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Power maximization for a multiple - input and multiple - output wireless power transfer system described by the admittance matrix

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Goal

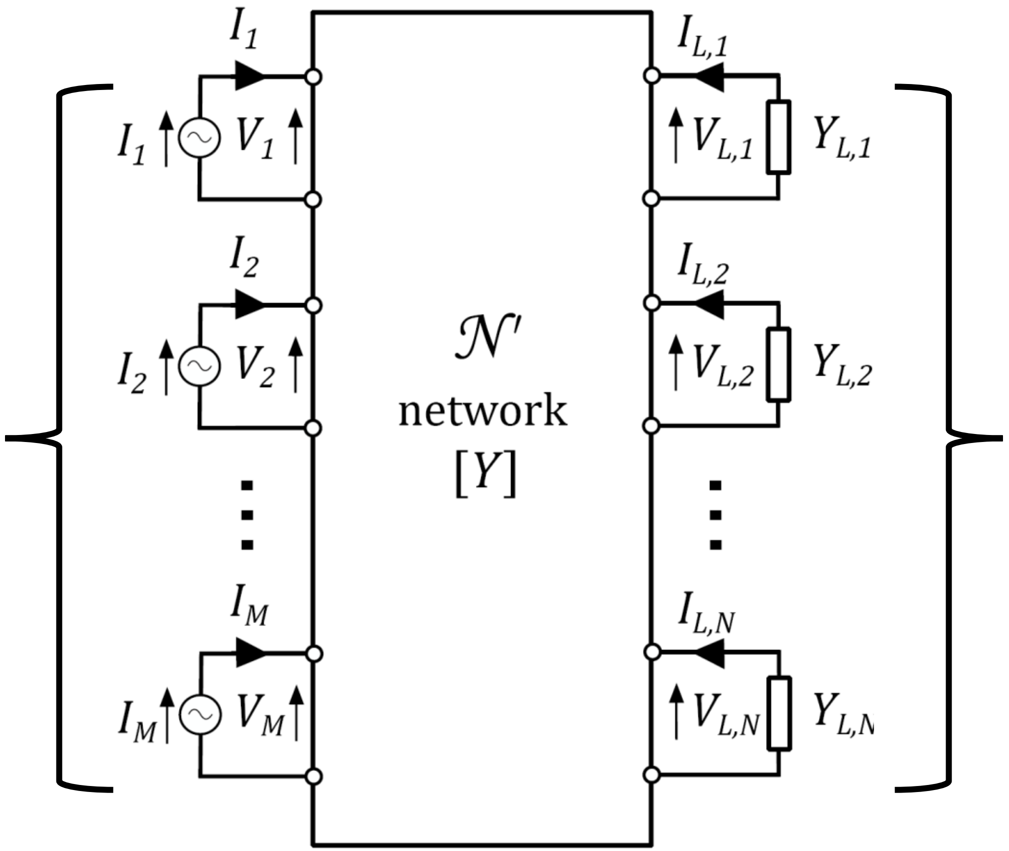
In this work, the optimal loads to maximize the power transfer for a wireless power transfer (WPT) system with *any* number of transmitters and receivers are determined.

This was already done for WPT systems characterized by their *impedance* matrix, but for certain applications (e.g. capacitive WPT), an *admittance* matrix approach is much more straightforward.

Describing the WPT system as a multiport

A multiport network $\mathcal{N}$ with M transmitters and N receivers.

The M input ports of the network are connected to M current sources.

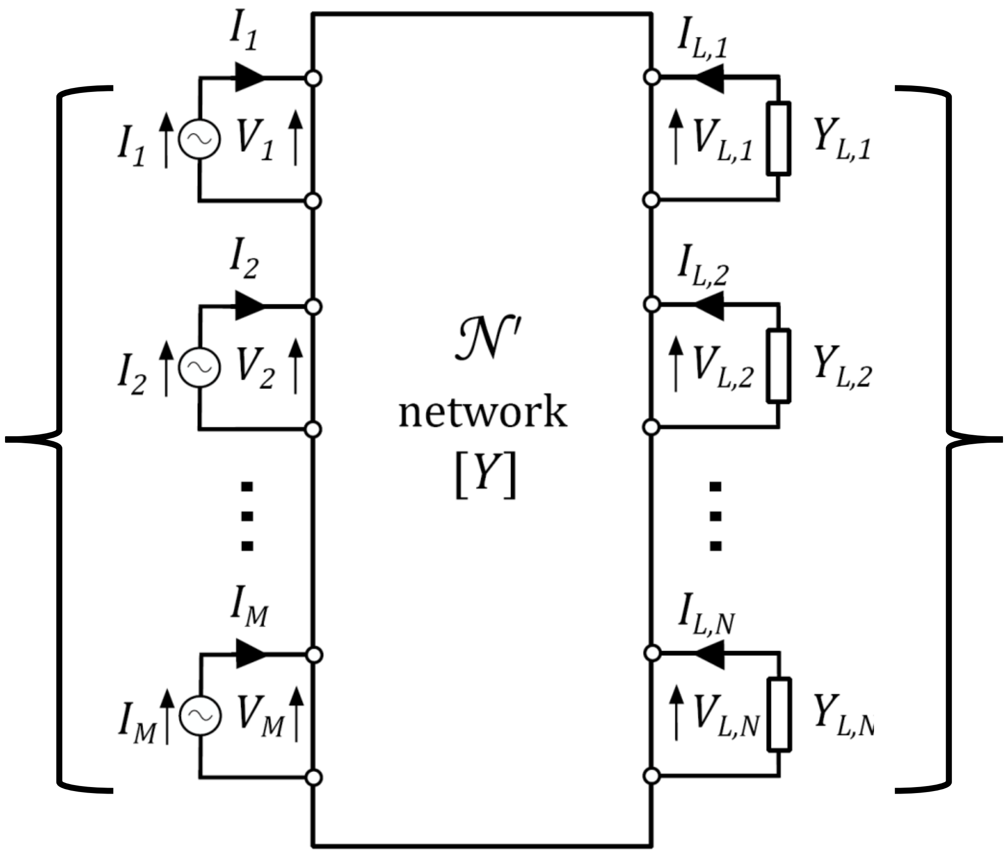


At the N output ports N load admittances $Y_{L,i}$ are present.

The relation between the voltages and the currents of the multiport can be described by an admittance matrix $\boldsymbol{Y}$ which can be partitioned in four submatrices:

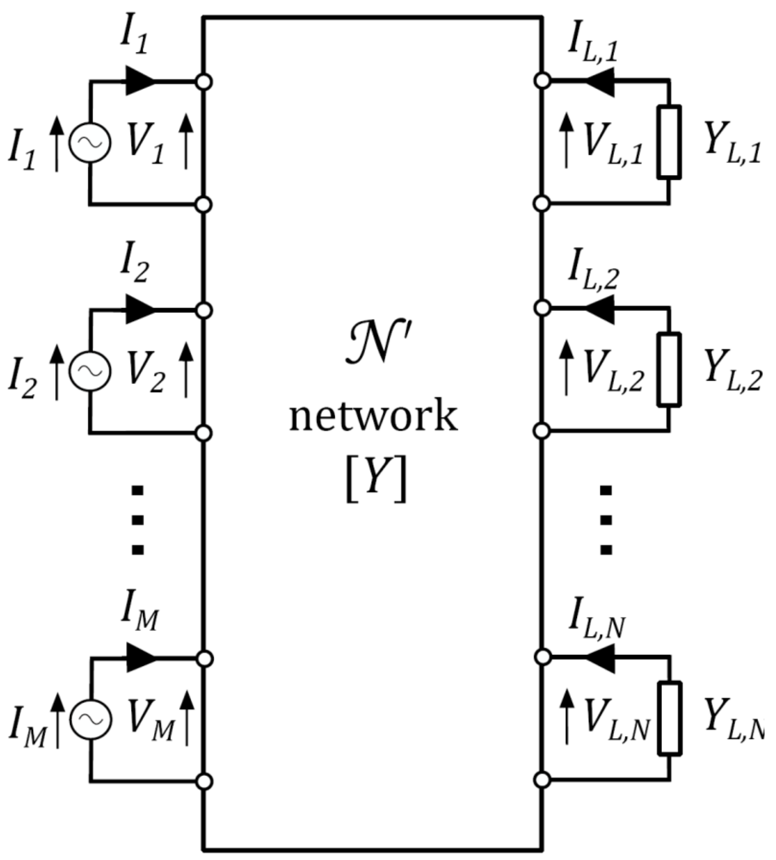
$$\begin{bmatrix} \boldsymbol{i_M} \\ \boldsymbol{i_N} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{MM} \boldsymbol{Y}_{MN} \\ \boldsymbol{Y}_{NM} \boldsymbol{Y}_{NN} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{v_M} \\ \boldsymbol{v_N} \end{bmatrix}$$

$$\boldsymbol{i_M} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_M \end{bmatrix}, \boldsymbol{v_M} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_M \end{bmatrix}$$



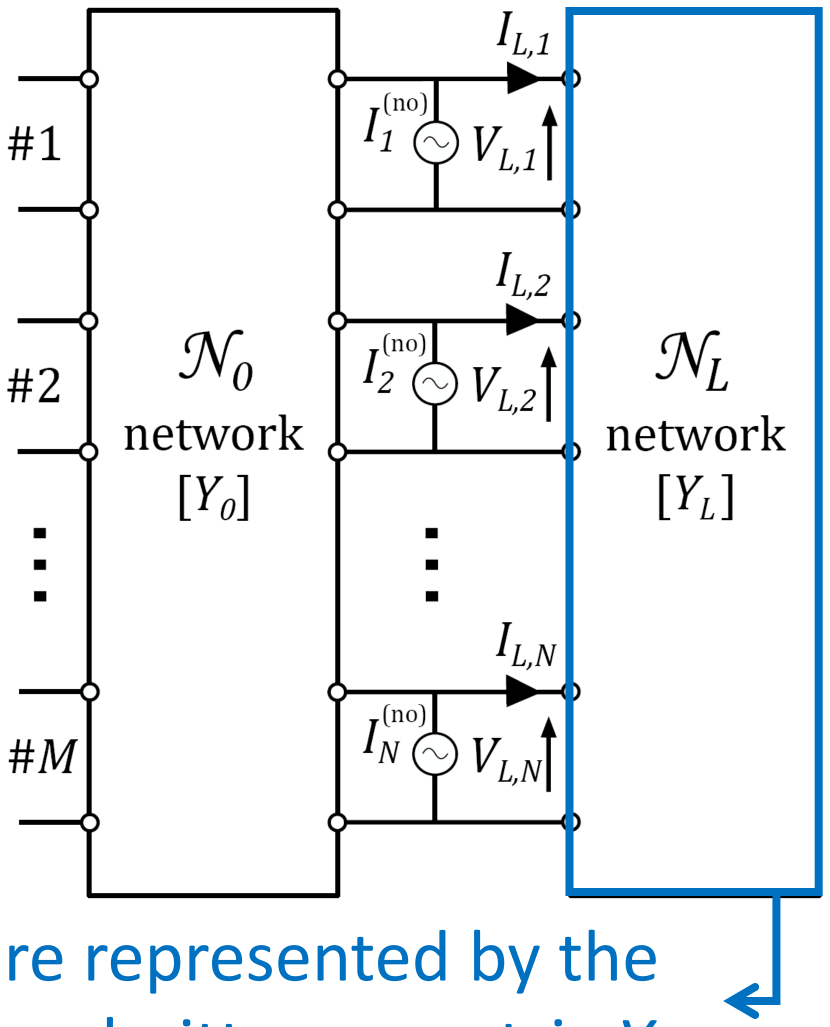
$$\boldsymbol{i_N} = \begin{bmatrix} I_{L,1} \\ I_{L,2} \\ I_{L,3} \\ \vdots \\ I_{L,N} \end{bmatrix}, \boldsymbol{v_N} = \begin{bmatrix} V_{L,1} \\ V_{L,2} \\ V_{L,3} \\ \vdots \\ V_{L,N} \end{bmatrix}$$

Norton equivalent circuit of the multiport



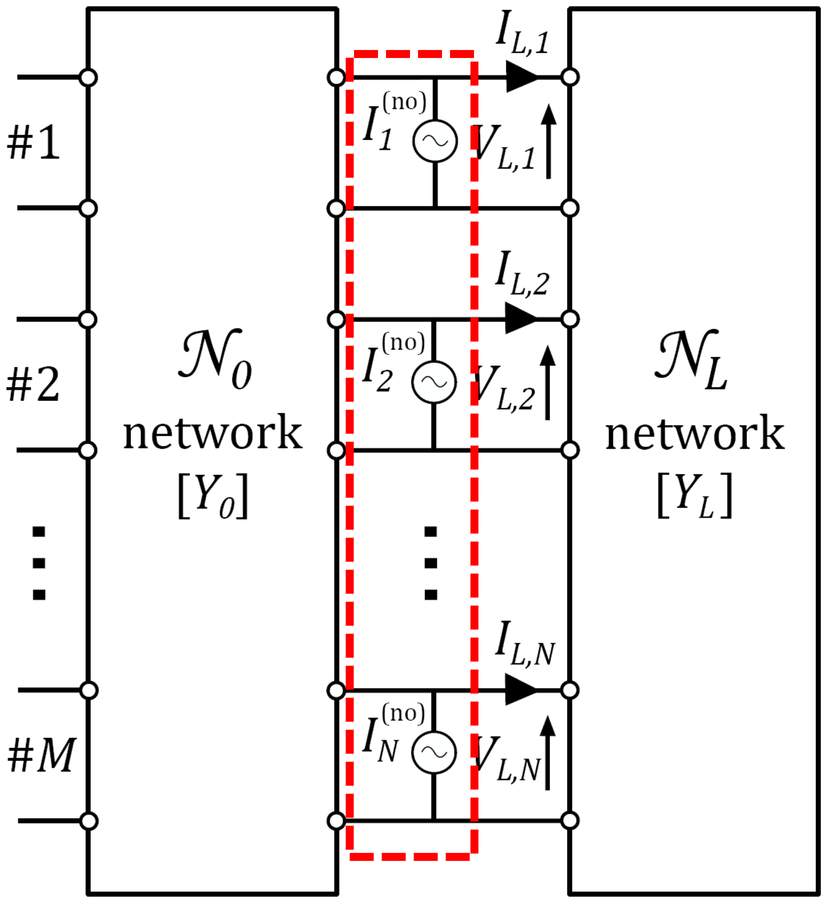
Norton’s theorem

M input ports are replaced by open circuits.



The *N* loads of the receiver are represented by the network $\mathcal{N}_L$ described by the admittance matrix Y_L.

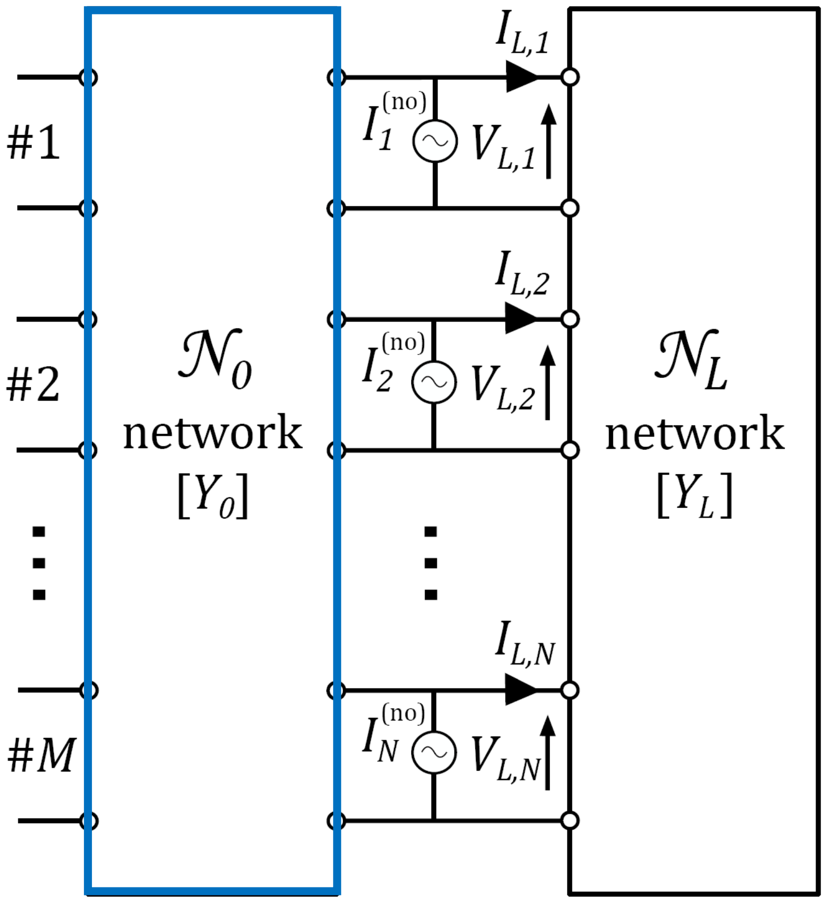
Norton equivalent circuit of the multiport



$I_i^{(no)}$ are the Norton currents, given by:

$$\boldsymbol{i_N} = \boldsymbol{Y}_{NM}\boldsymbol{Y}_{MM}^{-1}.\boldsymbol{i_M} \equiv \boldsymbol{i_N}^{(no)} = \begin{bmatrix} I_1^{(no)} \\ I_2^{(no)} \\ I_3^{(no)} \\ \vdots \\ I_N^{(no)} \end{bmatrix}$$

Norton equivalent circuit of the multiport



The Norton admittance matrix $\boldsymbol{Y}_0$ which characterizes network $\mathcal{N}_0$ is defined by

$$\boldsymbol{i}_N = \boldsymbol{Y}_0 . \boldsymbol{v}_N$$

As function of the original admittance matrix $\boldsymbol{Y}$,
$\boldsymbol{Y}_0$ is given by:

$$\boldsymbol{Y}_0 = \boldsymbol{Y}_{NN} - \boldsymbol{Y}_{NM} . \boldsymbol{Y}_{MM}^{-1} . \boldsymbol{Y}_{MN}$$

Optimal loads for power maximization

The goal of this work is to determine the loads that realize maximum power transfer from the *M* transmitters to the *N* receivers, i.e. that maximize the total output power P_{out} defined as

$$P_{out} = \sum_{i=1}^{N} P_i$$

with P_i the output power delivered to load $Y_{L,i}$.

Voltage condition for achieving maximum power transfer to loads [*]:

$$\boldsymbol{v}_N = (\boldsymbol{Y}_0 + \boldsymbol{Y}_0^+)^{-1} . \boldsymbol{i}^{(no)}$$

with $\boldsymbol{Y}_0^+$ the conjugate transpose of $\boldsymbol{Y}_0$.

This results in the current condition for the loads at the maximum power configuration:

$$\boldsymbol{i}_N = \boldsymbol{i}^{(no)} - \boldsymbol{Y}_0 . (\boldsymbol{Y}_0 + \boldsymbol{Y}_0^+)^{-1} . \boldsymbol{i}^{(no)}$$

$\Rightarrow$ The optimal loads are given by $Y_{L,i} = \dfrac{I_{L,i}}{V_{L,i}}$

with $V_{L,i}$ and $I_{L,i}$ the elements of $\boldsymbol{v}_N$ and $\boldsymbol{i}_N$.

[*] H. Baudrand, “On the generalizations of the maximum power transfer theorem,” Proceedings of the IEEE, 58, 10, Oct., pp. 1780-1, 1970.

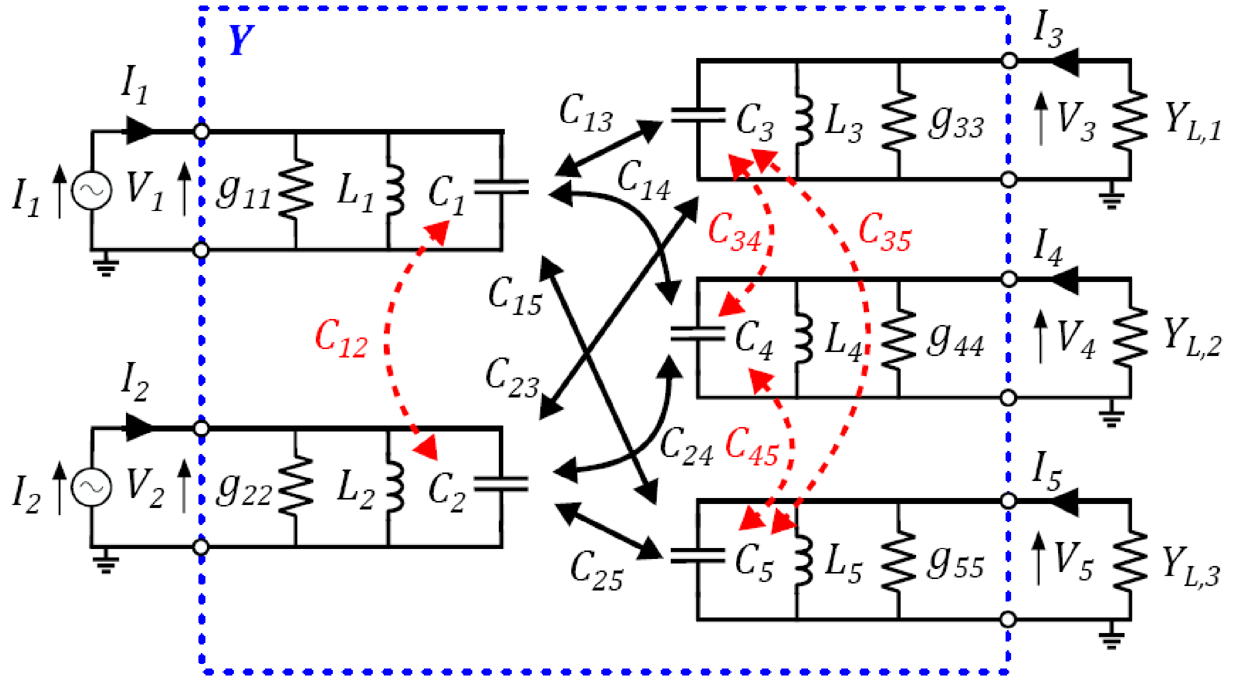
Overview procedure

The general procedure to find the loads for power maximization for a WPT system with *any* number of transmitters and receivers:

1. Establish (e.g., by measurement or simulation) the admittance matrix Y of the network.
2. Determine the Norton current sources $I_i^{(no)}$.
3. Set up the Norton admittance matrix Y_0.
4. Calculate the voltages v_N and currents i_N for the loads at the maximum power configuration.
5. Determine the optimal loads $Y_{L,i}$ from these voltages and currents.

Example: capacitive WPT with 2 transmitters and 3 receivers

Two current sources I_1 and I_2 power the system with operating angular frequency ω_0.



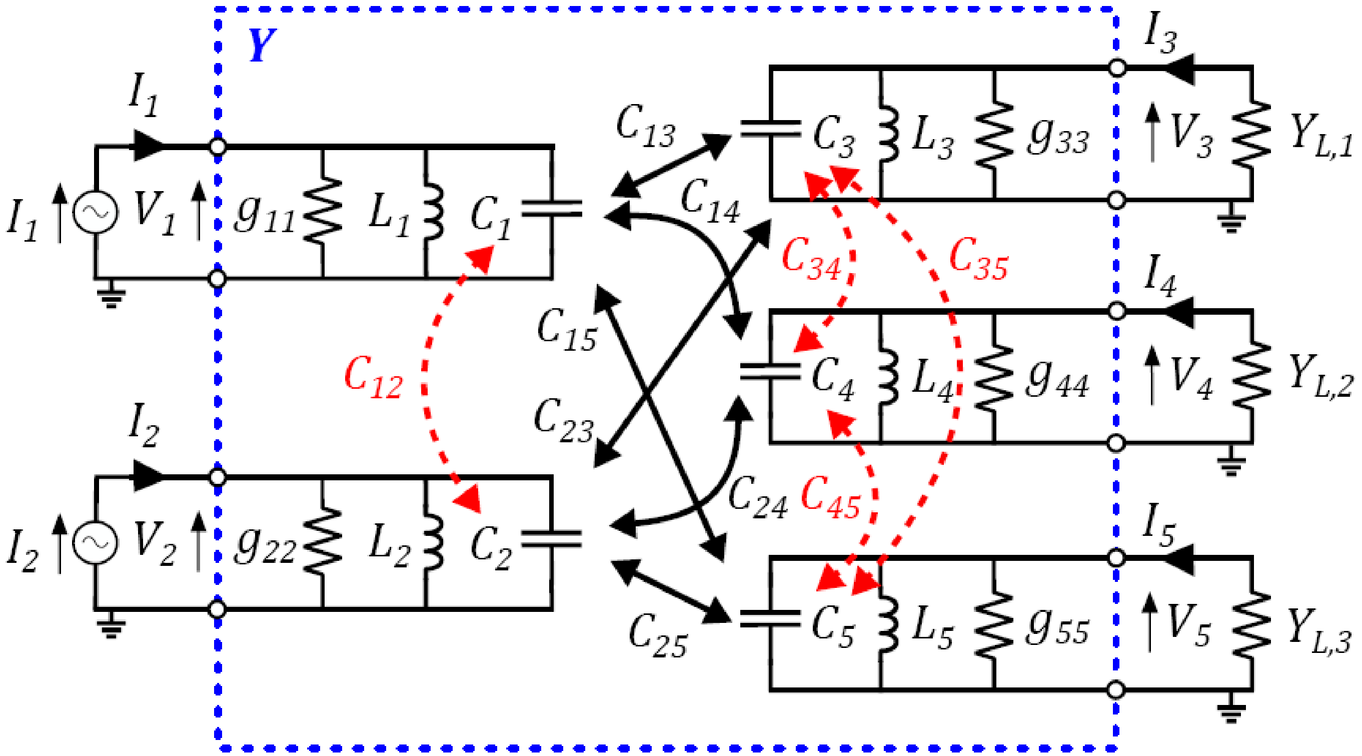
At the 3 output ports load admittances $Y_{L,1}$, $Y_{L,2}$ and $Y_{L,3}$ are connected.

The shunt conductances g_{jj} (j=1,…,5) describe the losses in the circuit.

The mutual capacitances C_{13}, C_{14}, C_{15}, C_{23}, C_{24} and C_{25} represent the **desired** electric coupling between the transmitter capacitances C_1, C_2, and the receiver capacitances C_3, C_4, C_5.

Undesired electric coupling is present between both transmitters, indicated by the mutual capacitance C_{12}.

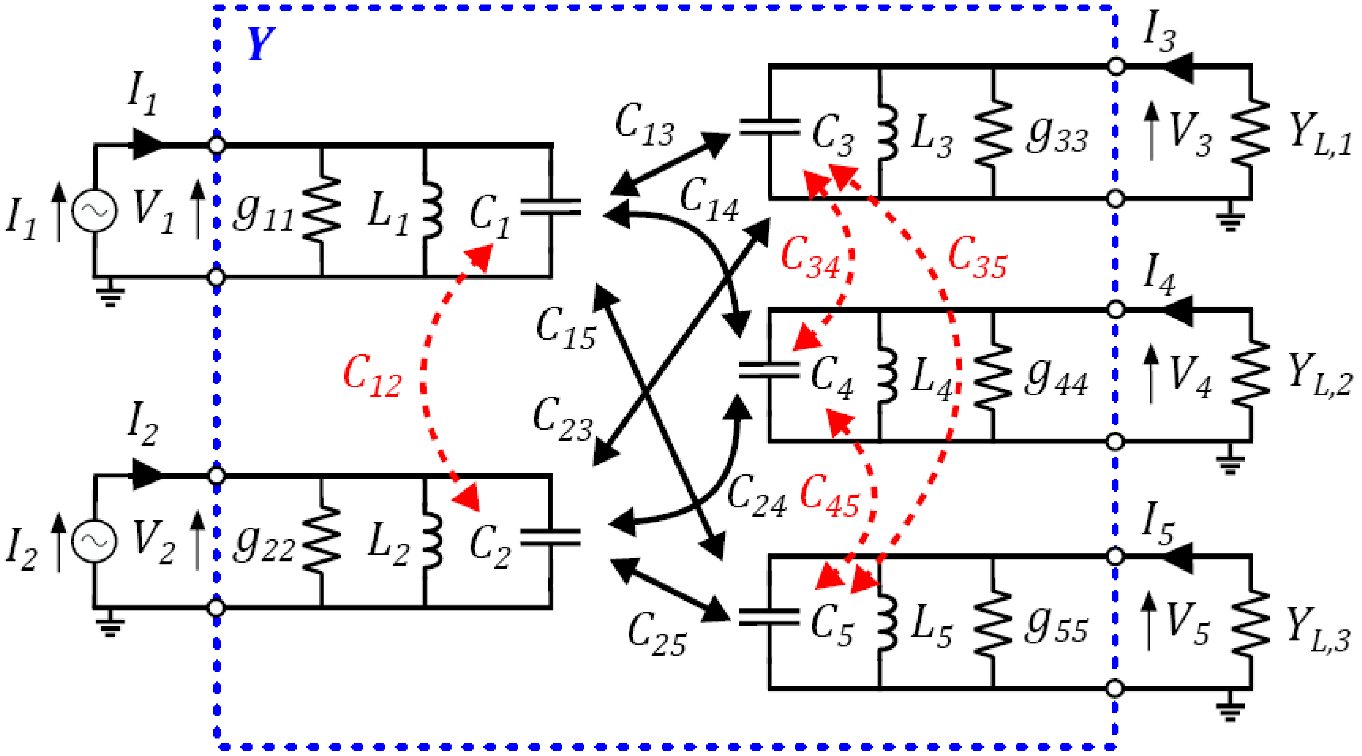
Also between the receivers, an undesired coupling is present: C_{34}, C_{35} and C_{45}.



In order to obtain a resonant scheme, the inductors L_j are added: $L_j = \frac{1}{\omega_0^2 C_j}$

The coupling factor k_{ij} is defined as $k_{ij} = \frac{C_{ij}}{\sqrt{C_i C_j}}$

(i,j=1,…,5)

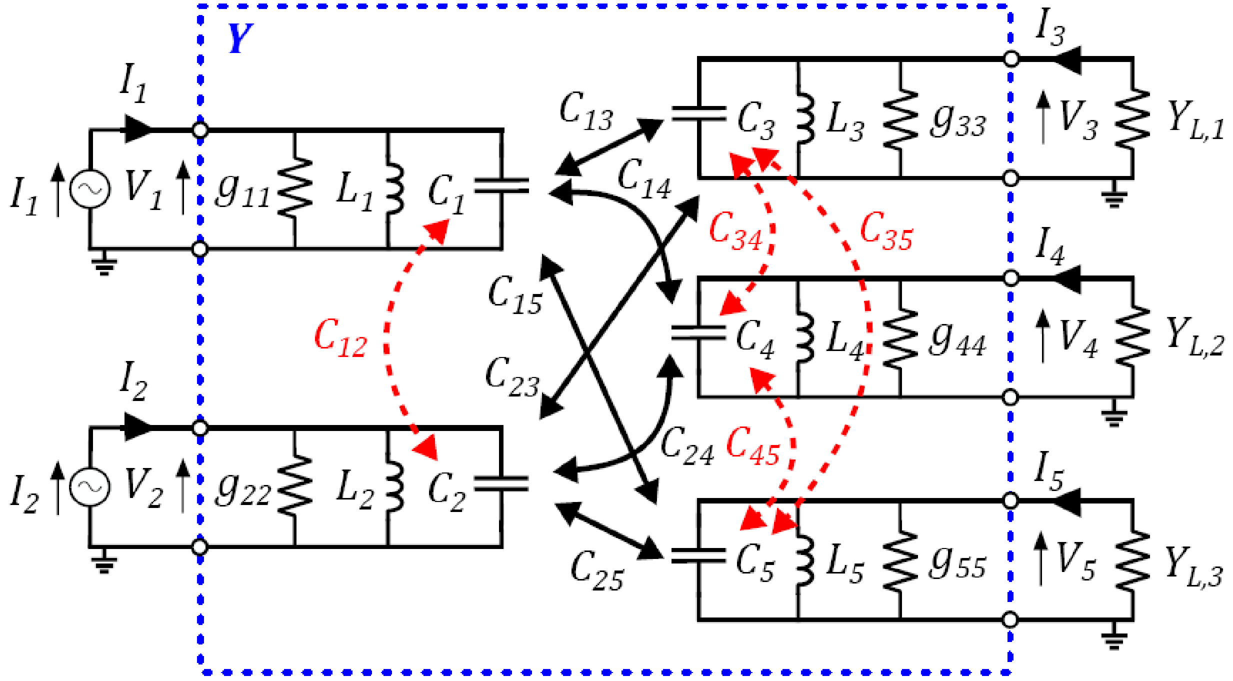


The entire multiport system (indicated by the dashed rectangle) is fully determined by the admittance matrix Y which is, at the resonance angular frequency ω_0, equal to:

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{Y}_{MM} \boldsymbol{Y}_{MN} \\ \boldsymbol{Y}_{NM} \boldsymbol{Y}_{NN} \end{bmatrix} =$$

$$= \begin{bmatrix} g_{11} & -jb_{12} & -jb_{13} & -jb_{14} & -jb_{15} \\ -jb_{12} & g_{22} & -jb_{23} & -jb_{24} & -jb_{25} \\ -jb_{13} & -jb_{23} & g_{33} & -jb_{34} & -jb_{35} \\ -jb_{14} & -jb_{24} & -jb_{34} & g_{44} & -jb_{45} \\ -jb_{15} & -jb_{25} & -jb_{35} & -jb_{45} & g_{55} \end{bmatrix}$$

with $b_{ij} = \omega_0 C_{ij}$.



In order to verify the analytical results, **circuital simulations** have been performed in SPICE with the following example values:

Quantity	Value	Quantity	Value
g_{11}	1.00 mS	C_1	350 pF
g_{22}	1.25 mS	C_2	300 pF
g_{33}	1.50 mS	C_3	250 pF
g_{44}	1.75 mS	C_4	225 pF
g_{55}	2.00 mS	C_5	200 pF
I_1	100 mA	f_0	10 MHz
I_2	200 mA		

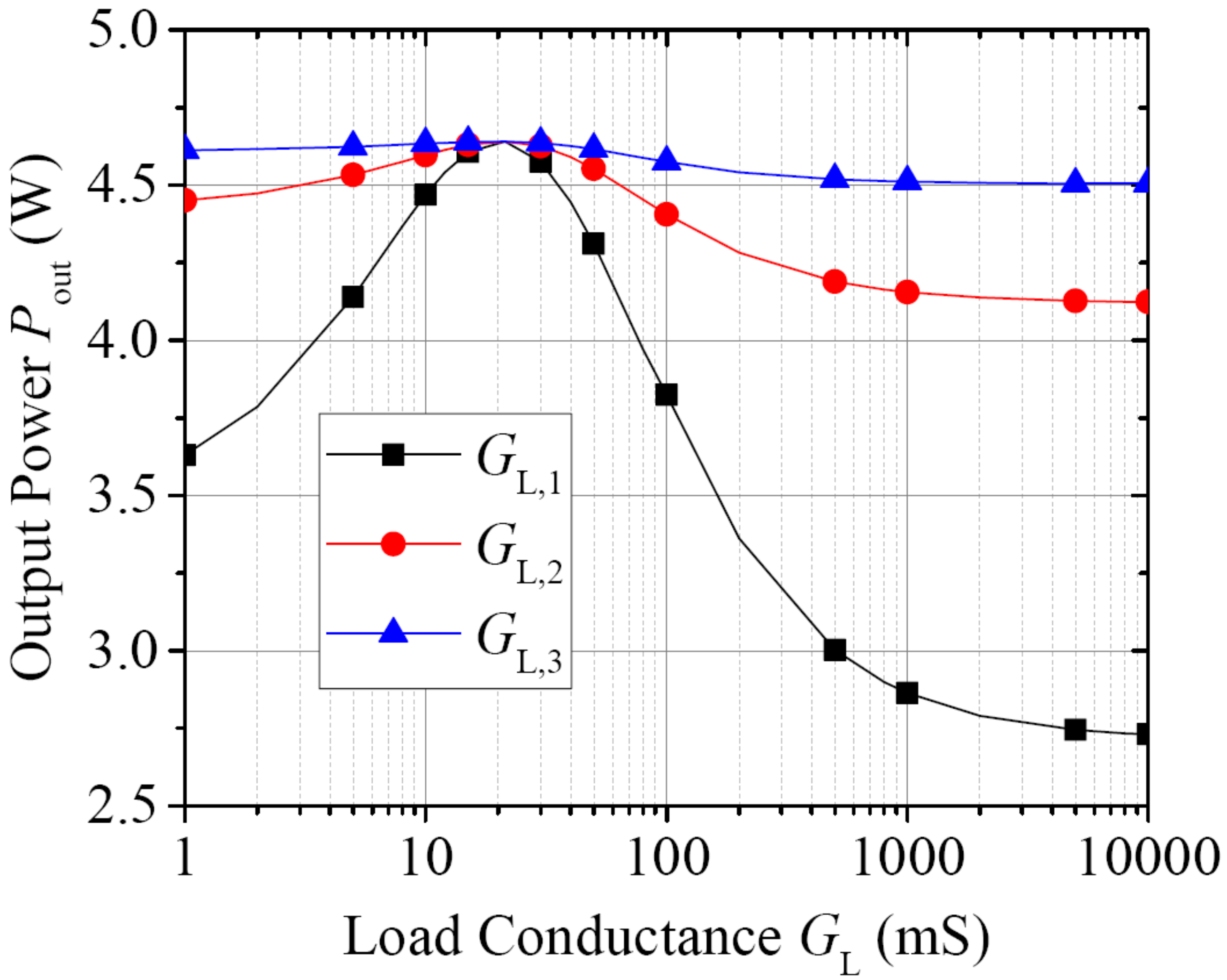
Desired coupling	Value	Undesired coupling	Value
k_{13}	30 %	k_{12}	10 %
k_{14}	25 %	k_{34}	5 %
k_{15}	20 %	k_{35}	2 %
k_{23}	25 %	k_{45}	5 %
k_{24}	20 %		
k_{25}	15 %		

Optimal terminating admittances according to the developed theory are:

$G_{L,1}$ (mS)	L_{L1} (µH)	$G_{L,2}$ (mS)	L_{L2} (µH)	$G_{L,3}$ (mS)	L_{L3} (µH)
21.4	524.0	23.0	443.0	22.6	370

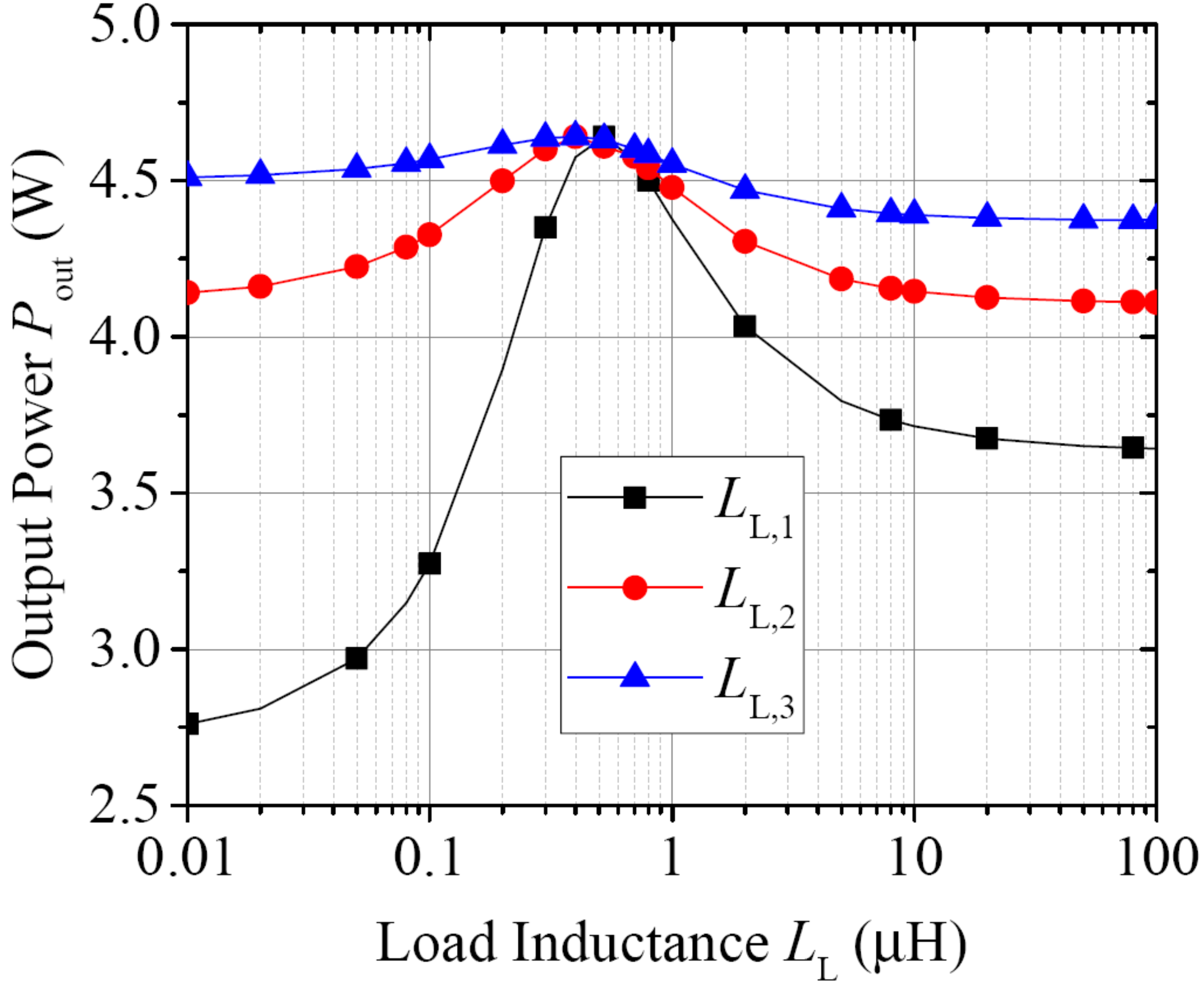
First, a simulation with the network terminated on the optimal admittances returns an output power of 4.64 W.

Next, simulations were performed by varying one **load conductance** $G_{L,i}$ at a time while keeping all the others constant at their optimal value.



The results confirm the data provided by the theory for this example.

Next, simulations were performed by varying one **load inductance** $L_{L,i}$ at a time while keeping all the others constant at their optimal value.



The results confirm the data provided by the theory for this example.

Conclusion

A general procedure was shown to easily determine the terminating loads that maximize power transfer for a WPT system

- with *any* number of transmitters or receivers,
- characterized by its *admittance* matrix.

Thank you for reading

Any questions? Mail me at ben.minnaert@odisee.be