Excitation of an Electromagnetic Field in a Large Nerve Fiber by an Array of Electric-Dipole Filaments

Vasiliy A. Es'kin, Alexander V. Kudrin, and Anna A. Popova

University of Nizhny Novgorod

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Outline

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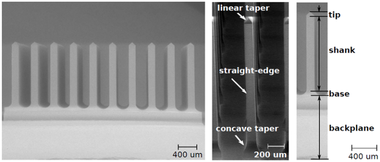
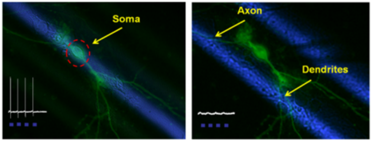
Motivation



- Optical neural stimulation by a glass optrode array[1)]



- Two-dimensional neuron stimulation by a micro-LED array[2)]

1) T.V.F. Abaya et al. Biomedical Optics Express. V. 3, No. 12. P. 3087–3104 (2012)
2) V. Poher et al. Journal of Physics D: Applied Physics. V. 41, No. 9. P. 094014 (2008)

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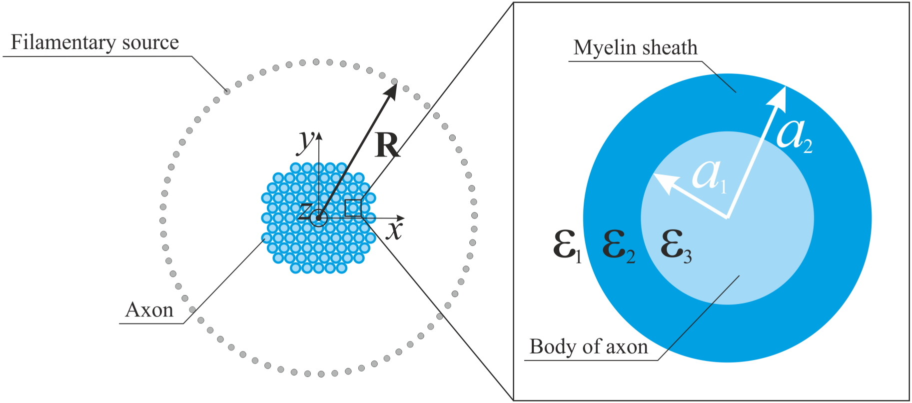
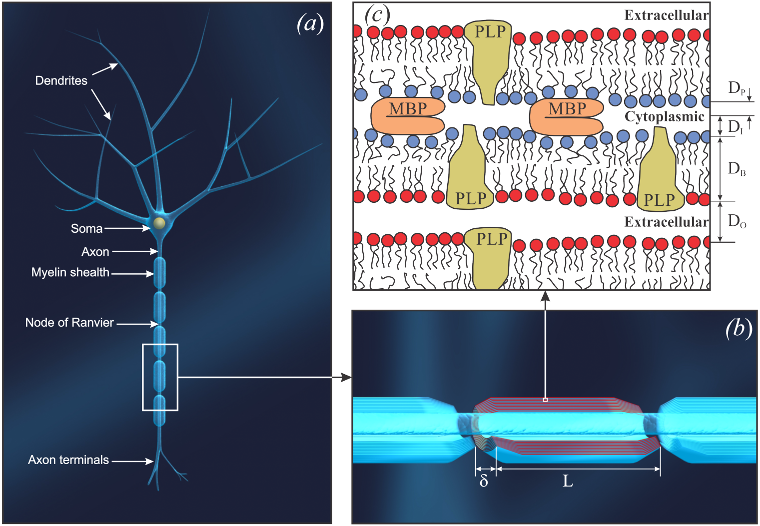
Geometry of the problem



Models of neuron and axon



$D_{\mathrm{p}} = 1$ nm

$D_{\mathrm{I}} = 1$ nm

$D_{\mathrm{B}} = 4.5$ nm

$D_{\mathrm{O}} = 3.5$ nm

$\delta = 1.5\ \mu$m

$L = 600\ \mu$m

$D_{\mathrm{Total}} = D_{\mathrm{p}} + D_{\mathrm{I}} + 2D_{\mathrm{B}} + D_{\mathrm{O}}$ is the thickness of the myelin layer[1)]

1) Y. Min et al. Proceedings of the National Academy of Sciences of the USA. V.106, No. 9. P. 3154–3159 (2009)

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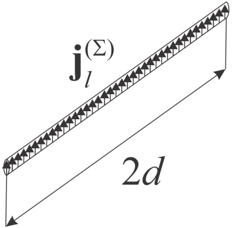
The electric current density of the lth source



$$\mathbf{j}_l^{(\Sigma)} = \frac{\delta(\rho_l)}{2\pi\rho_l} \sum_{s=1}^{S} \mathbf{j}_{l,s}^{(0)} \exp\left(-ik_0 p_s z_l\right) \left[U(z+d) - U(z-d)\right]$$

$$\mathbf{j}_{l,s}^{(0)} = i\omega \mathcal{P}_{l,s} (\mathbf{x}_0 \cos\theta_l + \mathbf{y}_0 \sin\theta_l)$$

(time dependence $\exp(i\omega t)$ is dropped)

The source representation

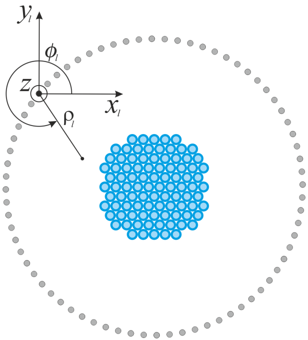
$$\mathbf{j}_l^{(\Sigma)} = \frac{k_0}{2\pi} \int_{-\infty}^{\infty} \mathbf{j}_l(\rho_l, p) \exp\left(-ik_0 p z\right) dp$$

$$\mathbf{j}_l(\rho_l, p) = \frac{\delta(\rho_l)}{\pi\rho_l} \sum_{s=1}^{S} \mathbf{j}_{l,s}^{(0)} \frac{\sin\left[k_0(p - p_s)d\right]}{k_0(p - p_s)}$$

Excited electromagnetic field

$$E_{z,l} = \frac{k_0^2}{2\pi}\int_{-\infty}^{\infty} p\frac{q_1}{\varepsilon_1} C_l^{(\mathrm{e})} H_1^{(2)}(k_0 q_1 \rho_l)\cos(\phi_l - \theta_l) e^{-ik_0 pz} dp$$

$$H_{z,l} = \frac{k_0^2}{2\pi}\int_{-\infty}^{\infty} q_1 C_l^{(\mathrm{e})} H_1^{(2)}(k_0 q_1 \rho_l)\sin(\phi_l - \theta_l) e^{-ik_0 pz} dp$$



$$C_l^{(\mathrm{e})} = 2\pi \sum_{s=1}^{S} \mathcal{P}_{l,s} \sin\left[k_0(p - p_s)d\right]/(p - p_s)$$

$$q_1 = \left(\varepsilon_1 - p^2\right)^{1/2}$$

Field representation in the coordinates related to the αth axon

$$\begin{bmatrix} E_{z;\alpha}(\mathbf{r}_\alpha) \\ H_{z;\alpha}(\mathbf{r}_\alpha) \end{bmatrix} = \sum_{m=-\infty}^{\infty} \frac{k_0}{2\pi} \int_{-\infty}^{\infty} \begin{bmatrix} E_{z;\alpha,m}(\rho_\alpha, p) \\ H_{z;\alpha,m}(\rho_\alpha, p) \end{bmatrix} e^{-im\phi_\alpha - ik_0 pz} dp$$

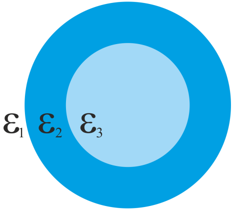
$$m = 0, \pm 1, \pm 2, \ldots$$

Field equations

$$\hat{L}_{\alpha,m} E_{z;\alpha,m} = 0, \quad \hat{L}_{\alpha,m} H_{z;\alpha,m} = 0,$$

$$\hat{L}_{\alpha,m} = \frac{\partial^2}{\partial \rho_\alpha^2} + \frac{1}{\rho_\alpha} \frac{\partial}{\partial \rho_\alpha} - \frac{m^2}{\rho_\alpha^2} + k_0^2 q_k^2,$$

$$k = 1, 2, 3, \quad q_k^2 = \varepsilon_k - p^2$$



Azimuthal harmonics of the electric and magnetic fields inside the αth axon

$$\begin{bmatrix} E_{z;\alpha,m} \\ H_{z;\alpha,m} \end{bmatrix} = \begin{bmatrix} B^{(1)}_{\alpha,m} \\ B^{(2)}_{\alpha,m} \end{bmatrix} q_3 J_m(k_0 q_3 \rho_\alpha), \quad \rho_\alpha \leq a_1$$

$$\begin{bmatrix} E_{z;\alpha,m} \\ H_{z;\alpha,m} \end{bmatrix} = \sum_{k=1}^{2} \begin{bmatrix} C^{(k)}_{\alpha,m} \\ \tilde{C}^{(k)}_{\alpha,m} \end{bmatrix} q_2 H^{(k)}_m (k_0 q_2 \rho_\alpha), \quad a_1 \leq \rho_\alpha \leq a_2$$

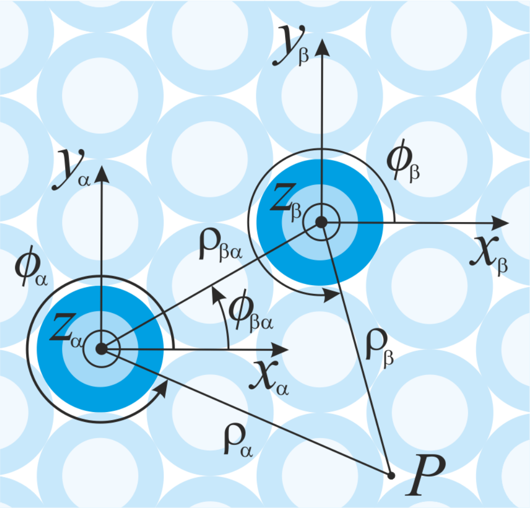
$B^{(1,2)}_{\alpha,m}$, $C^{(1,2)}_{\alpha,m}$, and $\tilde{C}^{(1,2)}_{\alpha,m}$ are the amplitude coefficients

Azimuthal harmonics of the electric and magnetic fields scattered by the αth axon

$$\begin{bmatrix} E^{(\mathrm{sc})}_{z;\alpha,m} \\ H^{(\mathrm{sc})}_{z;\alpha,m} \end{bmatrix} = \begin{bmatrix} D^{(1)}_{\alpha,m} \\ D^{(2)}_{\alpha,m} \end{bmatrix} q_1 H^{(2)}_m(k_0 q_1 \rho_\alpha)$$

$D^{(1)}_{\alpha,m}$ and $D^{(2)}_{\alpha,m}$ are the scattering coefficients

Graf's addition theorem



$$H_n^{(2)}\left(k_0 q_1 \rho_\beta\right) e^{-in\phi_\beta} = \sum_{m=-\infty}^{\infty} J_m\left(k_0 q_1 \rho_\alpha\right) H_{m-n}^{(2)}\left(k_0 q_1 \rho_{\beta\alpha}\right) e^{i(m-n)\phi_{\beta\alpha} - im\phi_\alpha}$$

Total field outside the αth axon

$$E_{z;\alpha,m} = E^{(\mathrm{sc})}_{z;\alpha,m} + E^{(\mathrm{ex})}_{z;\alpha,m}, \quad H_{z;\alpha,m} = H^{(\mathrm{sc})}_{z;\alpha,m} + H^{(\mathrm{ex})}_{z;\alpha,m}$$

$$E^{(\mathrm{ex})}_{z;\alpha,m}{=}q_1\mathcal{E}_{\alpha,m}J_m\left(k_0q_1\rho_\alpha\right),\ H^{(\mathrm{ex})}_{z;\alpha,m}{=}q_1\mathcal{H}_{\alpha,m}J_m\left(k_0q_1\rho_\alpha\right)$$

$$\begin{aligned}\begin{bmatrix}\mathcal{E}_{\alpha,m}\\ \mathcal{H}_{\alpha,m}\end{bmatrix} ={}& \sum_{\beta\neq\alpha}^{N_\mathrm{a}}\sum_{n=-\infty}^{\infty}\begin{bmatrix}D^{(1)}_{\beta,n}\\ D^{(2)}_{\beta,n}\end{bmatrix}H^{(2)}_{m-n}\left(k_0q_1\rho_{\beta\alpha}\right)e^{i(m-n)\phi_{\beta\alpha}} \\ &+\sum_{l=1}^{N_\mathrm{s}}\begin{bmatrix}E^{(0)}_l\\ H^{(0)}_l\end{bmatrix}\Bigg\{H^{(2)}_{m-1}(k_0q_1\rho_{l\alpha})e^{i(m-1)\phi_{l\alpha}+i\theta_l} \\ &-\begin{bmatrix}1\\ -1\end{bmatrix}H^{(2)}_{m+1}(k_0q_1\rho_{l\alpha})e^{i(m+1)\phi_{l\alpha}-i\theta_l}\Bigg\}\end{aligned}$$

where $E^{(0)}_l = k_0pC^{(\mathrm{e})}_l/(2\varepsilon_1)$, $H^{(0)}_l = ik_0C^{(\mathrm{e})}_l/2$
N_a is the number of the axons in the nerve fiber
N_s is the number of the filamentary sources

System of equations for the scattering coefficients

$$D^{(1)}_{\alpha,m} = S^{ee}_m \mathcal{E}_{\alpha,m} + S^{eh}_m \mathcal{H}_{\alpha,m}$$
$$D^{(2)}_{\alpha,m} = S^{he}_m \mathcal{E}_{\alpha,m} + S^{hh}_m \mathcal{H}_{\alpha,m}$$

S^{ee}_m, S^{eh}_m, S^{he}_m, and S^{hh}_m are the elements of the scattering matrix of a single cylindrical scatterer

Outline

Representation of the required fields

An auxiliary magnetic field $H_z^{(\mathrm{d})}$ has the same spatial distribution as that of the desired electric field

Expanding $H_z^{(\mathrm{d})}$ in terms of eigenwaves of a homogeneous medium

$$H_z^{(\mathrm{d})} = \int_0^{\infty} H_z^{(\mathrm{r})}(\rho, \phi, q)\,dq$$

$$H_z^{(\mathrm{r})}(\rho, \phi, q) = \sum_{m=-M_e}^{M_e} a_m(q) q J_m(k_0 q \rho) e^{-im\phi}$$

$$a_m(q) = \frac{k_0^2}{2\pi} \int_0^{2\pi} d\phi \int_0^{\infty} H_z^{(\mathrm{d})} J_m(k_0 q \tilde{\rho}) e^{im\phi} \tilde{\rho}\, d\tilde{\rho}$$

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$$a_m(q) = \frac{k_0^2}{2\pi} \int_0^{2\pi} d\phi \int_0^\infty H_z^{(\mathrm{d})} J_m(k_0 q \tilde{\rho}) e^{im\phi} \tilde{\rho} d\tilde{\rho}$$

Replacing the integral by the quadrature formula

$$H_z^{(\mathrm{d})} \approx \sum_{s=1}^{S_{\max}} A_s H_z^{(\mathrm{r})}(\rho, \phi, q_s)$$

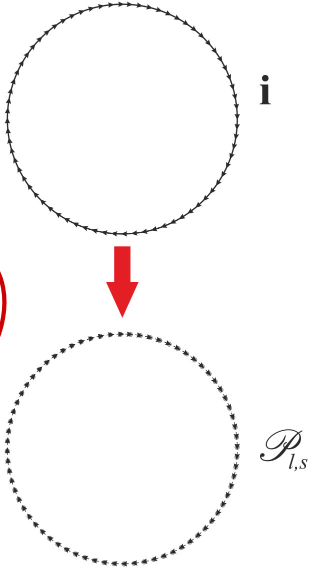
Dipole moments of the sources

$$\mathbf{i}(\phi, q_s) = \boldsymbol{\phi}_0 c A_s H_z^{(\mathrm{r})}(R, \phi, q_s)/(4\pi)$$

$$\mathcal{P}_{l,s} = c A_s H_z^{(\mathrm{r})}(R, \phi_{l0}, q_s) \Delta L/(4\pi\omega)$$

$$\mathbf{j}_{l,s}^{(0)} = i\omega \mathcal{P}_{l,s}(\mathbf{x}_0 \cos\theta_l + \mathbf{y}_0 \sin\theta_l)$$

$$\theta_l = \phi_{l0}$$



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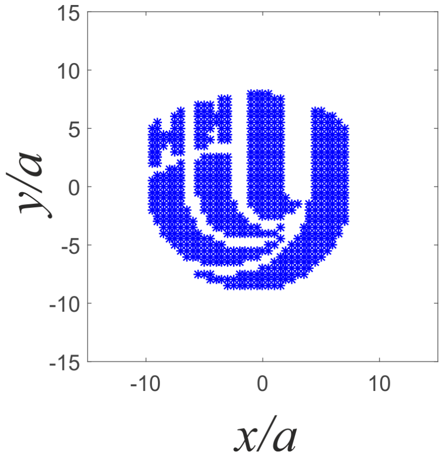
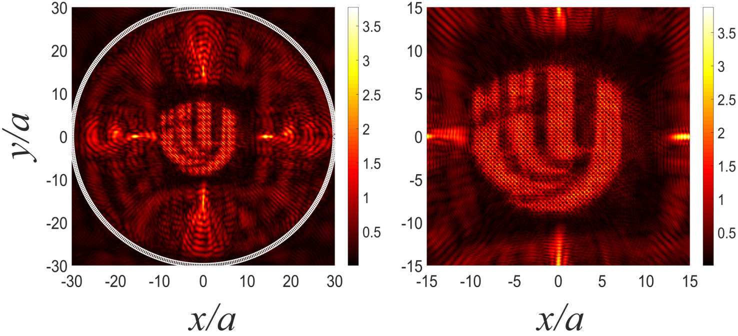
Parameters of a system

- The nerve fiber consisting of 92 tightly packaged axons
- The inner and outer radii of the myelin sheath were taken equal to a_1=1.5 μm and a_2=2.5 μm, respectively
- The array of filamentary sources consisted of 497 elements located on a cylindrical surface of radius $R = 30a_2$
- Each source operated at a frequency corresponding to the wavelength λ_0=2010 nm and had the half-length d=$2\times10^4\lambda_0$
- The distance ΔL between the nearest sources was equal to $\lambda_0/2$
- The maximum absolute value of the azimuthal index used in the calculations of the scattered fields was equal to $M = 15$
- The dielectric permittivities of water and myelin were calculated for λ_0=2010 nm: $\varepsilon_1 = \varepsilon_3 = 1.68 - i0.0026$ and $\varepsilon_2 = 1.97 - i0.0008$

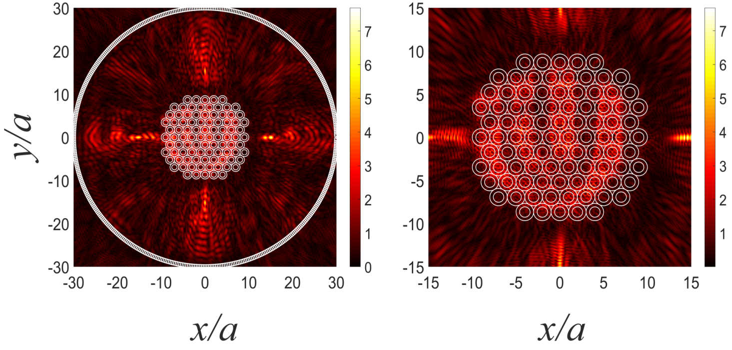
Desired distribution of the longitudinal electric field and its pattern specified in discrete form



Field excited by the found electric-dipole filaments in a homogeneous medium with dielectric permittivity ε_1



Field excited by the found electric-dipole filaments in a homogeneous medium with dielectric permittivity ε_1 in the presence of a nerve fiber



Outline

Conclusions

- We have solved the problem of excitation of an electromagnetic field with the specified spatial distribution by a set of electric-dipole filaments in the presence of a nerve fiber
- It has been demonstrated that the field of such sources with their dipole moments chosen appropriately can have a spatial structure that provides a selective impact on specified regions of the nerve fiber

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Thanks for your attention!

Email to vasiliy.eskin@gmail.com with any questions