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Teaching radiofrequency power measurements

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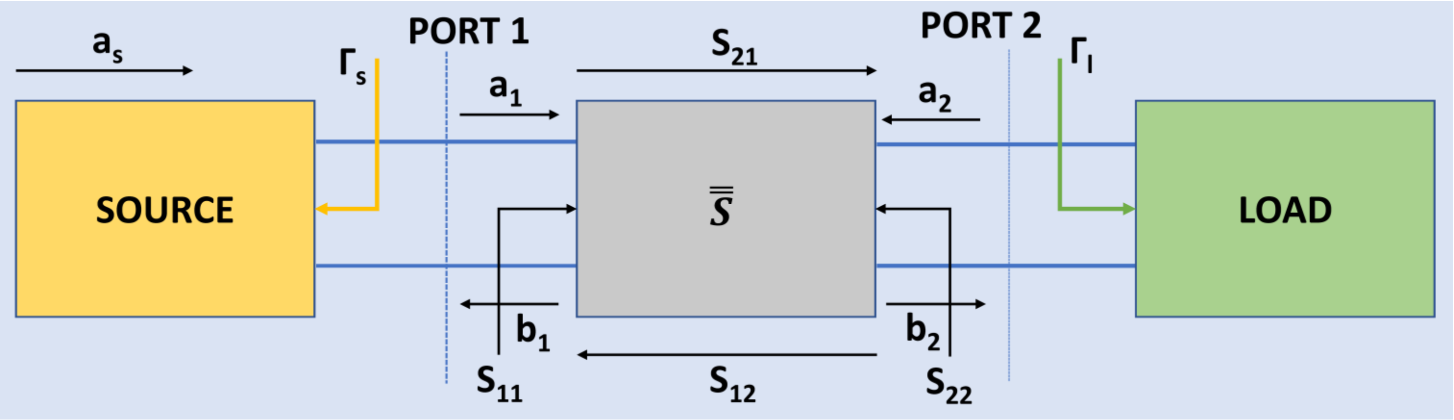
Learning objectives

- The topic of power measurement is introduced emphasizing the fact that power generators and meters are calibrated to indicate the power delivered to, or absorbed by, a perfect Z_0 (e.g. 50 Ω) load
- Then, it is discussed what happens when this perfect Z_0 load assumption is not satisfied by introducing the mismatch correction
- The mismatch correction is analyzed in terms of uncertainty governed by the U-shaped probability density function (PDF)
- The U-shaped PDF gives the opportunity to discuss *state-of-knowledge* PDFs (*) whose use is ubiquitous in modern measurement uncertainty quantification

(*) A state-of-knowledge PDF does not result from direct observation of the physical world; it reflects the state of knowledge of the experimenter about the physical phenomenon.

Power transfer between a source and a load



- A source of power is connected to a load through a connection represented by its $\bar{\bar{S}}$-parameters network
- The connection may be a cable, an adapter, an attenuator, an amplifier or a combination thereof
- Let Γ_S and Γ_L represent the output reflection coefficients of the source and of the load, respectively. Symbols a_i and b_i, $i = 1,2$, represent incident and reflected waves, respectively

Analysis – 1

- Through the analysis of the network the following set of equations is easily derived

$$\begin{cases} a_1 = a_S + b_1\Gamma_S \\ a_2 = \Gamma_L b_2 \\ b_1 = S_{11}a_1 + S_{12}a_2 \\ b_2 = S_{21}a_1 + S_{22}a_2 \end{cases}$$

- a_S represents the wave that the source would deliver to a load whose impedance is Z_0 ($b_1 = 0$, the "matched load")

Analysis – 2

- By solving the system of equations the incident wave to the load is derived

$$b_2 = \frac{S_{21} a_S}{(1 - \Gamma_S S_{11})(1 - \Gamma_L S_{22}) - \Gamma_S \Gamma_L S_{12} S_{21}}$$

- The incident power, the reflected power and the absorbed power are immediately obtained as $|b_2|^2$, $|b_2 \Gamma_L|^2$ and $|b_2|^2(1 - |\Gamma_L|^2)$

Direct connection

- In the case of a direct connection between the source and the load we have $S_{11} = S_{22} = 0$ and $S_{21} = S_{12} = 1$, therefore

$$b_2 = \frac{a_S}{1 - \Gamma_S \Gamma_L}$$

- Matched load ($\Gamma_L = 0$): $P_0 = |a_S|^2$
- Conjugate matched load ($\Gamma_L = \Gamma_S^*$): $P_S = \frac{|a_S|^2}{1 - |\Gamma_S|^2}$

Calibration

- Conjugate match is rarely implemented in a RF measurement setup.
- Conversely:
    - *RF power generators* are calibrated to indicate the power delivered to the matched load P_0
    - *RF power meters* are calibrated to indicate the power that would be absorbed by the matched load P_0
- This is quite important to know in order to correctly interpret the operation and specifications of power sources and power meters

Multiple reflections

- In RF literature the term $(1 - \Gamma_S \Gamma_L)^{-1}$ is often referred to as "correction for multiple reflections"
  - First incident wave → $w_0 = a_S$
  - Second incident wave → $w_1 = a_S \Gamma_S \Gamma_L$
  - Third incident wave → $w_2 = a_S (\Gamma_S \Gamma_L)^2$
  - ...
- Summing over the infinite incident waves

$$b_2 = a_S \sum_{n=0}^{\infty} (\Gamma_S \Gamma_L)^n = \frac{a_S}{1 - \Gamma_S \Gamma_L}$$

Calibration factor of a power meter

- The power absorbed by a power meter from a source is

$$P_L = \frac{P_0(1 - |\Gamma_L|^2)}{|1 - \Gamma_L \Gamma_S|^2}$$

- Power P_L is not entirely absorbed by the sensing element of the power meter (thermistor, diode or thermocouple), a small portion of it is lost in radiation, metallic and dielectric losses
- The fraction absorbed by the sensing element is the one sensed, and it is given by $P_M = \eta P_L$ (η is the efficiency of the power meter), therefore

$$P_0 = \frac{P_M |1 - \Gamma_L \Gamma_S|^2}{\eta(1 - |\Gamma_L|^2)} = P_M \frac{m}{c_F}$$

- $m = |1 - \Gamma_L \Gamma_S|^2$ is the mismatch correction, $c_F = \eta(1 - |\Gamma_L|^2)$ is the calibration factor of the power meter (provided by the manufacturer of or by the laboratory which calibrates the power meter)

Mismatch uncertainty

- The mismatch correction m depends on both the magnitude and phase of the product $\Gamma_S\Gamma_L$
- Such rich information is generally unavailable, therefore, the mismatch correction cannot be generally applied
- The maximum warranted magnitude of Γ_S and Γ_L can however be deduced from specifications, calibrations, or other sources of information about the performance of the source and power meter thus permitting to evaluate the minimum and maximum warranted values of m

$$m^- = (1 - |\Gamma_S\Gamma_L|)^2 \text{ and } m^+ = (1 + |\Gamma_S\Gamma_L|)^2$$

Mismatch uncertainty

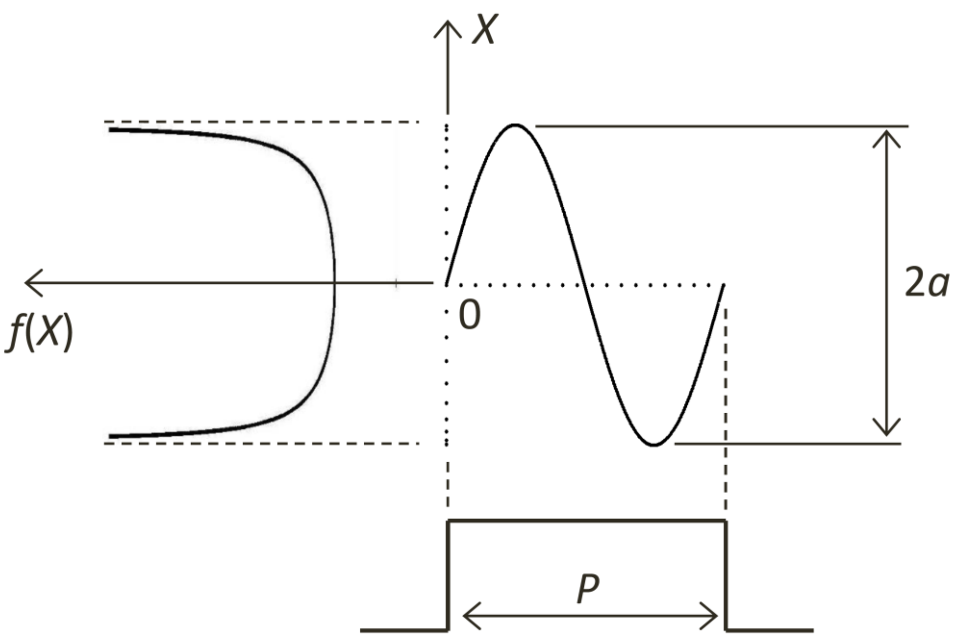
- Modern evaluation of uncertainty is based on a probabilistic (rather than deterministic) approach
- How can we deal with mismatch uncertainty rather than mismatch error?
- Consider

$$m = 1 - 2|\Gamma_S \Gamma_L| \cos\varphi + |\Gamma_S \Gamma_L|^2$$

  where $\varphi = \arg\{\Gamma_S \Gamma_L\}$
- φ is unknown but surely comprised between 0 and π or between $-\pi$ and 0
- φ is interpreted as a random variable and since no information is available about its probability distribution then a uniform PDF is assigned to it (ignorance is uniform!)

U-shaped probability density function (PDF)

- If φ is a uniformly distributed random variable in $[0, \pi]$ then $a\cos\varphi$ is a random variable having a symmetric U-shaped PDF with expected value 0 and standard deviation $\frac{a}{\sqrt{2}}$



- The U-shaped PDF has the following mathematical expression

$$f(X) = \frac{1}{\pi\sqrt{a^2 - X^2}}$$

where $|X| < a$ ($f(X) = 0$, otherwise)

Application of the U-shaped PDF to mismatch uncertainty

- Mismatch uncertainty is usually dealt with in log units (decibel is a very common unit in RF measurements), then

$$M = 10\log_{10} m$$

- The expected value of M is zero and the standard uncertainty is approximately

$$\frac{8{,}686|\Gamma_S \Gamma_L|}{\sqrt{2}}$$

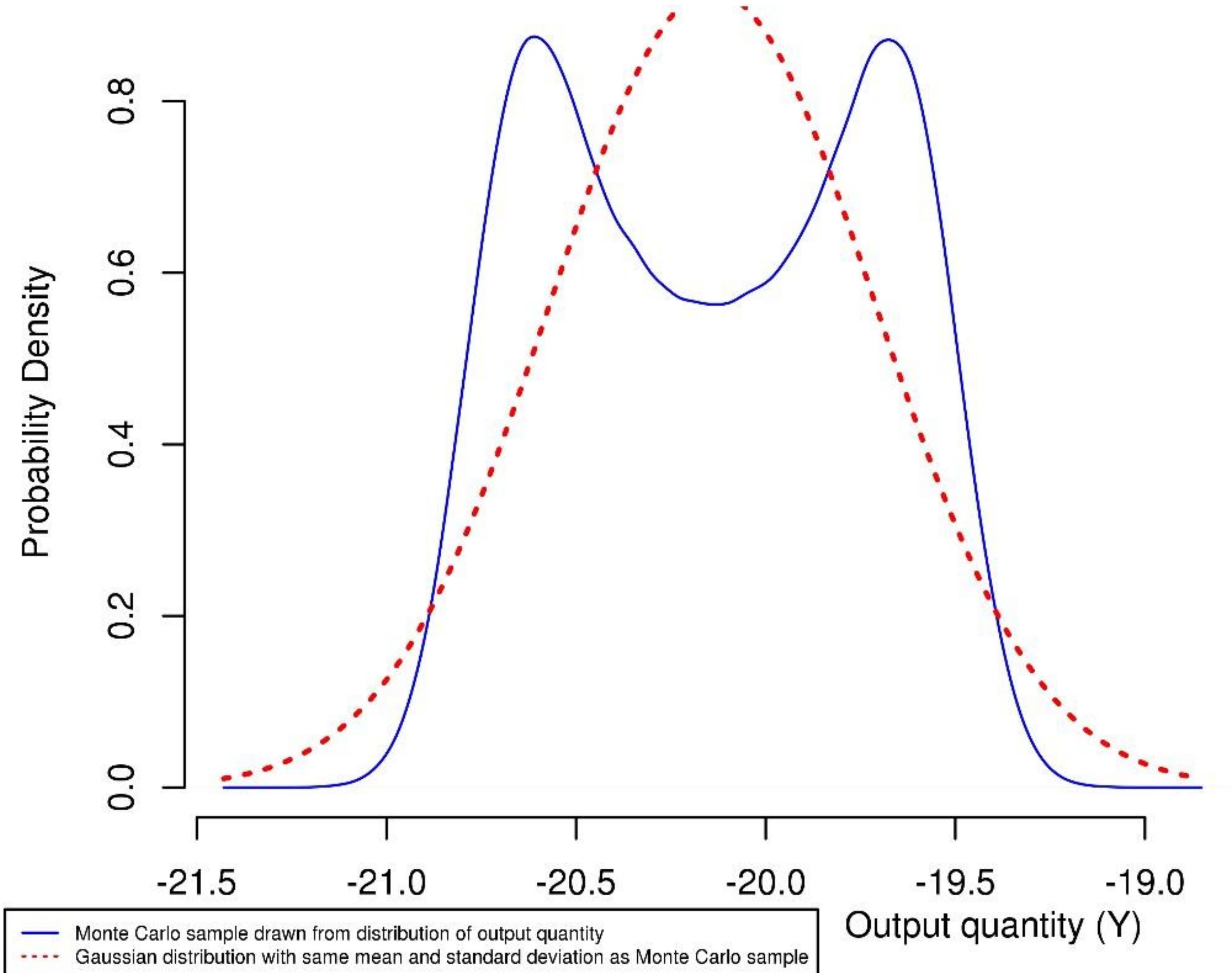
Example

- The reading of a power meter when connected to a power source is −20,37 dBm
- The calibration factor of the power meter, as provided by its certificate of calibration, is −0,23 dB with an expanded uncertainty of 0,26 dB (coverage factor $k = 2$, corresponding to a coverage probability of 95 % assuming a normal PDF)
- From specifications of both the power source and power meter we deduce $|\Gamma_S| = 1/3$ and $|\Gamma_L| = 1/5$, hence the standard mismatch uncertainty is 0,41 dB
- The estimate of the source power is therefore −20,37 dBm + 0,23 dB = −20,14 dBm and the combined standard uncertainty is

$$u = \sqrt{\left(\frac{0{,}26}{2}\right)^2 + (0{,}41)^2}\ \mathrm{dB} = 0{,}43\ \mathrm{dB}$$

Expanded uncertainty

- In order to evaluate the expanded uncertainty, it is to be acknowledged that the non-normal (U-shaped) mismatch contribution dominates over the normal contribution associated with the calibration factor
- Therefore, the central limit theorem is not applicable
- From a Monte Carlo uncertainty analysis, it is found (see next slide) that the PDF is quite different from the normal one (red dashed line) and much more like a U-shaped PDF (bleu continuous line)
- The coverage factor corresponding to the stipulated 95 % coverage probability is 1,6 and the expanded uncertainty is 0,70 dB
- The plot was obtained by using NIST Uncertainty Machine, a free internet resource made available by NIST for Monte Carlo uncertainty analysis, see https://uncertainty.nist.gov/



Probability Density
0.0
0.2
0.4
0.6
0.8
-21.5
-21.0
-20.5
-20.0
-19.5
-19.0
Output quantity (Y)
Monte Carlo sample drawn from distribution of output quantity
Gaussian distribution with same mean and standard deviation as Monte Carlo sample

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