non-radiative WPT based on inductive coupling: variables of interest and figures of merit
- Resonant inductive WPT links with multiple transmitters:
    - Statement of the problem and analytical solution
    - Validation
- Conclusion

Resonant inductive WPT

Single Input Single Output configuration: a single transmitter is wirelessly connected to a single load



Operating frequency	Mutual inductance	Quality factor
$\omega_0 = \frac{1}{\sqrt{L_i C_i}}$	$M = k\sqrt{L_1 L_2}$	$Q_i = \frac{\omega_0 L_i}{R_i}$



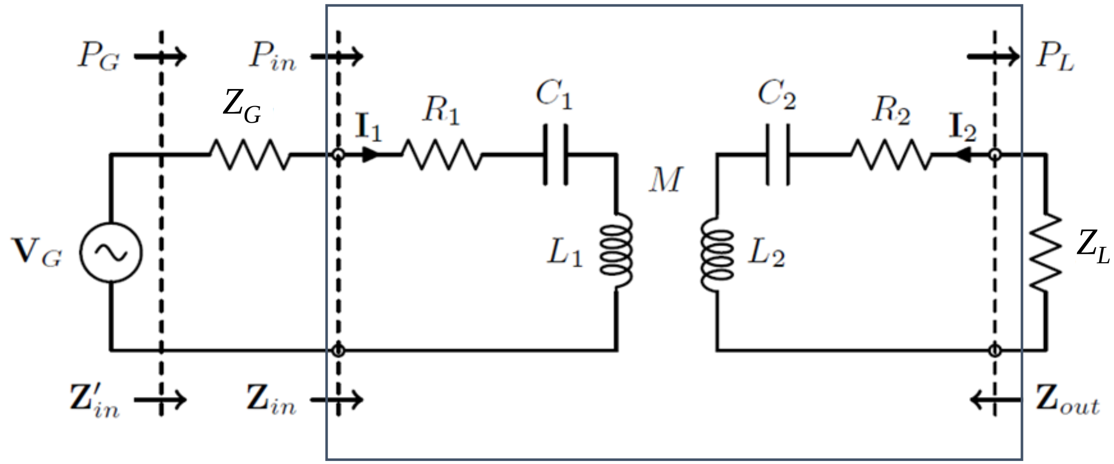
Two-port Equivalent circuit

Resonant inductive WPT

$$\mathbf{V} = \mathbf{Z}\mathbf{I}$$

$$\mathbf{Z} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}$$

$$z_{ij} = r_{ij} + jx_{ij}, (i, j = 1,2)$$



It is assumed that a voltage generator is on port 1 and that a load Z_L is on port 2.

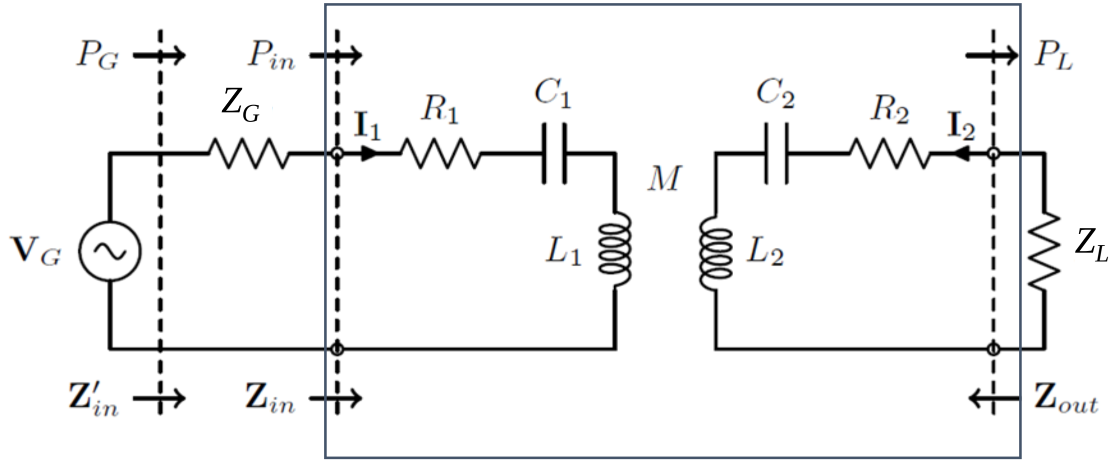
$$Z_{in} = \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

$$Z_{out} = \frac{V_2}{I_2}\bigg|_{V_G=0} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_G}$$

$$\omega_0 = \frac{1}{\sqrt{L_i C_i}} \qquad M = k\sqrt{L_1 L_2} \qquad Q_i = \frac{\omega_0 L_i}{R_i}$$

Dionigi, M., Mongiardo, M., Perfetti, R.: Rigorous network and full-wave electromagnetic modeling of wireless power transfer links. Microwave Theory and Techniques, IEEE Transactions on 63(1), 65–75 (2015). DOI 10.1109/TMTT.2014.2376555

Resonant inductive WPT: figures of merit



$$\omega_0 = \frac{1}{\sqrt{L_i C_i}} \quad (i = 1{,}2)$$

With regard to the use of the two-port network for WPT applications, two are the main figures of merit:

- ➔ the active power delivered to the load **(PDL)**;
- ➔ the power transfer efficiency **(PTE)**, η, defined as:

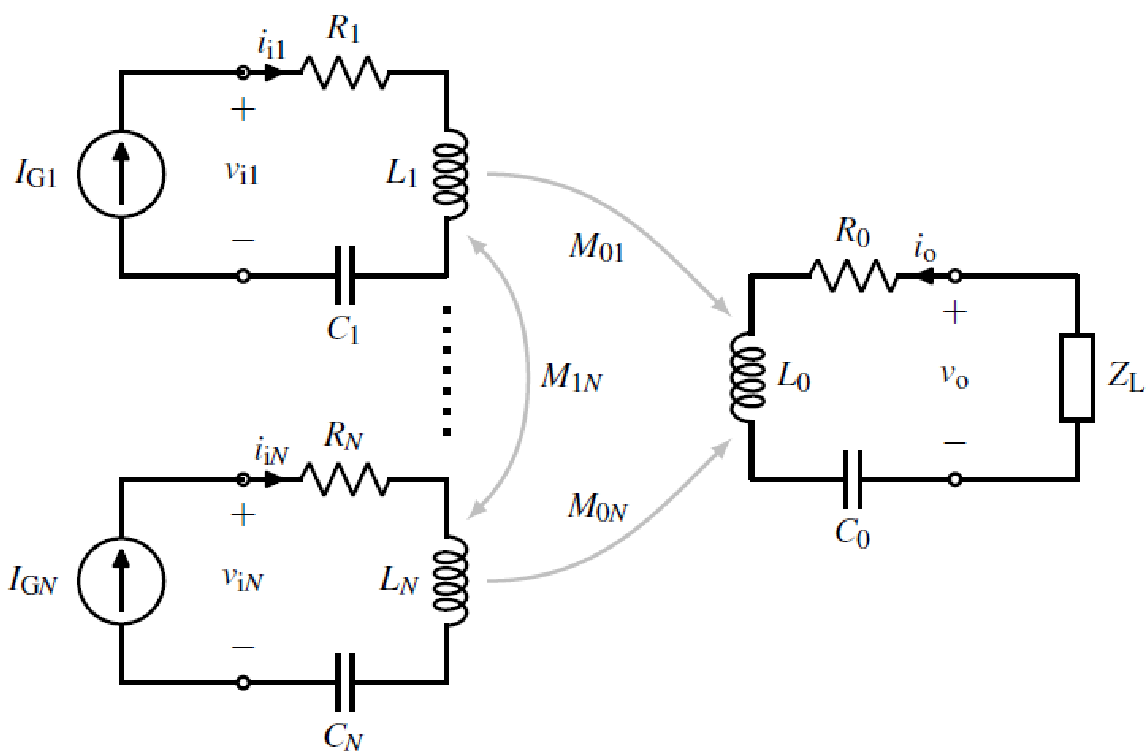
$$\eta = \frac{P_L}{P_{in}}$$

Resonant inductive WPT: MISO configuration

Multiple Input Single Output (MISO) configuration

The use of multiple transmitters suitably distributed in space can be exploited for obtaining a nearly constant efficiency even for a moving receiver

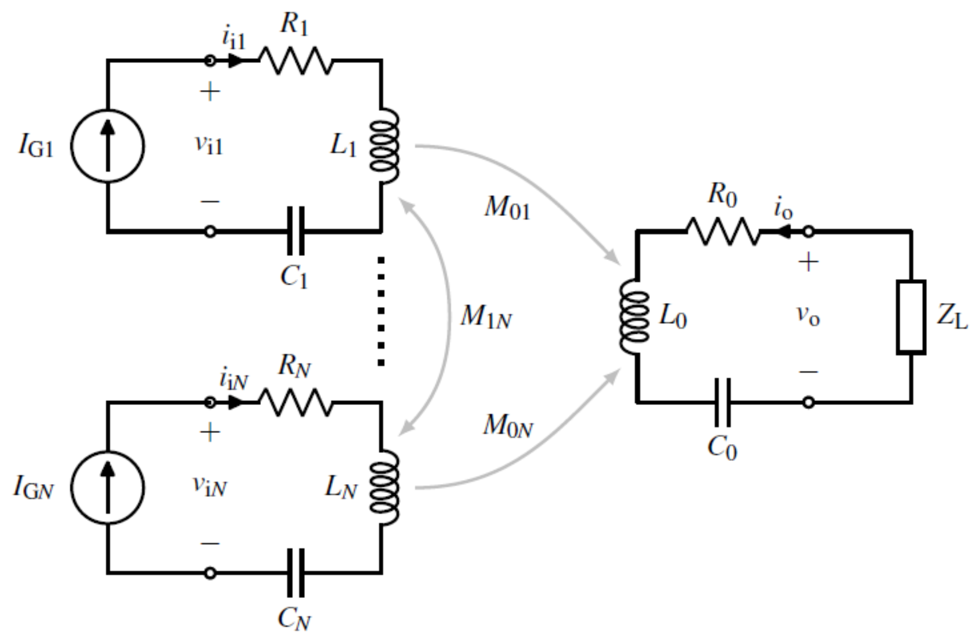
It is assumed that the link consists of (N+1) resonators:
N transmitters and a single load



Equivalent circuit of a resonant inductive WPT link using a MISO configuration

Resonant inductive WPT: MISO configuration

The WPT system is represented as an (N +1)–port strictly passive reciprocal network, described in terms of an impedance matrix Z.
The impedance matrix of the (N+1)–port network is:

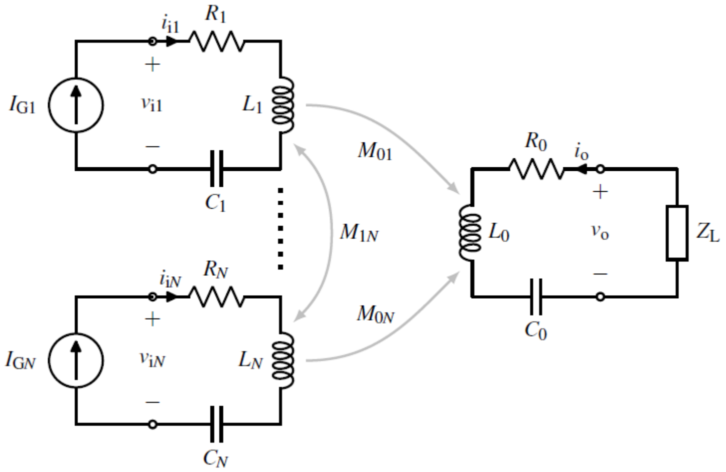


$$\mathbf{Z} = \left[\begin{array}{c|c} Z_0 & \mathbf{Z}_c^T \\ \hline \mathbf{Z}_c & \mathbf{Z}_i \end{array}\right] = \left[\begin{array}{c|ccc} R_0 & j\omega M_{01} & \cdots & j\omega M_{0N} \\ \hline j\omega M_{01} & R_1 & \cdots & j\omega M_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ j\omega M_{0N} & j\omega M_{1N} & \cdots & R_N \end{array}\right]$$

Resonant inductive WPT: MISO configuration

By introducing the normalization matrix **n**

$$\mathbf{n} = \begin{bmatrix} \frac{1}{\sqrt{\omega L_0}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\sqrt{\omega L_N}} \end{bmatrix}$$



it is possible to obtain the normalized impedance matrix of the network:

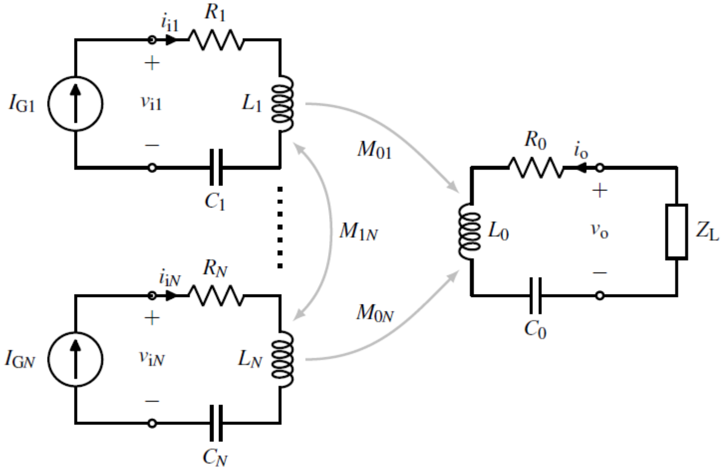
$$\mathbf{z} = \mathbf{nZn} = \left[\begin{array}{c|c} z_0 & \mathbf{z}_c^T \\ \hline \mathbf{z}_c & \mathbf{z}_i \end{array}\right] = \left[\begin{array}{c|cccc} \frac{1}{Q_0} & jk_{01} & \cdots & jk_{0N} \\ \hline jk_{01} & \frac{1}{Q_1} & \cdots & jk_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ jk_{0N} & jk_{1N} & \cdots & \frac{1}{Q_N} \end{array}\right]$$

$$Q_n = \frac{\omega_n L_n}{R_n} \qquad (n = 0, \ldots, N),$$

$$k_{nm} = \frac{M_{nm}}{\sqrt{L_n L_m}} \qquad (n, m = 0, \ldots, N; n \neq m)$$

Resonant inductive WPT: MISO configuration

The power transfer efficiency is defined as the ratio of the active power delivered to the load (P_o) to the sum of the total power entering the network (P_i)



$$\eta = \frac{P_o}{\sum_{n=1}^{N} P_{i,n}} = \frac{P_o}{P_i}$$

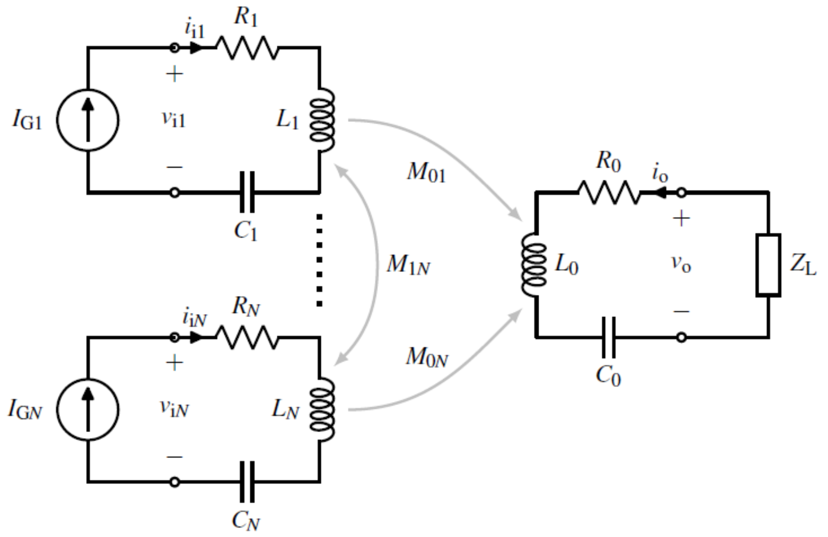
By introducing the currents vector **i** and the matrices **A** and **B**

$$\mathbf{i} = \begin{bmatrix} i_0 \\ i_{i1} \\ i_{i2} \\ \vdots \\ i_{iN} \end{bmatrix} \qquad \mathbf{A} = \left[\begin{array}{c|c} z_o + z_o^* & \mathbf{z}_c^T \\ \hline \mathbf{z}_c^* & \mathbf{0} \end{array}\right] \qquad \mathbf{B} = \left[\begin{array}{c|c} 0 & \mathbf{z}_c^H \\ \hline \mathbf{z}_c & \mathbf{z}_i + \mathbf{z}_i^H \end{array}\right]$$

$$P_o = -\frac{1}{4}\mathbf{i}^H \mathbf{A} \mathbf{i} \qquad P_i = \frac{1}{4}\mathbf{i}^H \mathbf{B} \mathbf{i}$$

Resonant inductive WPT: MISO configuration

$$\eta = \frac{P_o}{\sum_{n=1}^{N} P_{i,n}} = \frac{P_o}{P_i}$$



$$\mathbf{A} = \left[\begin{array}{c|c} z_0 + z_0^* & \mathbf{z}_c^T \\ \hline \mathbf{z}_c^* & \mathbf{0} \end{array}\right] \qquad \mathbf{B} = \left[\begin{array}{c|c} 0 & \mathbf{z}_c^H \\ \hline \mathbf{z}_c & \mathbf{z}_i + \mathbf{z}_i^H \end{array}\right]$$

$$\eta = \frac{P_o}{P_i} = -\frac{\mathbf{i}^H \mathbf{A} \mathbf{i}}{\mathbf{i}^H \mathbf{B} \mathbf{i}}.$$

GENERALIZED EIGENVALUE PROBLEM

$$-\mathbf{A}\mathbf{i} = \eta \mathbf{B}\mathbf{i}$$

By solving the problem, the maximum power transfer efficiency of the network can be derived

The corresponding eigenvector is the currents vector realizing the maximum power transfer efficiency

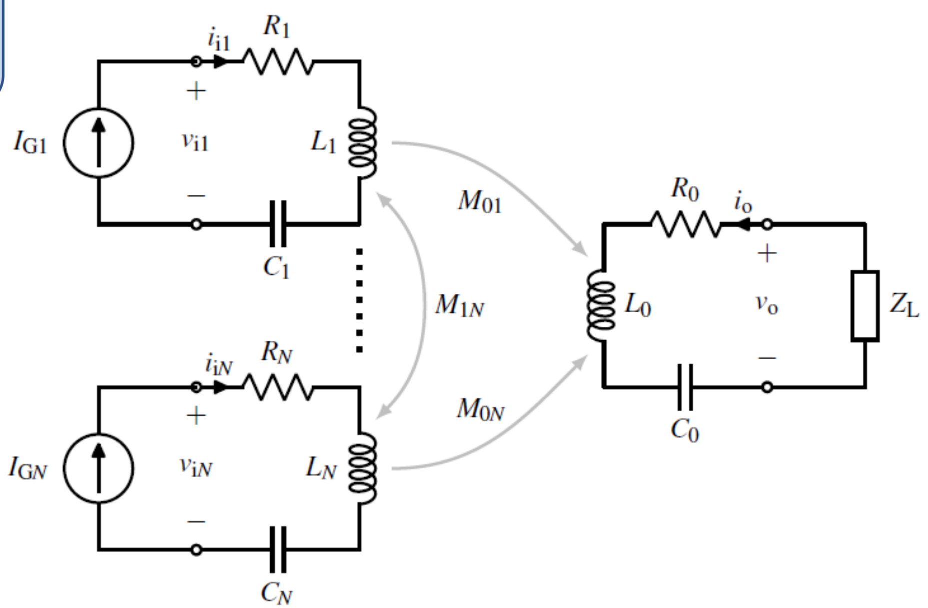
Resonant inductive WPT: MISO configuration

GENERALIZED EIGENVALUE PROBLEM

$$-\mathbf{A}\mathbf{i} = \eta \mathbf{B}\mathbf{i}$$

$$\eta_{\mathrm{MAX}} = \frac{\alpha - 1}{\alpha + 1}$$

$$\alpha = \sqrt{1 + Q_0 \sum_{n=1}^{N} k_{0n}^2 Q_n}$$



INPUT CURRENTS REALIZING THE MAXIMUM EFFICIENCY

$$\mathbf{i}_{in} = j\frac{k_{0n}Q_n}{\alpha - 1} i_o \qquad (n = 1, \ldots, N)$$

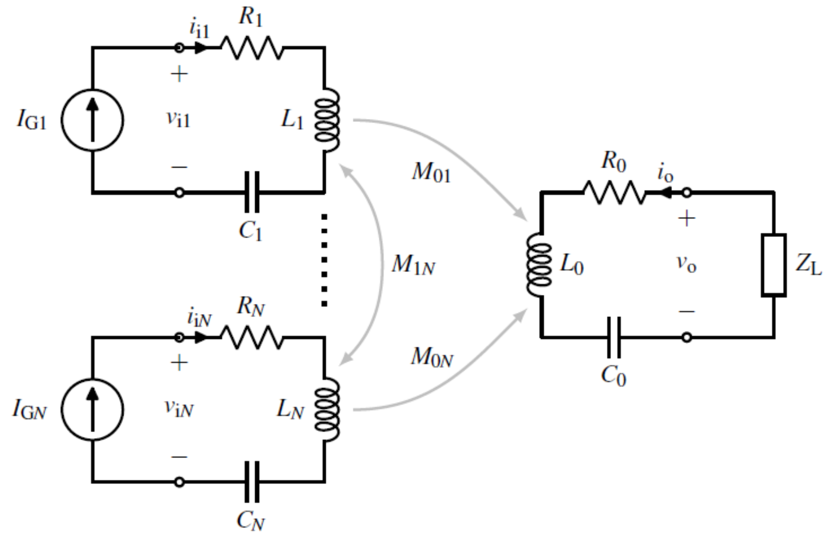
Resonant inductive WPT: MISO configuration

GENERALIZED EIGENVALUE PROBLEM

$$-\mathbf{Ai} = \eta \mathbf{Bi}$$

$$\eta_{\mathrm{MAX}} = \frac{\alpha - 1}{\alpha + 1}$$

$$\alpha = \sqrt{1 + Q_0 \sum_{n=1}^{N} k_{0n}^2 Q_n}$$



$$\mathbf{i}_{in} = j\frac{k_{0n}Q_n}{\alpha - 1} i_o \qquad (n = 1, \ldots, N)$$

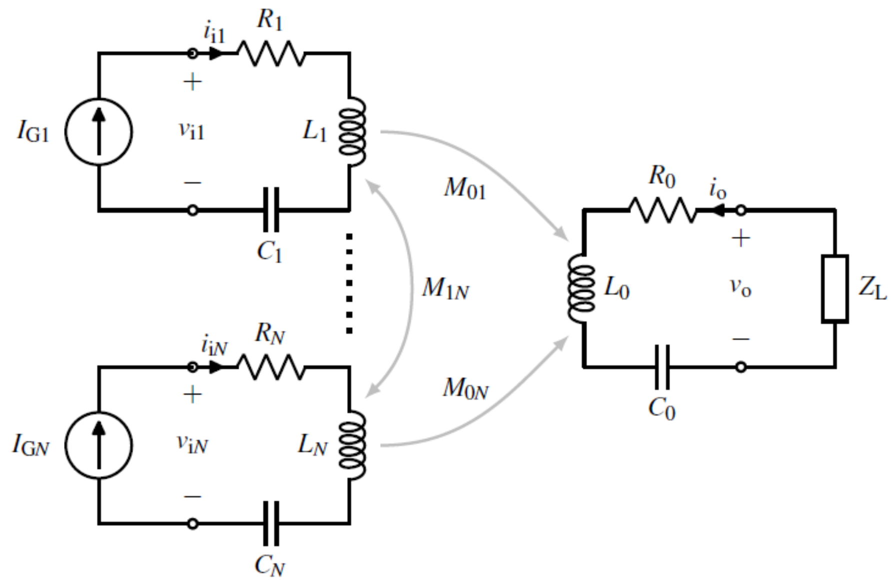
INPUT VOLTAGES CORRESPONDING TO THE OPTIMAL CONFIGURATION OF THE NETWORK

$$v_o = -\frac{\alpha}{Q_0} i_o,$$

$$\mathbf{v}_{in} = -\frac{1}{\alpha - 1}\left[\sum_{\substack{m=1 \\ m \neq n}}^{N} k_{0m} k_{nm} Q_m - j\,\alpha k_{0n}\right] i_o$$

Resonant inductive WPT: MISO configuration

$$\alpha = \sqrt{1 + Q_0 \sum_{n=1}^{N} k_{0n}^2 Q_n}$$



OPTIMAL INPUT CURRENTS

$$\mathbf{i}_{in} = j\frac{k_{0n}Q_n}{\alpha - 1} i_o \qquad (n = 1, \dots, N)$$

OPTIMAL INPUT VOLTAGES

$$\mathbf{v}_{in} = -\frac{1}{\alpha - 1}\left[\sum_{\substack{m=1\\m\neq n}}^{N} k_{0m}k_{nm}Q_m - j\,\alpha k_{0n}\right] i_o$$

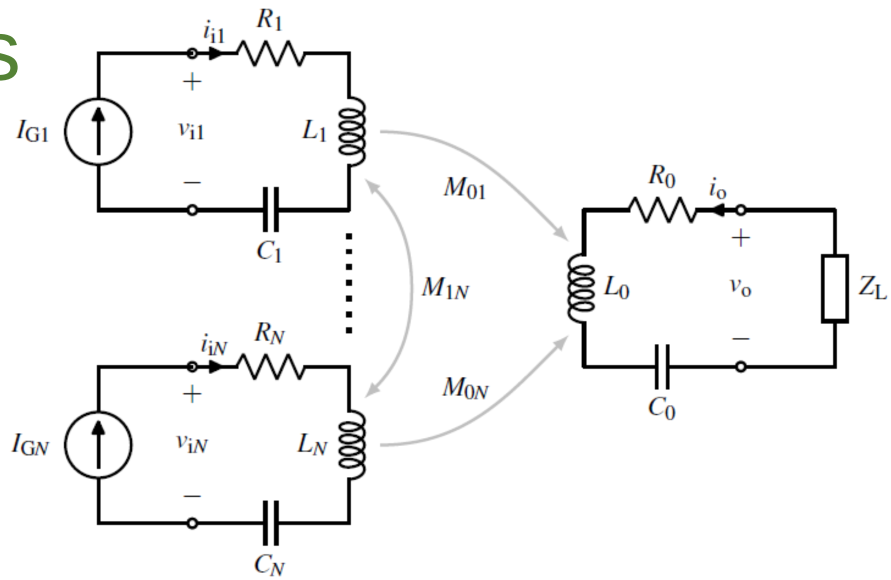
OPTIMAL LOAD IMPEDANCE

$$Z_L = \omega_0 L_0 z_L = R_0 \alpha$$

MAXIMUM REALIZABLE EFFICIENCY

$$\eta_{MAX} = \frac{\alpha - 1}{\alpha + 1}$$

Discussion of the Results



MAXIMUM EFFICIENCY

$$\eta_{MAX} = \frac{\alpha - 1}{\alpha + 1}$$

$$\alpha = \sqrt{1 + Q_0 \sum_{n=1}^{N} k_{0n}^2 Q_n}$$

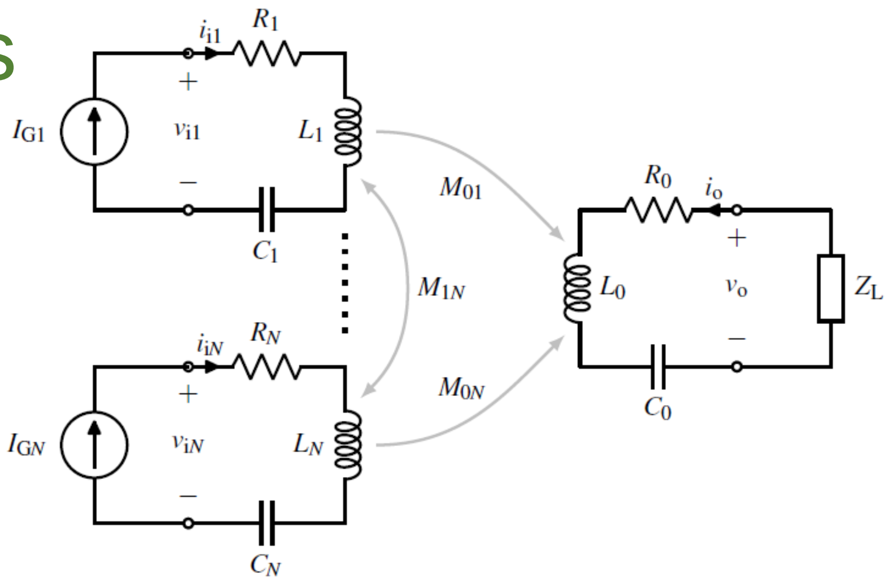
➡ The maximum efficiency of the link does not depend on possible couplings between the transmitters, this implies that these couplings can be always compensated

Discussion of the Results

OPTIMAL CURRENTS

$$\mathbf{i}_{in} = j\,\frac{k_{0n}Q_n}{\alpha - 1}\,i_o \qquad (n = 1,\ldots,N)$$

$$\alpha = \sqrt{1 + Q_0 \sum_{n=1}^{N} k_{0n}^2 Q_n}$$



➡ The optimal currents:

✓ do not depend on the couplings between the TXs,

✓ are always in phase independently of all the couplings,

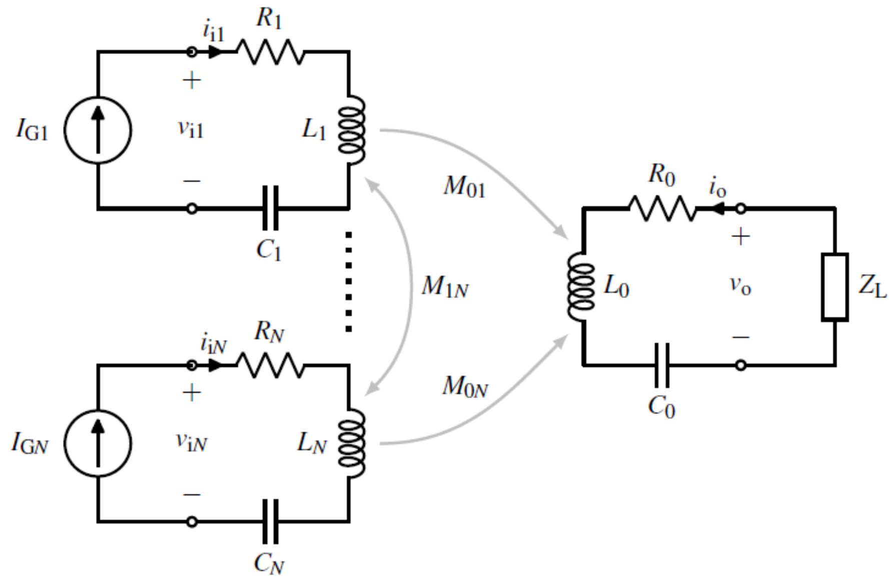
✓ have an amplitude that depends on the couplings between the TXs and the RX

Discussion of the Results

OPTIMAL LOAD IMPEDANCE

$$Z_L = \omega_0 L_0 z_L = R_0 \alpha$$

$$\alpha = \sqrt{1 + Q_0 \sum_{n=1}^{N} k_{0n}^2 Q_n}$$



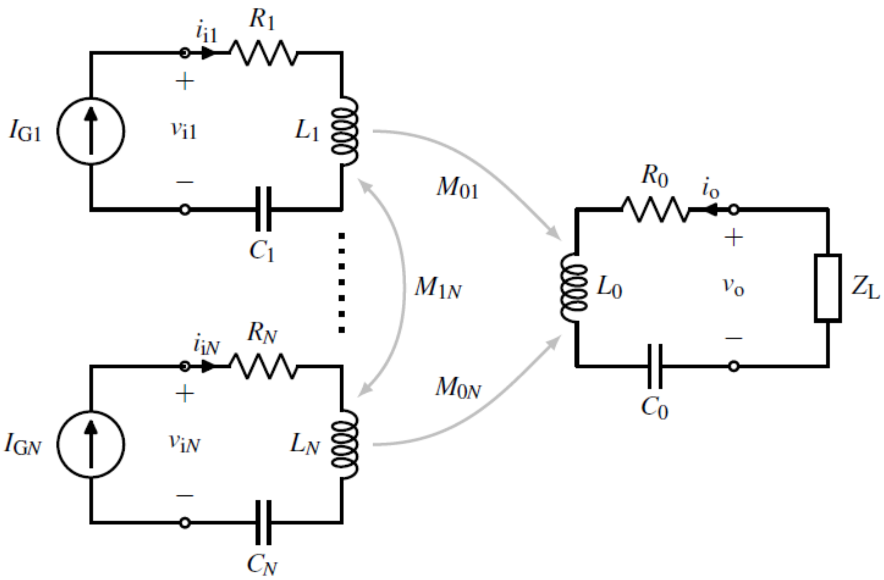
➡ The expression of the optimal load does not depend on the coupling between the TXs provided that the network is powered by the optimal currents

Discussion of the Results

OPTIMAL INPUT VOLTAGES

$$\mathbf{v}_{in} = -\frac{1}{\alpha - 1}\left[\sum_{\substack{m=1\\ m\neq n}}^{N} k_{0m}k_{nm}Q_m - j\,\alpha k_{0n}\right] i_o$$

$$\alpha = \sqrt{1 + Q_0 \sum_{n=1}^{N} k_{0n}^2 Q_n}$$



➡ The optimal input voltages

✓ depend on the couplings between the TXs,

✓ are in phase for uncoupled TXs,

✓ by adding to the input ports suitable series compensating reactances the optimal input voltages are in-phase and independent of the coupling between the TXs

Resonant inductive WPT: MISO configuration

$$\alpha = \sqrt{1 + Q_0 \sum_{n=1}^{N} k_{0n}^2 Q_n}$$

OPTIMAL INPUT CURRENTS

$$\mathbf{i}_{in} = j\,\frac{k_{0n} Q_n}{\alpha - 1}\, i_o \qquad (n = 1, \ldots, N)$$

OPTIMAL INPUT VOLTAGES

$$\mathbf{v}_{in} = -\frac{1}{\alpha - 1}\left[\sum_{\substack{m=1 \\ m \neq n}}^{N} k_{0m} k_{nm} Q_m - j\,\alpha k_{0n}\right] i_o$$

OPTIMAL LOAD IMPEDANCE

$$Z_{\mathrm{L}} = \omega_0\, L_0\, z_{\mathrm{L}} = R_0\, \alpha$$

MAXIMUM REALIZABLE EFFICIENCY

$$\eta_{\mathrm{MAX}} = \frac{\alpha - 1}{\alpha + 1}$$

According to the derived formulas it can be concluded that it is possible to obtain the maximum efficiency by simply acting on the load impedance only for the case where the TXs are uncoupled and have the same coupling with the RX. In all the other cases it is necessary to act on the input side by suitably choosing either the input voltages or currents.

Validation

In order to validate the theory the case of a resonant inductive WPT link using three transmitters has been analyzed through circuital simulations

L_0 (μH)	L_1 (μH)	L_2 (μH)	L_3 (μH)	Q	f_0 (MHz)
4.59	4.13	3.67	3.21	270.27	6.78
k_{01}	k_{02}	k_{03}	k_{12}	k_{13}	k_{23}
0.15	0.13	0.11	0.09	0.07	0.05

AWR A National Instruments Company™

NI AWR Design Environment

Validation

L_0 (µH)	L_1 (µH)	L_2 (µH)	L_3 (µH)	Q	f_0 (MHz)
4.59	4.13	3.67	3.21	270.27	6.78

k_{01}	k_{02}	k_{03}	k_{12}	k_{13}	k_{23}
0.15	0.13	0.11	0.09	0.07	0.05

Theoretical values: maximum realizable efficiency and parameters of the optimal voltage generators having series impedances $Z_{Gi} = R_{Gi} + jX_{Gi}$

η_{max}	$R_{L,opt}$	V_{G1}	V_{G2}	V_{G3}	R_{G1}	R_{G2}	R_{G3}
0.97	44.37	0.305	0.264	0.224	39.9	35.5	31.1

Reactances to be added for coupled TXs		
$X_{G1}(C_{G1})$	$X_{G2}(C_{G2})$	$X_{G3}(C_{G3})$
-22.8 (1.03)	-22.9 (1.03)	-21.2 (1.11)

Validation

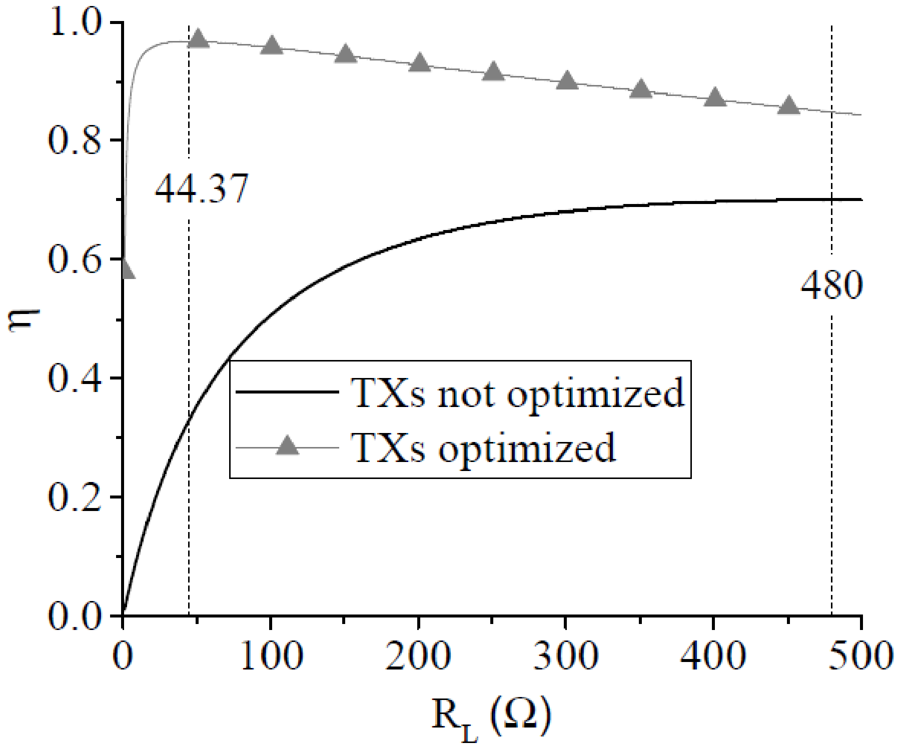
Theoretical values: maximum efficiency and optimal generators

η_{max}	$R_{L,opt}$	V_{G1}	V_{G2}	V_{G3}	R_{G1}	R_{G2}	R_{G3}
0.97	44.37	0.305	0.264	0.224	39.9	35.5	31.1

Results obtained for the analyzed numerical example when the TXs are not coupled.

The figures compares the results obtained for:
1) the case where the network is powered by in-phase unitary voltage generators, curve referred as TX not optimized,

2) the case where the network is powered by voltage generators with amplitudes and series real impedances calculated according to the theory, curve referred to as TX optimized.



Validation

Theoretical values: maximum efficiency and optimal generators

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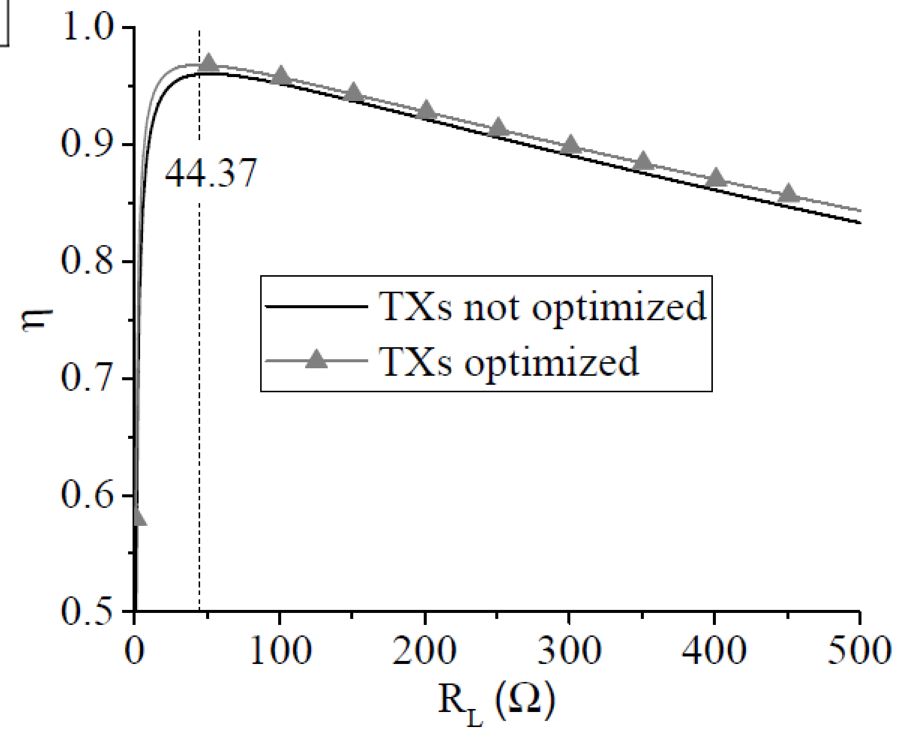
Results obtained for the analyzed numerical example when the TXs are coupled.
The figure compares the results obtained for:

1) the case where the network is powered by in-phase unitary voltage generators, curve referred as TX not optimized ;
2) The case where the network is powered by voltage generators with amplitudes and series complex impedances calculated according to the theory, curve referred to as TX optimized.



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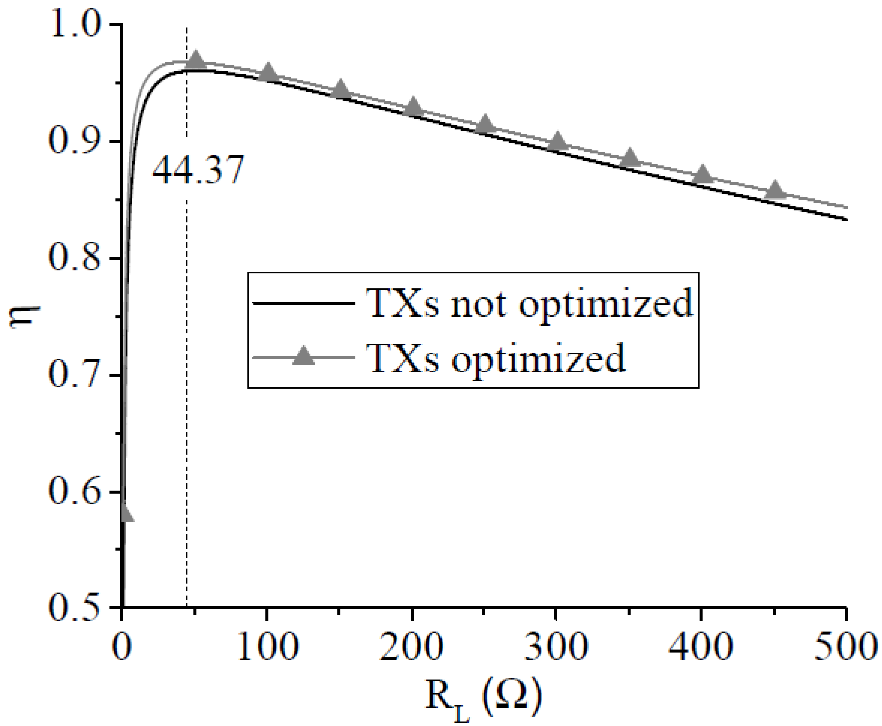

Conclusion

The case of a *resonant inductive WPT link using multiple transmitters* has been analyzed.

By solving a generalized *eigenvalue problem*, the general solution for a network with any number of transmitters and any combination of couplings has been reported and discussed.

The expression of the maximum efficiency of the link described as an (N +1)–port has been derived.

It has been shown that the
maximum efficiency can be obtained by suitably choosing
the load impedance provided that the network is powered by the optimal currents

Conclusion

According to the derived formulas:

➡ The maximum efficiency of the link does not depend on possible couplings between the transmitters, this implies that these couplings can be always compensated.

➡ The optimal currents: do not depend on the couplings between the TXs, are always in phase independently of all the couplings, have an amplitude that depends on the couplings between the TXs and the RX.

➡ The optimal input voltages: depend on the couplings between the TXs, are in phase for uncoupled TXs, have an amplitude that depends on both the couplings between the TXs and the RX and the couplings between the TXs,

➡ The coupling among the TXs can be always compensated by using suitable compensating reactances.

Thank you for reading!
Any questions?
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