UNIVERSITY OF MANITOBA

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Lemma

SPATIAL PRIOR FOR QUANTITATIVE BREAST CANCER MICROWAVE IMAGING: A COMPARISON BETWEEN NON-ITERATIVE EIGENFUNCTION-BASED INVERSION AND SAMPLING METHODS

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Outline

1. Introduction to Biomedical Microwave Imaging
2. Inverse Scattering Problem
3. Qualitative Methods for generating Spatial Priors
4. Incorporation of Spatial Priors within Contrast Source Inversion
5. 2D Example via Discontinuous Galerkin Method CSI

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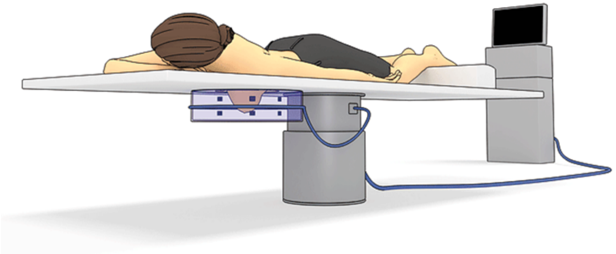
Biomedical Microwave Imaging

Microwave Imaging is very attractive as a cooperative diagnostic technique.



Breast cancer imaging

Bone fracture risk monitoring

Brain stroke monitoring

- ✓ **non-ionizing radiations**
- ✓ **low-cost and portable devices**

- X **Resolution**
- X **Solution of an inverse scattering problem**

[*] Nikolova, "Microwave Biomedical Imaging," John Wiley & Sons, Inc., pp. 1–22, 2014.

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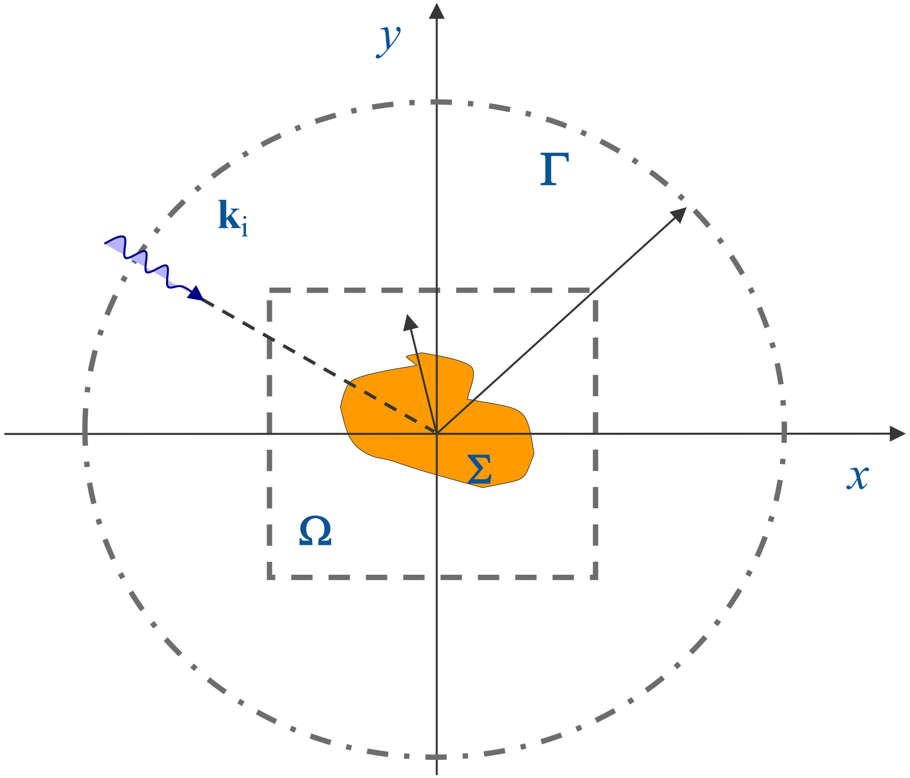
Inverse Scattering Problem
(2D scalar case)



$$E_i\,(\boldsymbol{r},\boldsymbol{r_t}), E_s\,(\boldsymbol{r_m},\boldsymbol{r_t})$$

$$\boldsymbol{r} \in \Omega, \quad \boldsymbol{r_t}, \boldsymbol{r_m} \in \Gamma$$

$$\Downarrow$$

$$\chi(\boldsymbol{r}) = \frac{\varepsilon(\boldsymbol{r})}{\varepsilon_b} - 1$$

contrast function, which encodes target properties (e.m. parameters, shape)

Inverse Scattering Problem

(2D scalar case)

$$E_s(\boldsymbol{r_m},\boldsymbol{r_t}) = \int_{\Omega} G_b(\boldsymbol{r_m},\boldsymbol{r}')\,\chi(\boldsymbol{r}')E_{tot}(\boldsymbol{r}',\boldsymbol{r_t})\,d\boldsymbol{r}' = \mathcal{A}_e[\chi E_{tot}(\boldsymbol{r},\boldsymbol{r_t})]$$

'data' equation

$$W(\boldsymbol{r},\boldsymbol{r_t}) - \chi(\boldsymbol{r})E_i(\boldsymbol{r},\boldsymbol{r_t}) = \chi(\boldsymbol{r})\int_{\Omega} G_b(\boldsymbol{r},\boldsymbol{r}')\,W(\boldsymbol{r}',\boldsymbol{r_t})\,d\boldsymbol{r}' = \chi(\boldsymbol{r})\mathcal{A}_{\boldsymbol{i}}[\chi E_{tot}(\boldsymbol{r},\boldsymbol{r_t})]$$

'state' equation

non-linear and ill-posed inverse problem

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Qualitative methods

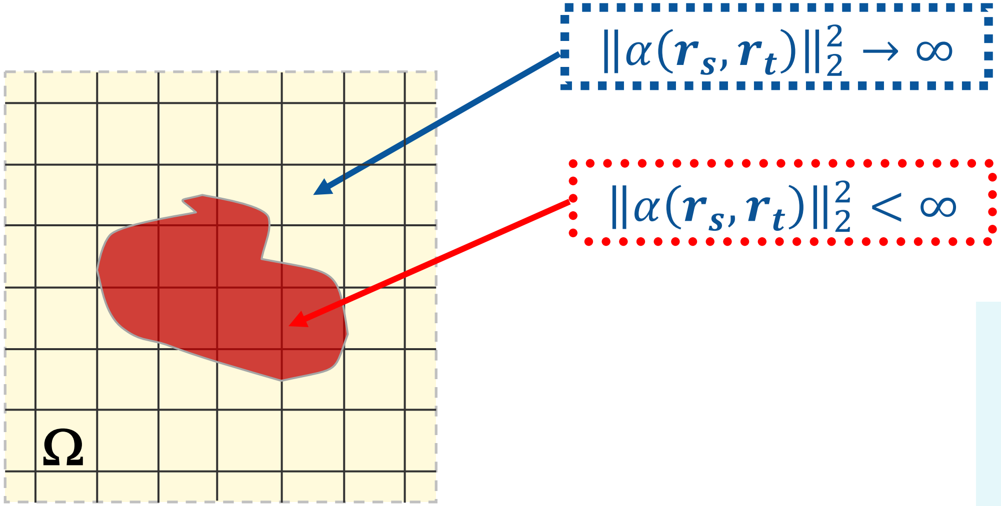
- **LOCATION AND SHAPE RECONSTRUCTION**
  - sufficient in several applications
  - useful prior information
- **A HIGH FLEXIBILITY**
  - dielectric and metallic objects
  - no approximations or prior information
- **COMPUTATIONAL EFFICIENCY**
  - straightforward implementation
  - “quasi-real” time execution

Linear Sampling Method

In each point $\boldsymbol{r}_s \in \Omega$, LSM solves the far-field equation:

$$\int_\Gamma \alpha(\boldsymbol{r}_s, \boldsymbol{r}_t) E_s(\boldsymbol{r}_m, \boldsymbol{r}_t)\, d\boldsymbol{r}_t = G_b(\boldsymbol{r}_m, \boldsymbol{r}_s)$$

The energy of the (regularized) solution allows to retrieve the target support.



$\|\alpha(\boldsymbol{r}_s, \boldsymbol{r}_t)\|_2^2 \rightarrow \infty$

$\|\alpha(\boldsymbol{r}_s, \boldsymbol{r}_t)\|_2^2 < \infty$

$$\mathfrak{I}_{LSM}(\boldsymbol{r}_s) = \|\alpha(\boldsymbol{r}_s, \boldsymbol{r}_t)\|_2^2$$

LSM support indicator

[*] I. Catapano, L. Crocco, and T. Isernia, On simple methods for shape reconstruction of unknown scatterers, IEEE TAP, 2007

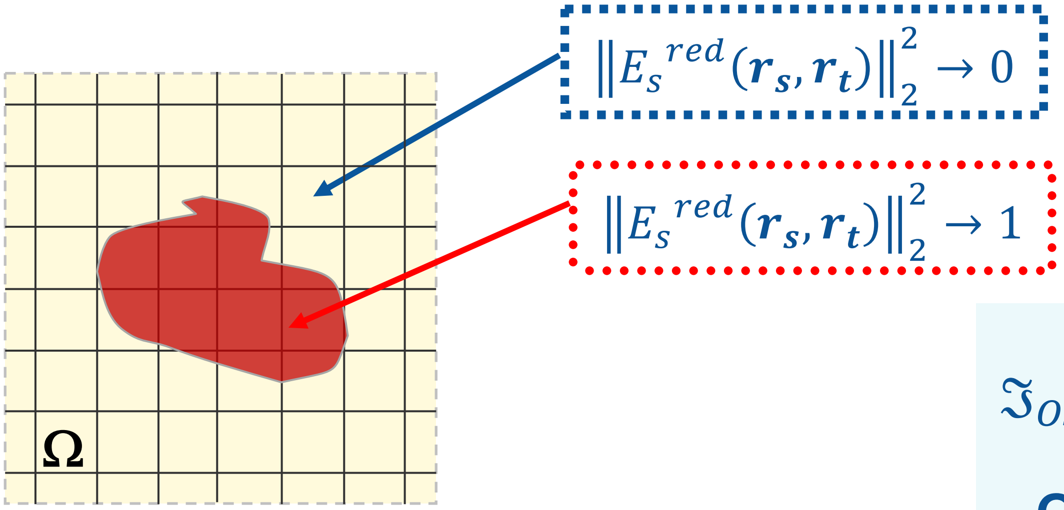
Orthogonality Sampling Method

OSM tests the <u>orthogonality relationship</u> between the far-field pattern ${E_s}^{\infty}$ and the Green's function ${G_b}^{\infty}$.

$$ {E_s}^{red}(\boldsymbol{r_s}, \boldsymbol{r_t}) = \frac{1}{\gamma} \langle {E_s}^{\infty}, {G_b}^{\infty} \rangle_{\Gamma(\hat{\boldsymbol{r}}_m)} $$

γ is a constant

The energy of ${E_s}^{red}$ allows to retrieve the target support.



$$ \left\| {E_s}^{red}(\boldsymbol{r_s}, \boldsymbol{r_t}) \right\|_2^2 \to 0 $$

$$ \left\| {E_s}^{red}(\boldsymbol{r_s}, \boldsymbol{r_t}) \right\|_2^2 \to 1 $$

$$ \mathfrak{I}_{OSM}(\boldsymbol{r_s}) = \left\| {E_s}^{red}(\boldsymbol{r_s}, \boldsymbol{r_t}) \right\|_2^2 $$

OSM support indicator

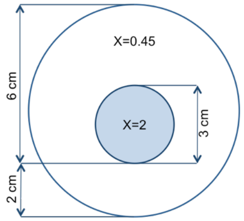
[*] R. Potthast, A study on orthogonality sampling, Inverse Prob., vol. 26, n. 7, 2010

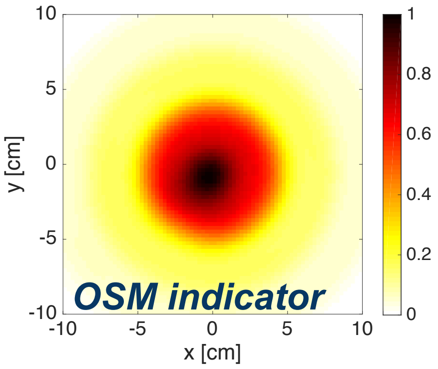
Orthogonality Sampling Method

Interestingly, the energy of E_s^{red} allows to identify **discontinuities** within the target.

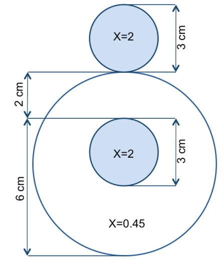
Fresnel experimental data inversions

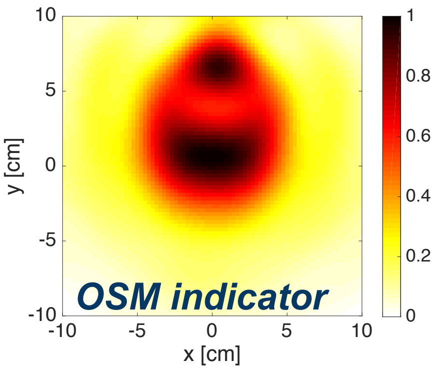
FoamDielInt, f=5GHz



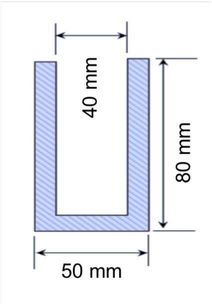


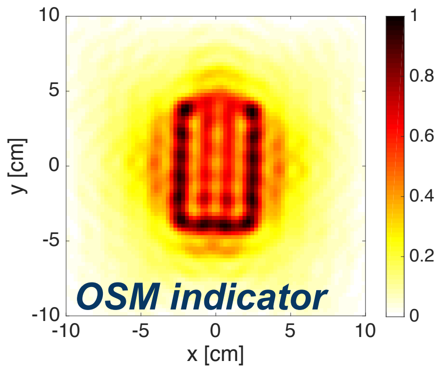
FoamTwinDielInt, f=4GHz





U-shaped target, f=12GHz





[*] Bevacqua et al., “Physical Insight Unveils New Imaging Capabilities of Orthogonality Sampling Method”, *IEEE TAP*, 2020.

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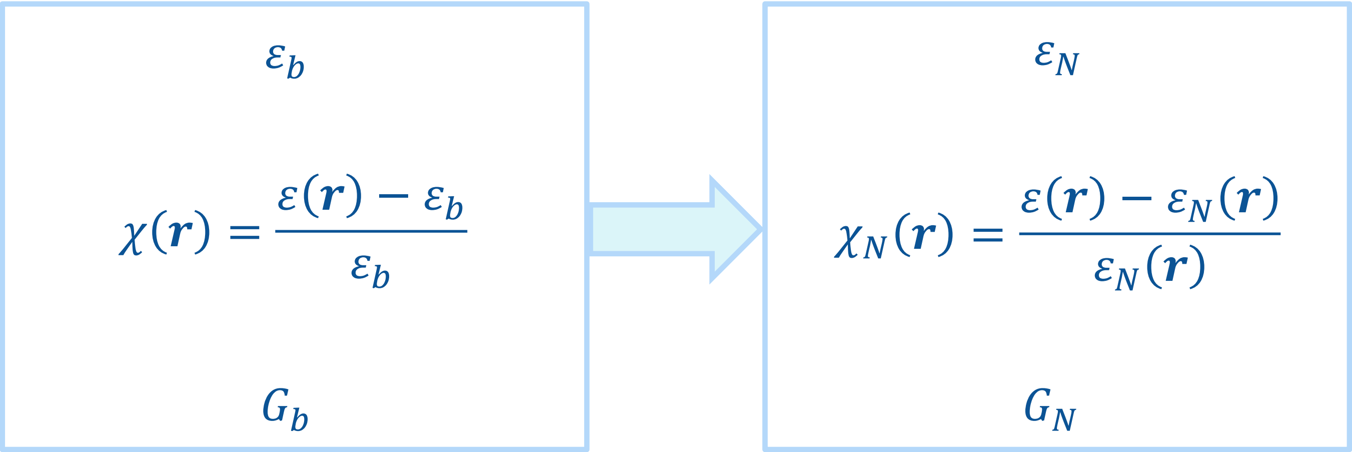
Incorporation of Spatial Priors within CSI

In Contrast Source Inversion (CSI), χ and W are estimated by minimizing:

$$\Phi(W,\chi) = \sum_t \frac{\|\chi E_i(\boldsymbol{r_t}) + \chi A_i[W(\boldsymbol{r_t})] - W(\boldsymbol{r_t})\|_2^2}{\|E_i(\boldsymbol{r_t})\|_2^2} + \sum_t \frac{\|E_s(\boldsymbol{r_t}) - A_e[W(\boldsymbol{r_t})]\|_2^2}{\|E_s(\boldsymbol{r_t})\|_2^2}$$

To integrate the prior information in CSI*:



ε_b

$$\chi(\boldsymbol{r}) = \frac{\varepsilon(\boldsymbol{r}) - \varepsilon_b}{\varepsilon_b}$$

G_b

ε_N

$$\chi_N(\boldsymbol{r}) = \frac{\varepsilon(\boldsymbol{r}) - \varepsilon_N(\boldsymbol{r})}{\varepsilon_N(\boldsymbol{r})}$$

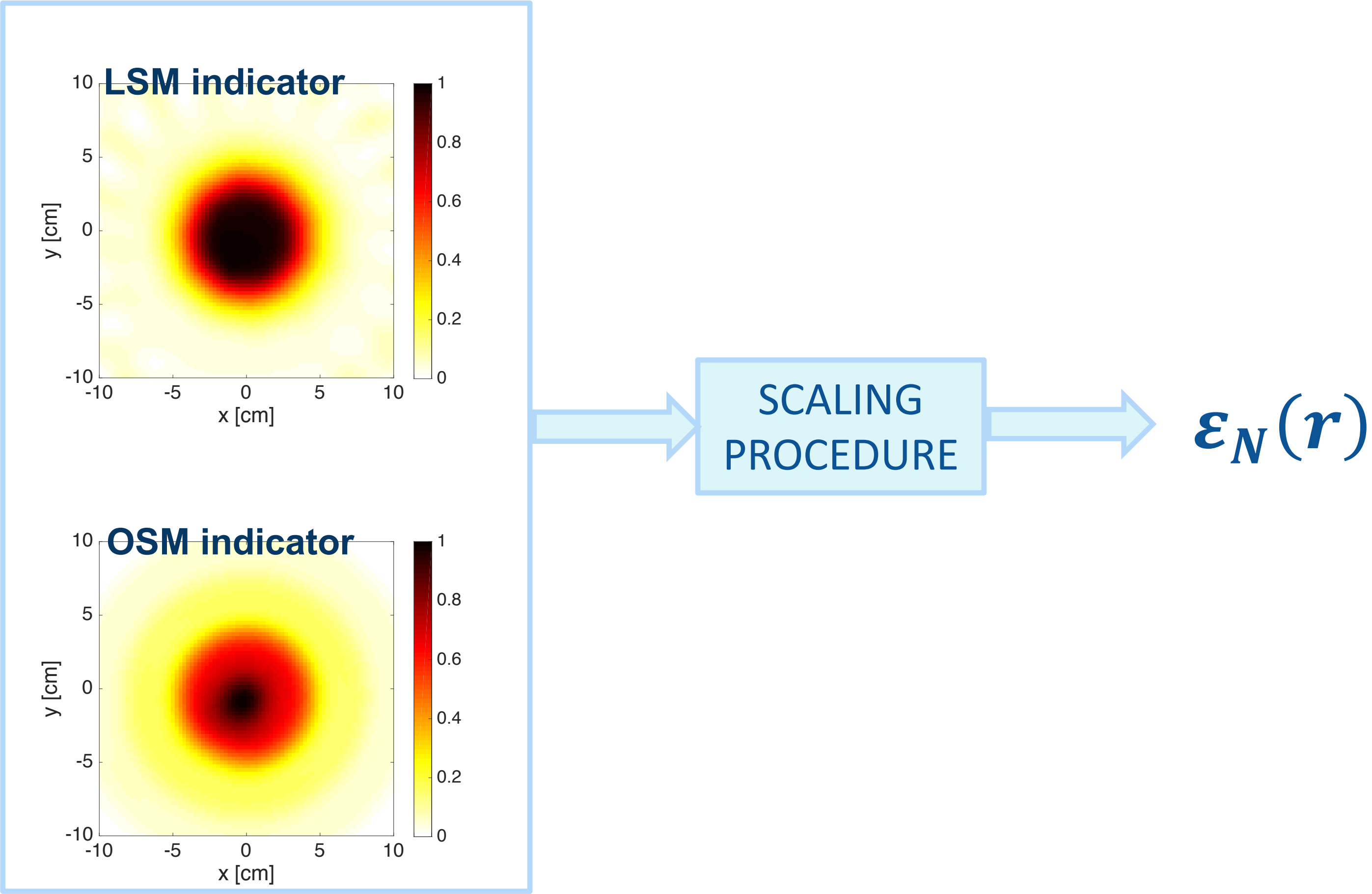
G_N

where G_N corresponds to the inhomogeneous background ε_N.

[*] Kurrant, et al., Integrating prior information into microwave tomography part 1: Impact of detail on image quality, Med. Physic 2017.

LSM and OSM for generating Spatial Prior Information

Different spatial priors $\varepsilon_N(\boldsymbol{r})$ can be generated by scaling the support indicators according to the expected value of permittivity and conductivity of the breast tissues.



SCALING PROCEDURE

$\boldsymbol{\varepsilon_N(r)}$

[*] Bevacqua et al., “Physical Insight Unveils New Imaging Capabilities of Orthogonality Sampling Method”, *IEEE TAP*, in print.

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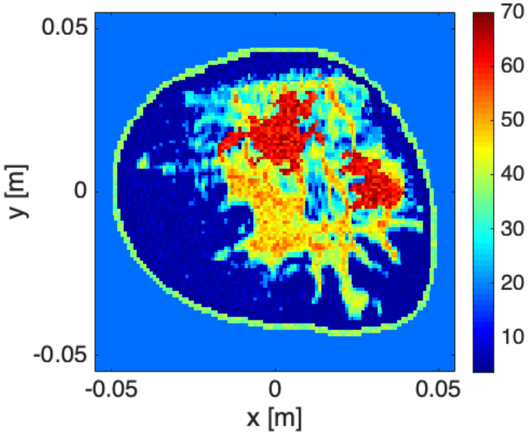
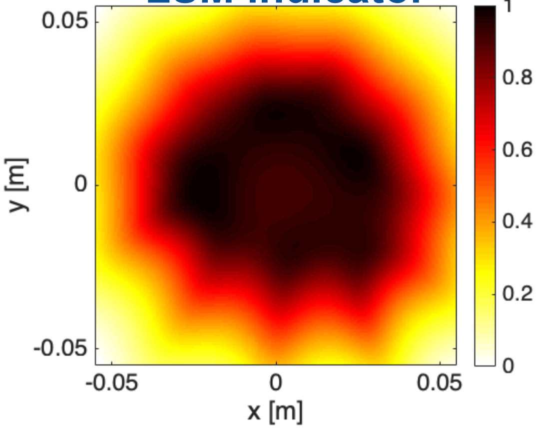
2D numerical example

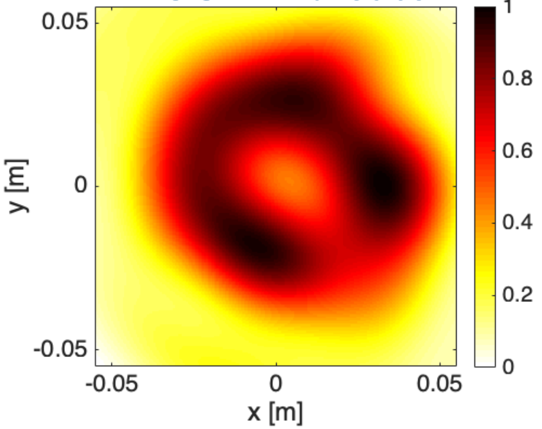
Permittivity of the adopted breast model*



LSM indicator



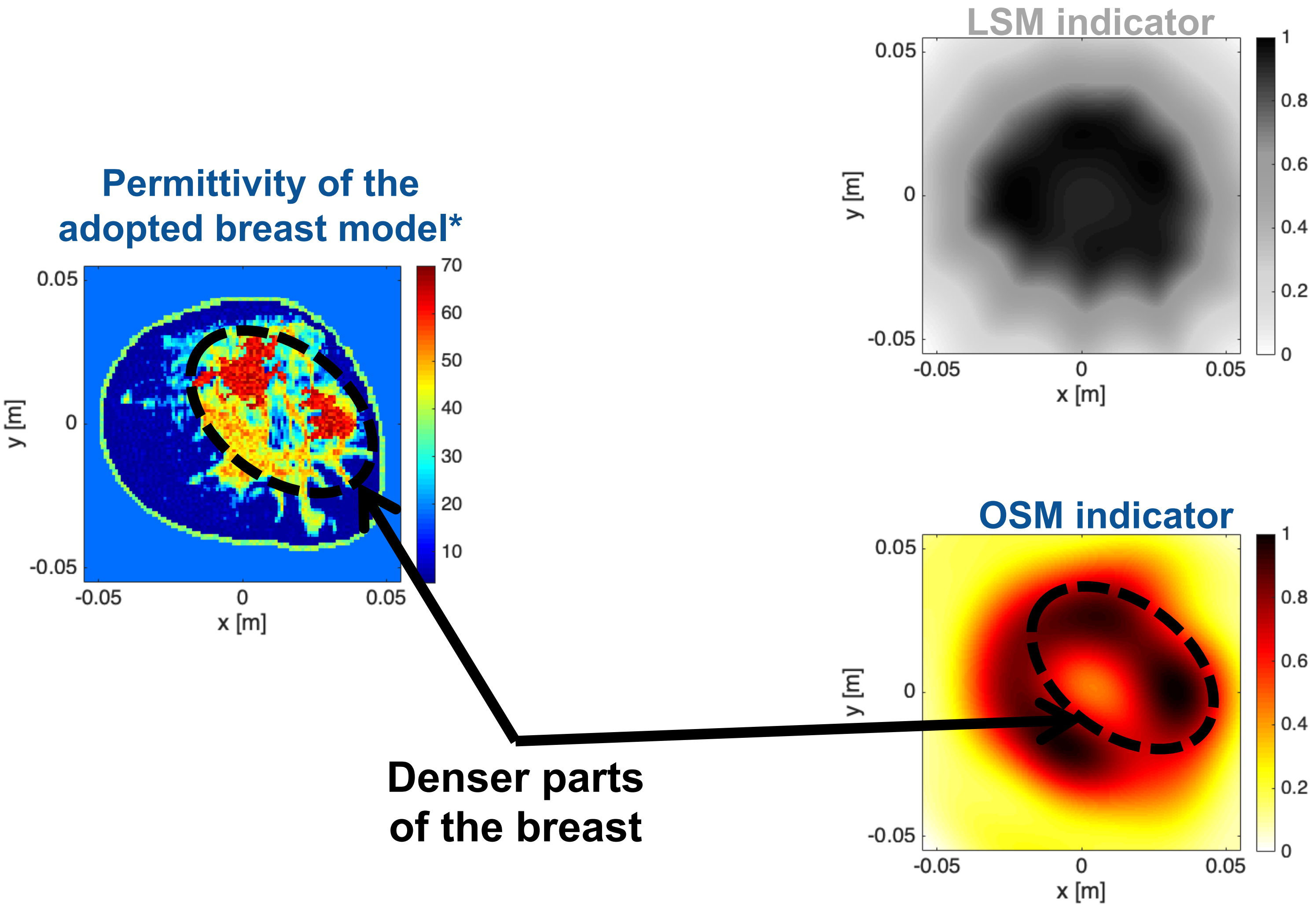
OSM indicator



INFO: $freq = 1\ GHz, R = 13\ cm,\ \ T = M = 24, SNR = 30dB, \varepsilon_b = 18, \sigma_b = 0$

[*] Anthropomorphic breast model repository for research and development of microwave breast imaging technologies. Scientific data, 2018.

2D numerical example



Permittivity of the adopted breast model*

LSM indicator

OSM indicator

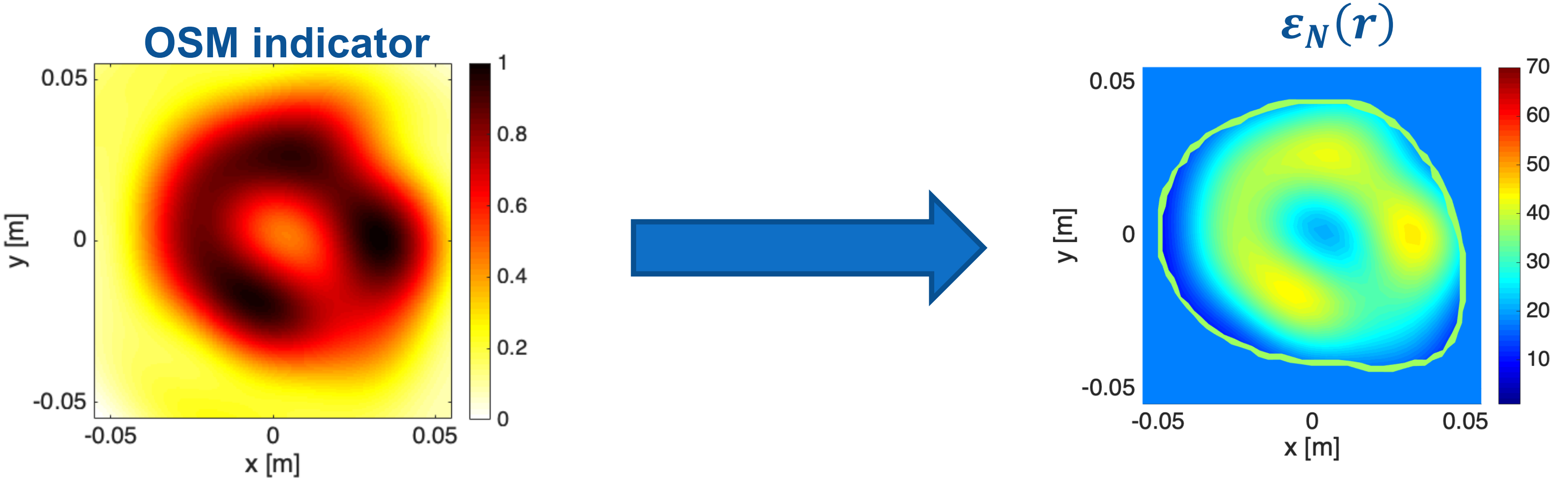
Denser parts of the breast

INFO: $freq = 1\ GHz, R = 13\ cm,\ \ T = M = 24, SNR = 30dB, \varepsilon_b = 18, \sigma_b = 0$

[*] Anthropomorphic breast model repository for research and development of microwave breast imaging technologies. Scientific data, 2018.

2D numerical example

The normalized OSM maps is modulated in amplitude according to the expected values of permittivity and conductivity of the breast tissues.



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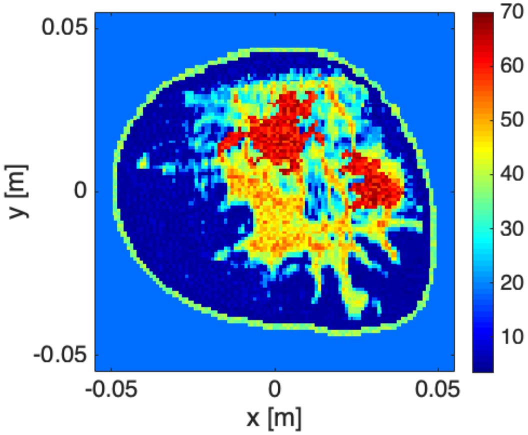
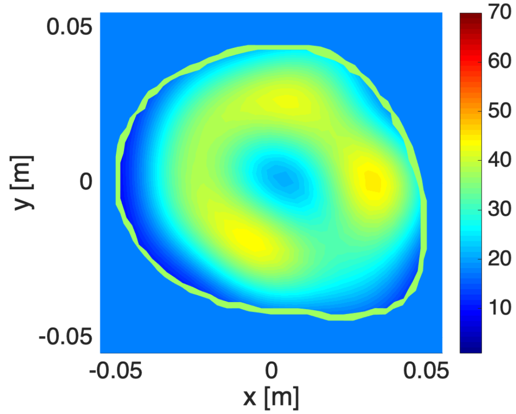
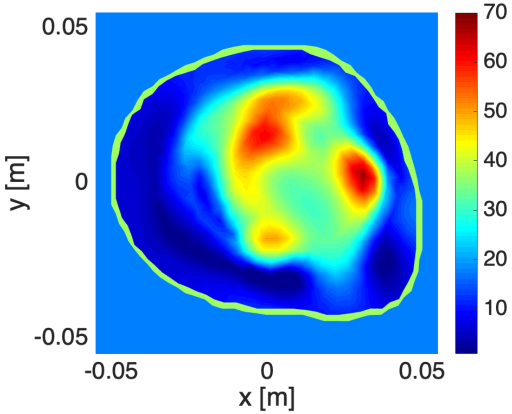


Permittivity of the adopted breast model*



$\boldsymbol{\varepsilon_N(r)}$

Retrieved permittivity



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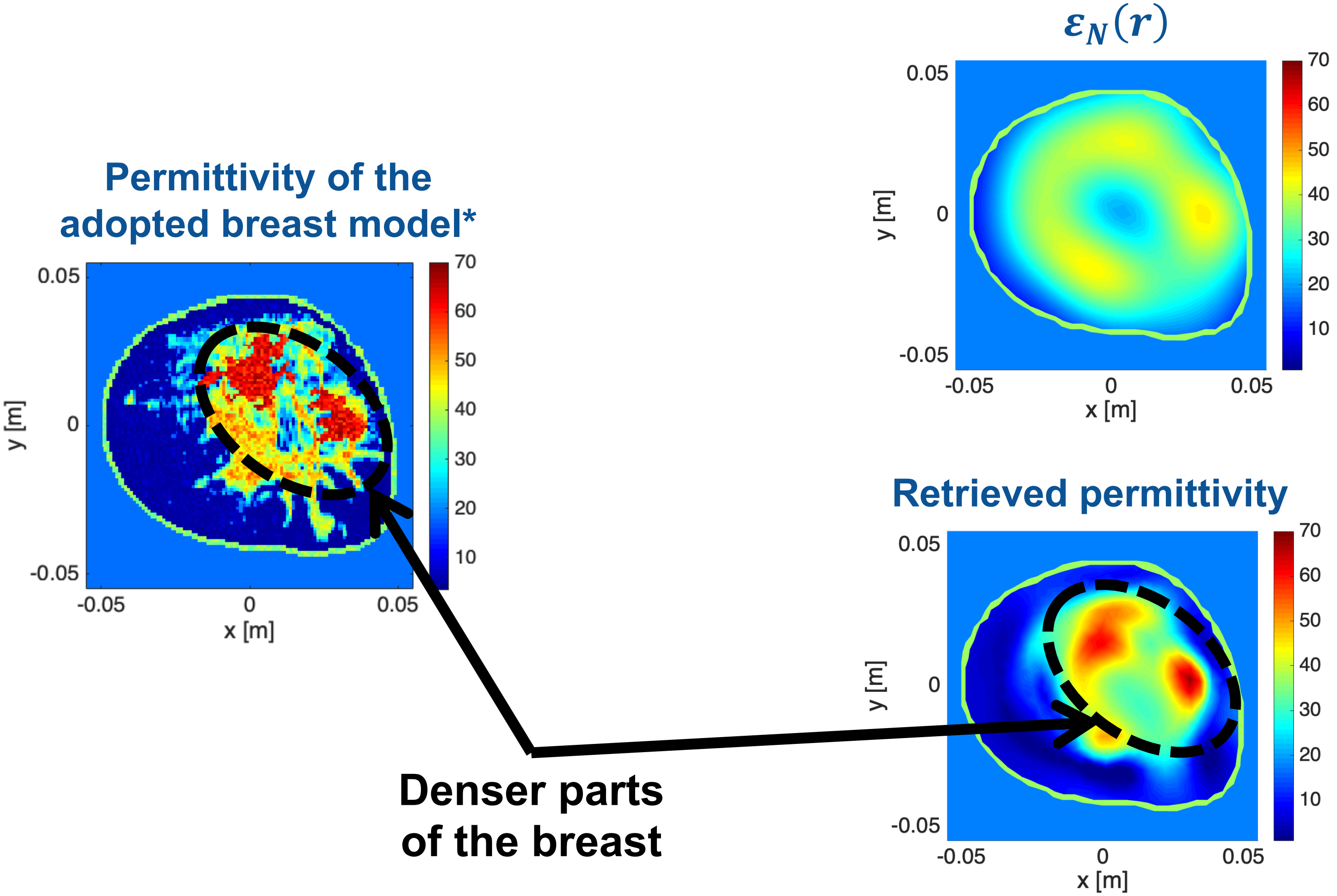

$\varepsilon_N(\boldsymbol{r})$

Permittivity of the adopted breast model*

Retrieved permittivity

Denser parts of the breast

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Conclusions

- Spatial priors to improve both the accuracy and resolution of microwave imaging reconstructions.
- A combination of linear/orthogonal sampling method maps to generate spatial priors in a simple and fast way.
- Incorporation with the CSI as inhomogeneous background.
- Preliminary 2D inversion against a heterogeneously dense breast model.
- Future works: comparisons with respect to the quantitative NIEI eigenfunction prior.*

[*] N. Abdollahi et al,,"Non-Iterative Eigenfunction-Based Inversion (NIEI) Algorithm for 2D Helmholtz Equation," PIER B, 2019.

Spatial Prior for Quantitative Breast Cancer Microwave Imaging: a Comparison Between Non-Iterative Eigenfunction-Based Inversion and Sampling Methods

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