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## Electromagnetic interaction with a monodisperse systems in sedimentation equilibrium

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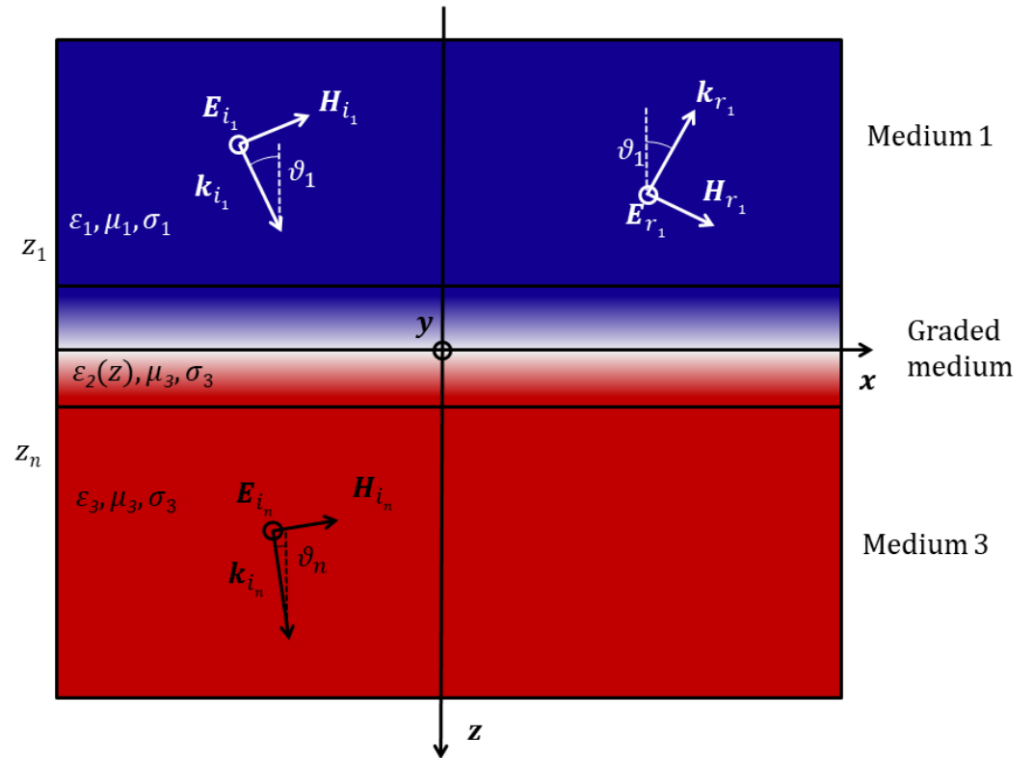
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# Introduction

Sedimentation is the phenomenon that Brownian particles attain a certain velocity under the action of an external field. Sedimentation in the gravity field causes the concentration differences of particles from the surface to the bottom of the dispersion system. The particle concentration increases with the increase in the depth of the dispersion system. These differences will lead the diffusion of particles from the bottom to the surface. When the diffusion velocity equals the settling velocity, the dispersion system reaches the equilibrium state, which is referred to as sedimentation equilibrium



# Why?

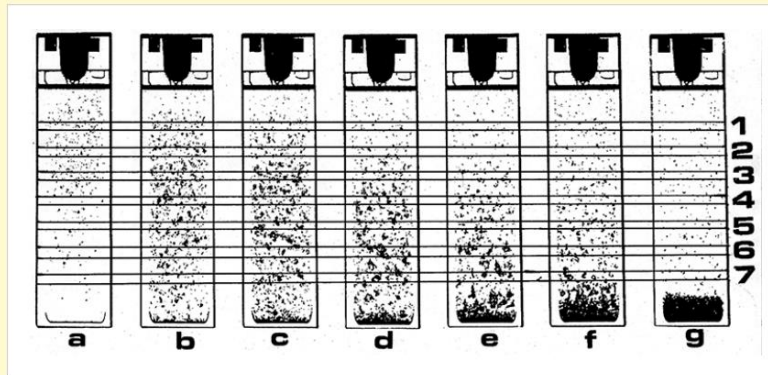
The work is motivated by the large number of applications in the biological and chemical fields

## Colloidal Aggregation Coupled with Sedimentation: A Comprehensive Overview

By Agustín E. González

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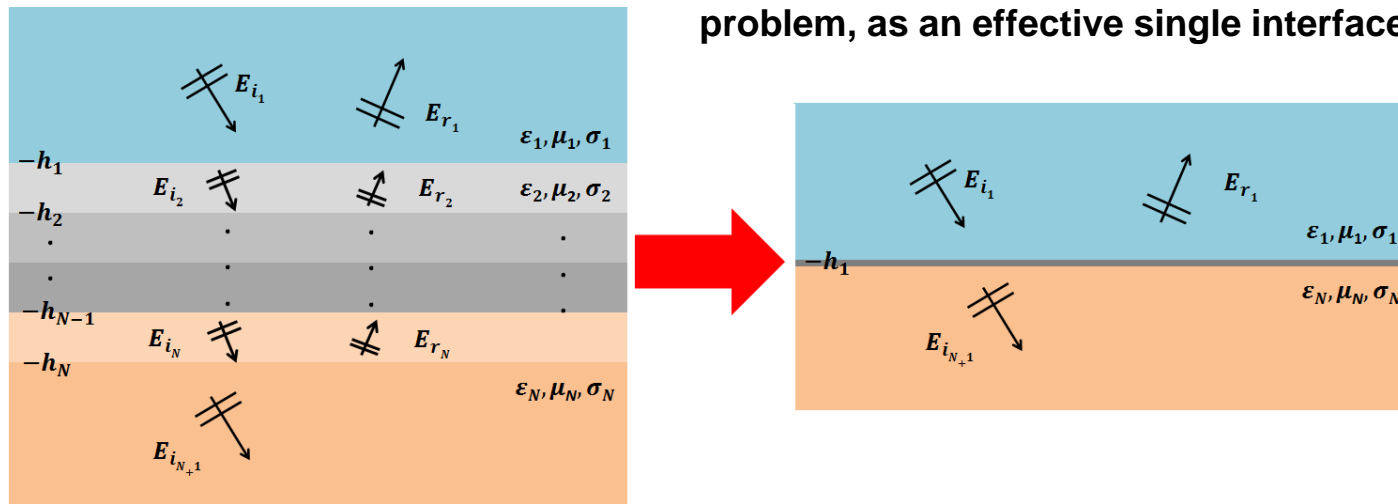
**Figure 6.**

The time evolution of a calcium carbonate aggregating suspension of spherical particles, for the following times: (a) 5 min and 20 s, (b) 6 min and 20 s, (c) 6 min and 40 s, (d) 7 min, (e) 7 min and 20 s, (f) 7 min and 40 s, and (g) 8 min (from Ref. [36]).

# Approach to the model

To deal an electromagnetic model about the electromagnetic interaction with a systems in sedimentation equilibrium, we assume the layers are characterized by a gradual gradient of the reflective index is modeled by a stratified medium. The transmission through the stratified medium is determined by the well-known formalism of the T-Matrix. This matrix has been firstly introduced by Abelès.

Thanks to the T-Matrix formalism, the layered medium can be taken into account, for what concerns of our problem, as an effective single interface



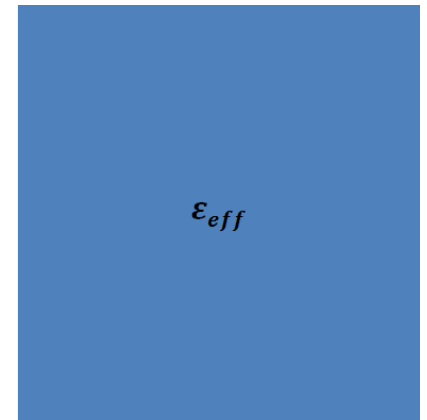
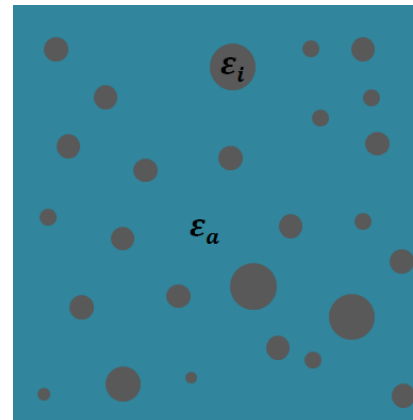
# Approach to the model

Each layer is constituted by a mixture of dielectric media, i.e. a dielectric host containing a particular volume fraction of spherical objects, in order to simulate a colloidal system. Working with a wavelength of the incident field more ten times greater than the maximum radius of the spherical object, we can modeling each layer with the homogenization models, in particular by the Maxwell-Wagner formula

Quasi-static approximation

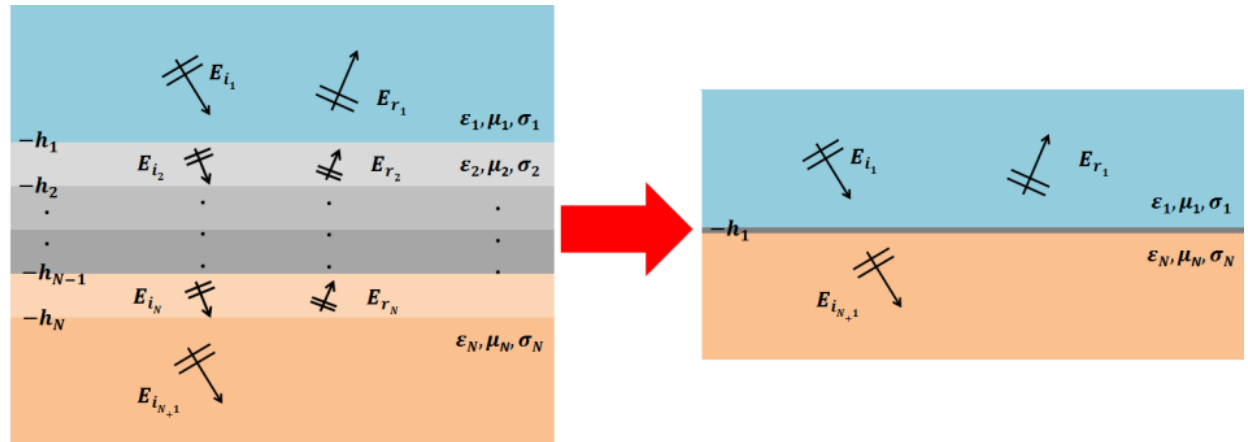
$$d \ll \lambda$$

The wavelength of the external electromagnetic field is much smaller than the geometric dimensions of the inclusion. So we can consider the inhomogeneous medium as a effective homogeneous media.



# Theoretical approach

Looking at the problem from the point of view of multiple reflections on the interfaces between the dielectrics, we expect that, in the  $j$ -th layer, two plane waves propagate, one in the forward direction, (superposition of all the secondary reflected waves), and the second one in backward direction (viveversa)



In order to obtain the Transmission Matrix we have to impose the boundary conditions on each interface

$$\mathbf{z}_0 \times (\mathbf{E}_{i_j} + \mathbf{E}_{r_j} - \mathbf{E}_{i_{j+1}} - \mathbf{E}_{r_{j+1}}) = 0 \quad \text{for} \quad z = z_j$$

$$\mathbf{z}_0 \times (\mathbf{H}_{i_j} + \mathbf{H}_{r_j} - \mathbf{H}_{i_{j+1}} - \mathbf{H}_{r_{j+1}}) = 0 \quad \text{for} \quad z = z_j$$

Substituting the equations of the fields in the B.C., we obtain the following linear system

$$\begin{cases} E_{0i_j} + E_{0r_j} = E_{0i_{j+1}} e^{-ik_{i_{j+1}n}(z_{j+1}-z_j)} + E_{0r_{j+1}} e^{ik_{i_{j+1}n}(z_{j+1}-z_j)} \\ E_{0i_j} - E_{0r_j} = \zeta_{j,j+1} \left[ E_{0i_{j+1}} e^{-ik_{i_{j+1}n}(z_{j+1}-z_j)} - E_{0r_{j+1}} e^{ik_{i_{j+1}n}(z_{j+1}-z_j)} \right] \end{cases}$$

# Theoretical approach

The pervious linear system can be written in matrix form

$$\begin{pmatrix} E_{0i_{N+1}}^E \\ 0 \end{pmatrix} = \prod_{\ell=N}^1 [M_\ell] \begin{pmatrix} E_{0i_1}^E \\ E_{0r_1}^E \end{pmatrix} = [M] \begin{pmatrix} E_{0i_1}^E \\ E_{0r_1}^E \end{pmatrix}$$



$$[M_j] = \frac{1}{2\zeta_j} \begin{bmatrix} (\zeta_j + 1)e^{i\phi_{j+1}} & (\zeta_j - 1)e^{i\phi_{j+1}} \\ (\zeta_j - 1)e^{-i\phi_{j+1}} & (\zeta_j + 1)e^{-i\phi_{j+1}} \end{bmatrix}$$

$$\begin{pmatrix} E_{0i_{N+1}}^H \\ 0 \end{pmatrix} = \prod_{\ell=N}^1 [N_\ell] \begin{pmatrix} E_{0i_1}^H \\ E_{0r_1}^H \end{pmatrix} = [N] \begin{pmatrix} E_{0i_1}^H \\ E_{0r_1}^H \end{pmatrix}$$

$$[N_j] = \frac{1}{2\chi_j} \begin{bmatrix} (\chi_j + 1)e^{i\phi_{j+1}} & (\chi_j - 1)e^{i\phi_{j+1}} \\ (\chi_j - 1)e^{-i\phi_{j+1}} & (\chi_j + 1)e^{-i\phi_{j+1}} \end{bmatrix}$$

with

$$R_E^E = \frac{E_{r_1}^E}{E_{i_1}^E} = -\frac{M_{21}}{M_{22}}$$

$$T_E^E = \frac{E_{i_{N+1}}^E}{E_{i_1}^E} = \frac{\det[M]}{M_{22}}$$

$$\zeta_j = \frac{\mu_j k_{i_{j+1}n}}{\mu_{j+1} k_{i_{jn}}}$$

$$R_H^E = \frac{H_{r_1}^E}{H_{i_1}^E} = R_E^E = -\frac{M_{21}}{M_{22}}$$

$$T_H^E = \frac{H_{i_{N+1}}^E}{H_{i_1}^E} = \frac{Z_1^* \det[M]}{Z_{N+1}^* M_{22}}$$

$$\chi_j = \frac{\varepsilon_j k_{i_{j+1}n}}{\varepsilon_{j+1} k_{i_{jn}}}$$



Under determine hypothesis, it is possible to consider each layer as an “effective medium” and to use the Maxwell-Wagner model to determinate its effective permittivity.

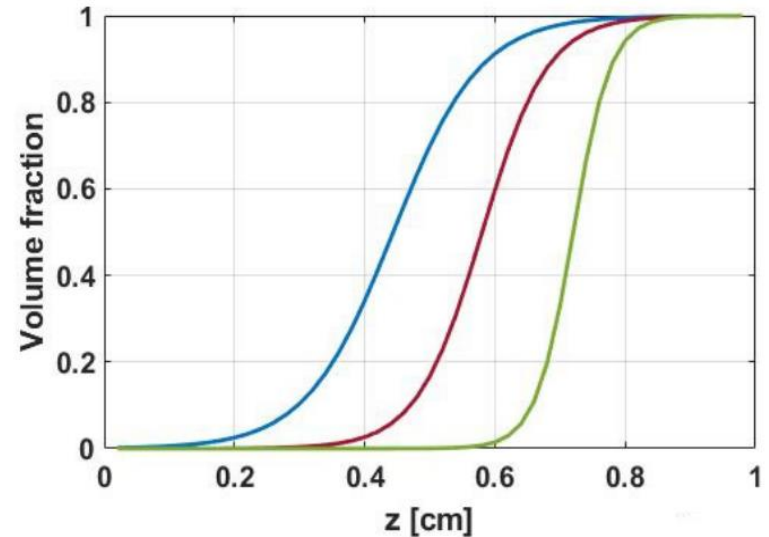
$$\varepsilon_{effj} = \varepsilon_{h_j} \frac{2\varepsilon_{h_j} + \varepsilon_{s_j} - 2\phi(\varepsilon_{h_j} - \varepsilon_{s_j})}{2\varepsilon_{h_j} + \varepsilon_{s_j} + \phi(\varepsilon_{h_j} - \varepsilon_{s_j})}$$

# Results

We suppose that the concentration particles varies according to the following law of the total force equilibrium

$$\frac{\partial \mu^{id}}{\partial z} + \frac{\partial \mu^{ex}}{\partial z} = \nu(\rho - \rho_F)g$$
$$\frac{1}{\phi} \frac{\partial \phi}{\partial z} + \beta \frac{\partial \mu^{ex}}{\partial \phi} \frac{\partial \phi}{\partial z} = -\frac{1}{L_g}$$

where  $\mu$  is the chemical potential which can be separated into an ideal contribution  $\mu_{ex}$  and an excess contribution  $\mu_{ex}$  of a type particle suspended in a continuum fluid,  $\rho$  is the particle mass density,  $\nu$  is the particle volume,  $g$  is the gravity acceleration, and  $L_g$  is the gravitational length.



Trend of the volume fraction along the stratified direction of a monodispersed systems in three different sedimentation equilibrium. We take the following parameters from [9]  $\rho = 1.19 \text{ g/cm}^3$ ,  $\rho_F = 1.04 \text{ g/cm}^3$ ,  $L_g = 2.94 \text{ mm}$ .



# Results

Applying an circular polarized plane wave of 1 V/m of intensity at wavelength  $\lambda = 1 \mu\text{m}$  and varying the incident angle from perpendicular ( $0^\circ$ ) to parallel ( $90^\circ$ ) direction, we obtained the following trend of the reflectivity for both polarization on a monodispersed systems in different sedimentation equilibrium degree

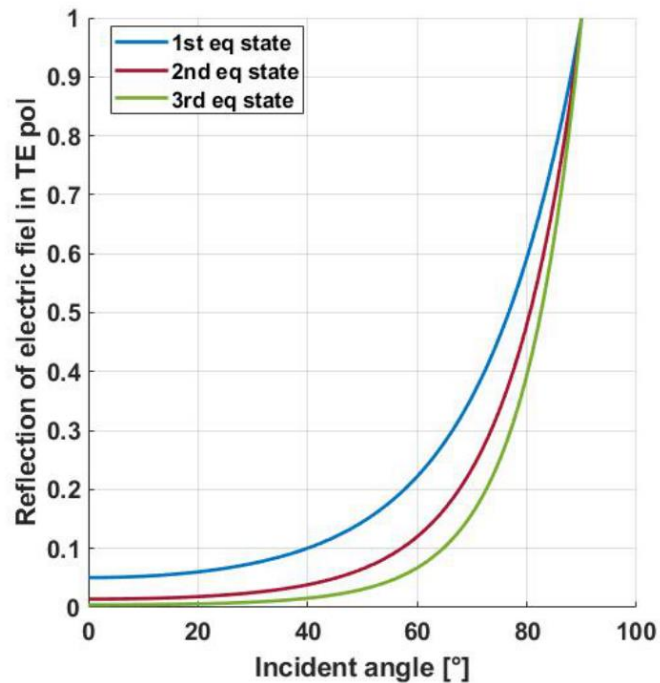


Fig. 3. Trend of the reflectivity for TE-polarization on a monodispersed systems as a function of the incident angle (between 0 and  $\pi/2$  rad) for three different sedimentation equilibrium state (see Fig. 2).

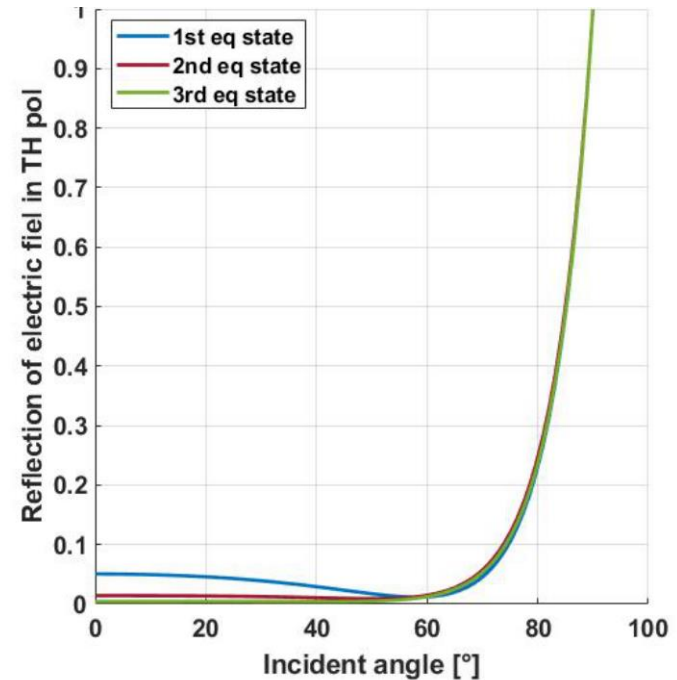


Fig. 4. Trend of the reflectivity for TH-polarization on a monodispersed systems as a function of the incident angle (between 0 and  $\pi/2$  rad) for three different sedimentation equilibrium state (see Fig. 2).

# Conclusions

In conclusion a new model to explain the electromagnetic interaction with a monodispersed systems in sedimentation equilibrium is been shown. From the preliminary results presented, the model well represents a monodispersive system in sedimentation equilibrium. Furthermore, it has been ascertained how such a system is more sensitive to the type of electromagnetic polarization which excites the system. In particular, it has been seen how TE polarization can be used to determine the sedimentation state of this complex system. This procedure could easily find application in various areas, including biology, chemical, and cultural heritage.

**END**

Thanks for the attention