Statistical model for MIMO propagation channel in cavities and random media

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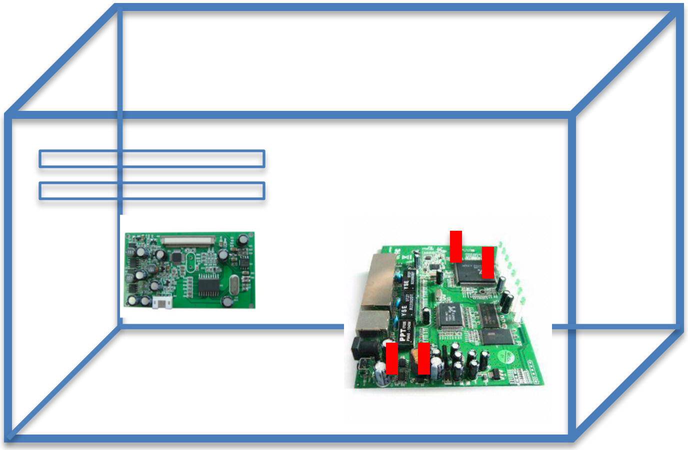
Introduction and motivation

- Determine the channel capacity of a MIMO interconnect in a cavity
- Consider noisy fields as a proxy for information transfer
- Statistical theories necessary to obtain noisy transfer functions.
- Impedance based formalism used: random coupling model
- Channel capacity distribution generated numerically
- Dependence on average loss parameter critical.

Example, small package antennas communicate within a resonant environment. How do we model the channel transfer functions?

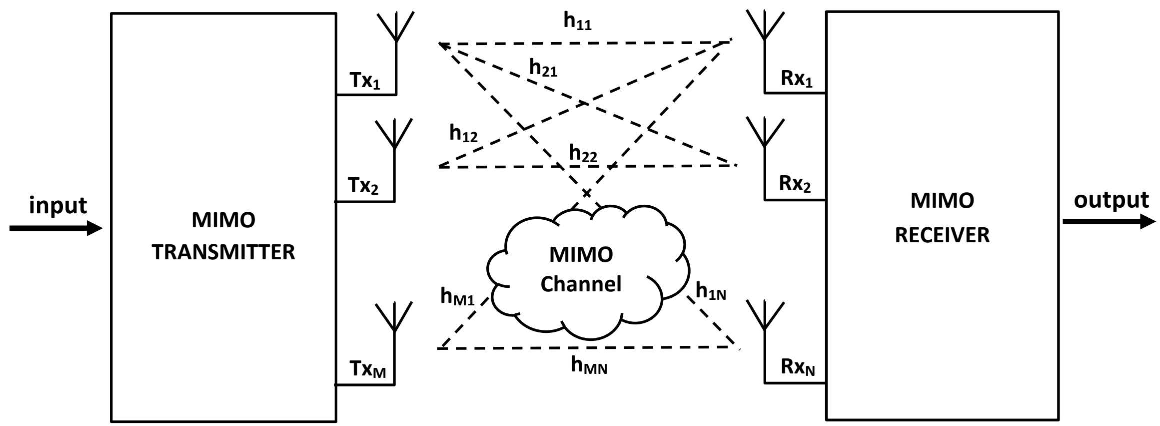
$$H = ?$$



- PCB scattering
- Multiple resonances
- Losses
- Antenna efficiency

Channel transfer matrix *H*

Linear relation between transmitted (*x*) and received (*y*) signals $y = Hx + n$

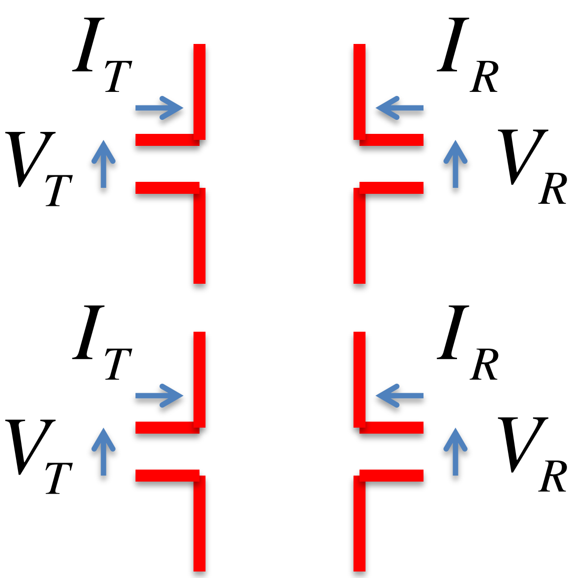


$$H = \begin{pmatrix} h_{11} & \cdots & h_{1T} \\ \vdots & \ddots & \vdots \\ h_{R1} & \cdots & h_{RT} \end{pmatrix}, \qquad h_{ij} \in C$$

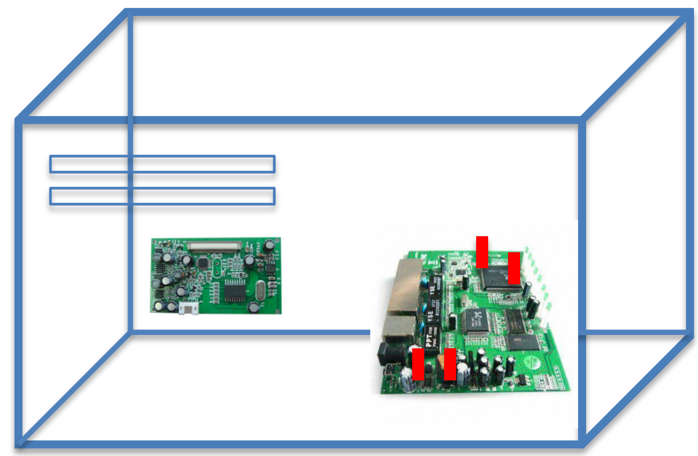
Multi-antenna communications in a box

Transfer function defined between port quantities at the antenna terminals

$$y = Hx$$



$$V_R = HV_T$$



$$Z^{cav} = \begin{bmatrix} Z_{TT}^{cav} & Z_{TR}^{cav} \\ Z_{RT}^{cav} & Z_{RR}^{cav} \end{bmatrix}$$

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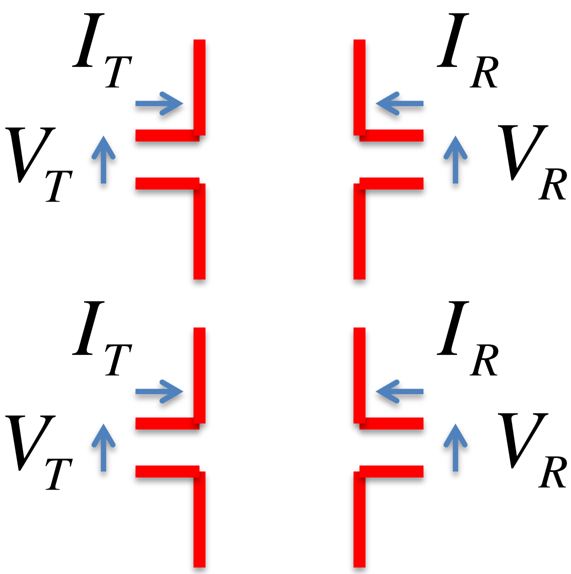

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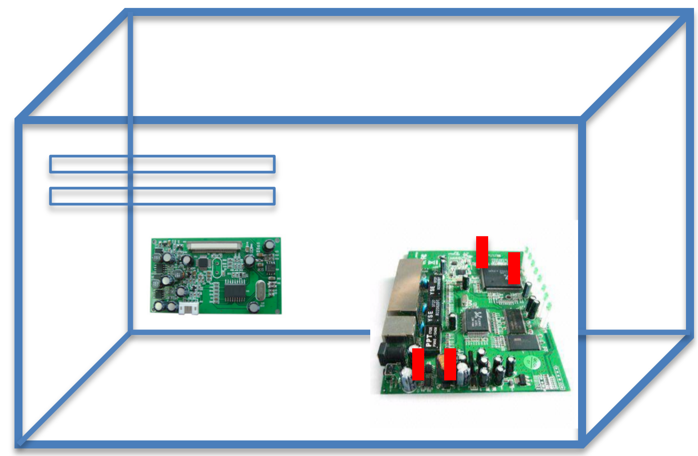

- Transfer matrix involves *sums* and *products* of impedance matrices

- Information theoretic *H* takes a simplified form for uncoupled MIMO channels.

- Leverage on RMT model of impedance matrix In irregular cavities – the Random Coupling Model

MIMO channel transfer matrix in resonant cavities

For a linear time invariant multi-port system

Cavity trans-impedance

$$\mathbf{H} = \frac{e^{-j\phi}}{R_r} \mathbf{C}_T^{-1/2} Z_{TR} \mathbf{C}_R^{-1/2}$$

$$\mathbf{C}_T = \frac{\Re\{Z_{TT}\}}{R_r} \qquad \mathbf{C}_R = \frac{\Re\{Z_{RR}\}}{R_r}$$

Impedance matrix of antennas radiating inside the cavity (*cav*): $Z_{\bullet}^{cav}$

?

Impedance matrix of antennas radiating in free-space (*rad*): $Z_{\bullet}^{rad}$

Random Coupling Model (RCM): a Random Matrix Theory model for cavity impedance matrix

$$Z^{cav} = i\,\mathrm{Im}\left(Z^{rad}\right) + \left[\mathrm{Re}\left(Z^{rad}\right)\right]^{1/2} \cdot \xi_\alpha \cdot \left[\mathrm{Re}\left(Z^{rad}\right)\right]^{1/2}$$

Coupling coefficients are GRVs

$$\left[\xi_\alpha\right]_{ps} \approx -\frac{i}{\pi}\sum_{m=1}^{M} \frac{\varphi_{pm}\cdot\varphi_{sm}}{\dfrac{k_0^2 - k_m^2}{\left\langle \Delta k_m^2 \right\rangle} + i\alpha}$$

RMT random spectrum:

Generated by eigenvalues of matrices from the Gaussian Orthogonal Ensemble (GOE).

Average loss parameter:

Measures the average Q-width of a resonant mode relative to average mode spacing Δk^2.

$$\alpha = \frac{k^2}{Q\Delta k^2} = \frac{B_Q}{\Delta k^2}$$

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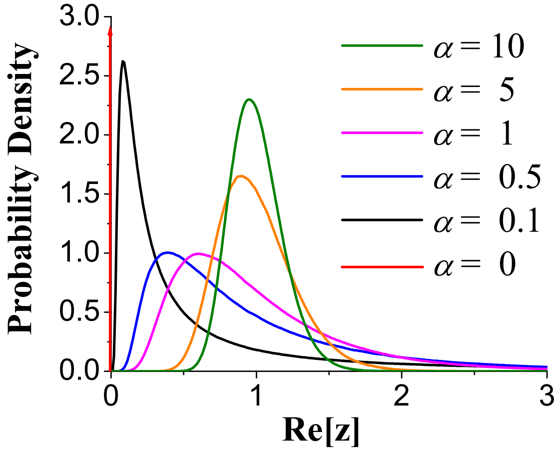
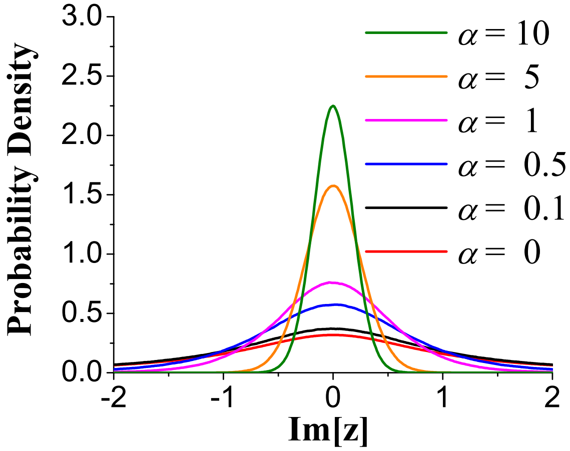
PDF of the normalized cavity impedance

An example of Monte Carlo computation…

$$z \approx \left[\xi_\alpha\right]_{11}; \quad \xi_\alpha = \left[\mathrm{Re}\left(Z^{rad}\right)\right]^{-1/2} \cdot \left[Z^{cav} - i\,\mathrm{Im}\left(Z^{rad}\right)\right] \cdot \left[\mathrm{Re}\left(Z^{rad}\right)\right]^{-1/2}$$





High loss: Gaussian; zero-loss: Lorenzian

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Each constitutive impedance of the channel transfer function can be treated with the RCM

$$\mathbf{H} = \frac{e^{-j\phi}}{R_r} \mathbf{C}_T^{-1/2} Z_{TR} \mathbf{C}_R^{-1/2}$$

RCM

$$Z_{TT}^{cav} = \mathrm{Im}\left(Z_{TT}^{rad}\right) + \left[\mathrm{Re}\left(Z_{TT}^{rad}\right)\right]^{1/2} \xi_\alpha \left[\mathrm{Re}\left(Z_{TT}^{rad}\right)\right]^{1/2}$$

$$Z_{TR}^{cav} = \left[\mathrm{Re}\left(Z_{TT}^{rad}\right)\right]^{1/2} \xi_\alpha \left[\mathrm{Re}\left(Z_{RR}^{rad}\right)\right]^{1/2}$$

$$Z_{RT}^{cav} = \left[\mathrm{Re}\left(Z_{RR}^{rad}\right)\right]^{1/2} \xi_\alpha \left[\mathrm{Re}\left(Z_{TT}^{rad}\right)\right]^{1/2}$$

$$Z_{RR}^{cav} = \mathrm{Im}\left(Z_{RR}^{rad}\right) + \left[\mathrm{Re}\left(Z_{RR}^{rad}\right)\right]^{1/2} \xi_\alpha \left[\mathrm{Re}\left(Z_{RR}^{rad}\right)\right]^{1/2}$$

Channel transfer matrix: high loss approximation

High average losses $\alpha > 1$ yield

$$Z_{TT} \approx Z_{TT}^{rad}, \qquad Z_{RR} \approx Z_{RR}^{rad}.$$

and

$$\Re\{\xi_{\alpha,(c)}\}, \Im\{\xi_{\alpha,(c)}\} \sim \mathcal{N}_{N_T}\left(\mathbf{0}, \frac{1}{\sqrt{2\pi\alpha}}\mathbf{I}\right)$$

whence the Gramian matrix $\mathbf{H}\mathbf{H}^{\dagger} \approx \xi_{\alpha}\xi_{\alpha}^{\dagger}$

takes the complex Wishart matrix form

$$\mathbf{H}\mathbf{H}^{\dagger} \sim \mathcal{CW}_{N_T}\left(N_R, \sqrt{\frac{2}{\pi\alpha}}\mathbf{I}\right)$$

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Eigenvalue distribution of the channel transfer matrix

Predicted by Marchenko-Pastur (MP) law

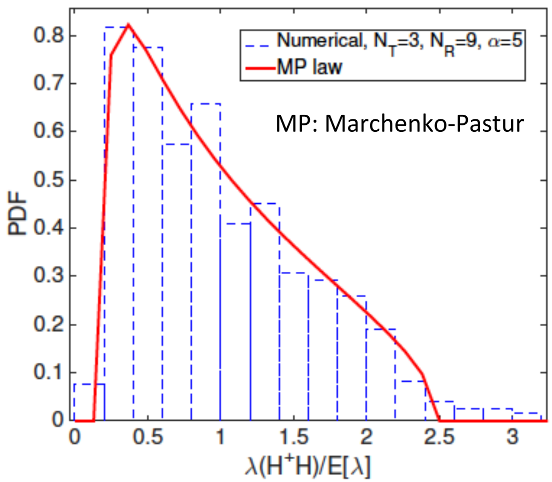
$$(1+\sqrt{\frac{\beta}{\alpha}})^2$$

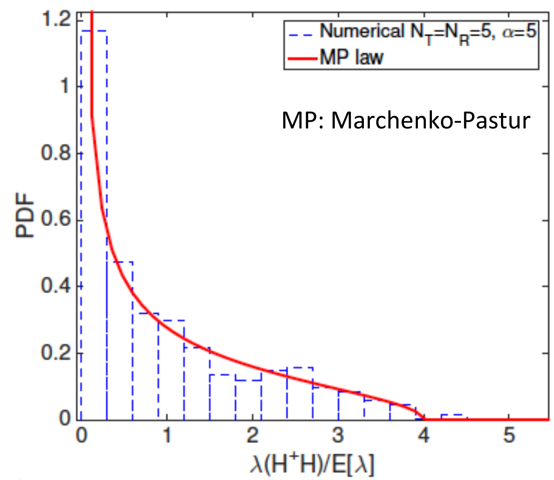
$$f_\beta(\lambda) = \left(1-\frac{\alpha}{\beta}\right)^+ \delta(\lambda) + \frac{\sqrt{(\lambda-a)^+}\sqrt{(b-\lambda)^+}}{2\pi\lambda\left(\frac{\beta}{\alpha}\right)}$$

$$(1-\sqrt{\frac{\beta}{\alpha}})^2$$

with

$$\beta = \frac{N_T}{N_R} \quad (z)^+ = \max(0,z)$$



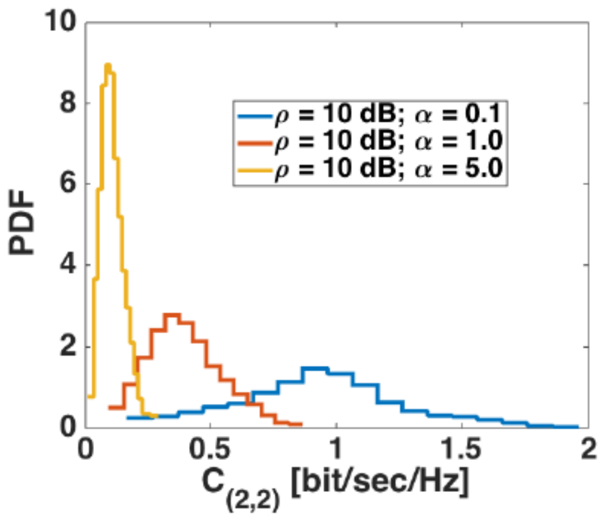


$$\langle\lambda\rangle \sim \frac{\beta}{\alpha}, \qquad \alpha > 1.$$

Channel capacity distribution function at variable losses

$$C = \log_2\left(\det\left(\mathbf{I} + \frac{\rho}{N_T}\mathbf{H}\mathbf{H}^{\dagger}\right)\right)$$

$$C = \sum_{n=1}^{N_C} \log_2\left(1 + \frac{\rho}{N_T}\lambda_n\right)$$



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Conclusion

- Physics based model of information theoretic transfer function found
- Model for MIMO systems operating in complex resonators/environments
- Radiation resistance becomes a random matrix – coupling with ergodic cavity eigenmodes
- Channel transfer matrix universally depends on a loss factor
- Channel capacity distribution calculated from Gramian eigenvalue distribution
- Results relevant in calculations of outage probability within real life electromagnetic environments

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Thank you for your attention

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