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The polarizability of an alternative sequence of isotropic and radially anisotropic multilayer sphere

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Introduction

Aims and Objective

1:
The polarizability of a multilayer sphere that consists of an alternative sequence of layers of isotropic and Spherically Radially Anisotropic (SRA) material has been investigated.
2:
Within each SRA layer, components of the tensor permittivity have different values in radial and tangential directions.
3:
The mathematical treatment for extracting electrostatic polarizability has been formulated in terms of scattered potentials.
4:
Obtained results have been used to describe the behavior of a whole sphere as a function of number of the isotropic and SRA alternating layers using a numerical approach.

Formation

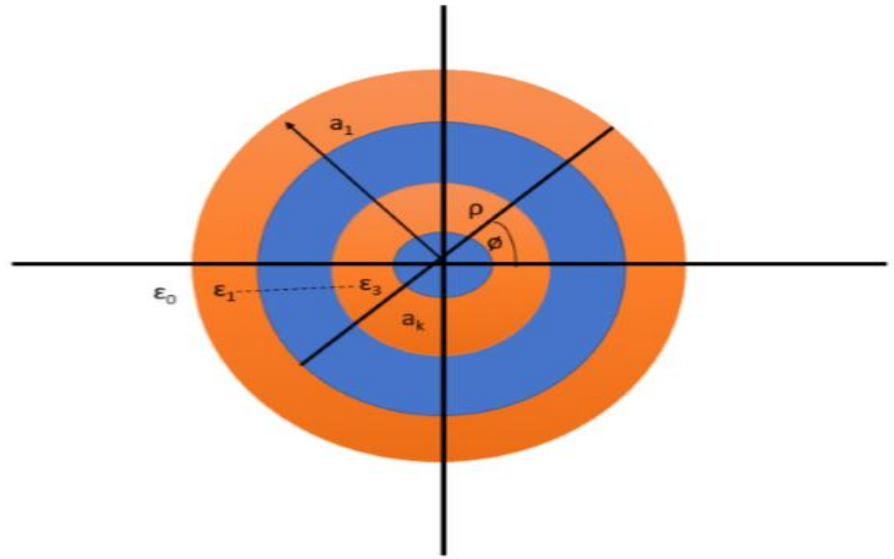

Figure 1. Geometry represents the alternative sequence of isotropic and anisotropic multilayer sphere immersed in free space.

Formation and Methodology

1:

Let us consider a multilayer inhomogeneous sphere with radius a_k and consists of an alternative sequence of isotropic and SRA layers, such that the core is isotropic.

2:

The Core is covered by SRA layers, then next layer is again isotropic and that is surrounded by another SRA layer and sequence continues.

3:

The radius of the outer layer is fixed and equal to a_1 and the internal radii.

$$a_k = \frac{N-(k-1)}{N} a_1$$

Formation and Methodology

With excitation from external electric field $\mathbf{E} = E\hat{\mathbf{z}}_\mathbf{0}$, solution that satisfies Laplace's equation in an arbitrary k_{th}

isotropic layer can be written as

$$\Phi_k = B_k r \cos\phi + C_k r^{-2} \cos\phi$$

whereas, for SRA layer it will be

$$\Phi_k = B_k r^{\nu} \cos\theta + C_k r^{-\nu-1} \cos\theta$$

where

$$\nu = \frac{1}{2}\left(-1 + \sqrt{1 + \frac{\varepsilon_t}{\varepsilon_r}}\right)$$

The boundary condition between any two adjacent layer k and k+1 must be

$$\begin{pmatrix} B_k \\ C_k \end{pmatrix} = [Q_k] \begin{pmatrix} B_{k+1} \\ C_{k+1} \end{pmatrix}$$

where

$$[Q_k] = \frac{1}{2\gamma_k \varepsilon_k} \begin{pmatrix} Q_{k11} & Q_{k12} \\ Q_{k21} & Q_{k22} \end{pmatrix}$$

and:

$$Q_{k11} = [2\varepsilon_{k+1} + \varepsilon_{k+1}]a_{k+1}^{\nu_{k+1}-1}$$
$$Q_{k12} = [2\varepsilon_k - (\nu_{k+1} + 1)\varepsilon_{k+1}]\, a_{k+1}^{-\nu_{k+1}-2}$$
$$Q_{k21} = [\varepsilon_k - \nu_{k+1}\varepsilon_{k+1}]\, a_{k+1}^{\nu_{k+1}+2}$$
$$Q_{k22} = [\varepsilon_k + (\nu_{k+1} + 1)\varepsilon_{k+1}]\, a_{k+1}^{-\nu_{k+1}+1}$$

Since the core is isotropic and the total number of layers is $k = 1, 2, 3......N$, which implies that for every layer with odd number in entire sequence such that $k = 1, 3......N-1$ we need to insert $\nu = 1$. For remaining layers $k = 2, 4......N$, ν will have value as in Eq. (5).

When we take into account all layers, we obtain the following relationship

$$\begin{pmatrix} B_0 \\ C_0 \end{pmatrix} = \prod_{k=0}^{N-1} [Q_k] \begin{pmatrix} B_N \\ C_N \end{pmatrix} = [Q] \begin{pmatrix} B_N \\ 0 \end{pmatrix}$$

with

$$[Q] = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}$$

$C_N = 0$, because the core does not contain any reflected field. The generalized expression of polarizability can be

$$\alpha_P = \frac{3V\varepsilon_0}{a_1^3} \frac{Q_{21}}{Q_{11}}$$

where, ε_0 is the vacuum's permittivity and V is the volume of the sphere, i.e., $V = \frac{4}{3}\pi r^3$. The effective permittivity can be derived with simple algebra and polarizability from Eq.

$$\varepsilon_{eff} = \varepsilon_0 + \frac{\frac{\alpha_P}{V}}{1 - \frac{\alpha_P}{3\varepsilon_0 V}}$$

As an illustration,we can write the polarizability by taking the case $N = 1$

$$\alpha_P = 3V\varepsilon_0 \frac{(\varepsilon_0 - \nu_1\varepsilon_1)}{(2\varepsilon_0 + \nu_1\varepsilon_1)}$$

For the case $N = 2$, we obtain the polarizability of two concentric isotropic and SRA spheres

$$\frac{\alpha_P = 2V\varepsilon_0\left\{(A_1 * A_2) + (A_3 * A_4)\right\}\left(a_2{}^3/a_1^{2\nu_1+1}\right)}{\left\{(A_5 * A_2) + (A_6 * A_4)\right\}\left(a_2{}^3/a_1^{2\nu_1+1}\right)}$$

where

$$A_1 = (\varepsilon_0 - \nu_1 \varepsilon_1)$$
$$A_2 = (2\varepsilon_1 + \nu_2 \varepsilon_2)$$
$$A_3 = (\varepsilon_0 + (\nu_1 + 1)\varepsilon_1)$$
$$A_4 = (\varepsilon_1 - \nu_2 \varepsilon_2)$$
$$A_5 = (2\varepsilon_0 + \nu_1 \varepsilon_1)$$
$$A_6 = (2\varepsilon_0 - (\nu_1 + 1)\varepsilon_1)$$

Similarly, we can find the polarizability of a multilayer inhomogeneous sphere with consecutive isotropic and SRA layers and consisting of an arbitrary number of layers.

Numerical Results and Discussion

We have implemented above equation of polarizability of inhomogeneous multilayer sphere as function of number of layers, using Matlab code.

We have fixed the following parameters

Radius of inner core = a_2=0.70;
Outer shell a_1=1;
Anisotropic ratio ={1,4}
Permittivity of two concentric layers={2,4}

In Fig. 2, we can see that, for the case when permittivity of cover layer $\varepsilon_2 = 4$ has more weight as compared to the opposite case when permittivity of coverlayer $\varepsilon_2 = 2$ initially. Then afterward with an increase in the number of layers in both cases polarizability approaches to zero. For the case

N =2, the same trend is observed for outer layer permittivity, $\varepsilon_2 = 4$and $\varepsilon_2 = 2$ Respectively, and has a constant value. In Fig. 3, we investigated the behavior of inhomogeneous isotropic multilayer sphere. In this case, when permittivity of cover layer $\varepsilon_2 = 4$ and permittivity of inner layer $\varepsilon_1 = 2$, polarizability has more weight as compared to the other case, when permittivities of cover and inner layers are $\varepsilon_2 = 2$ and $\varepsilon_1 = 4$ respectively. Hence with an increasing number of layers in both cases polarizability approaches to a negative value. When N = 2, a similar pattern is noticed for both cases of outer layer permittivity, $\varepsilon_2 = 4$ and $\varepsilon_2 = 2$ respectively.

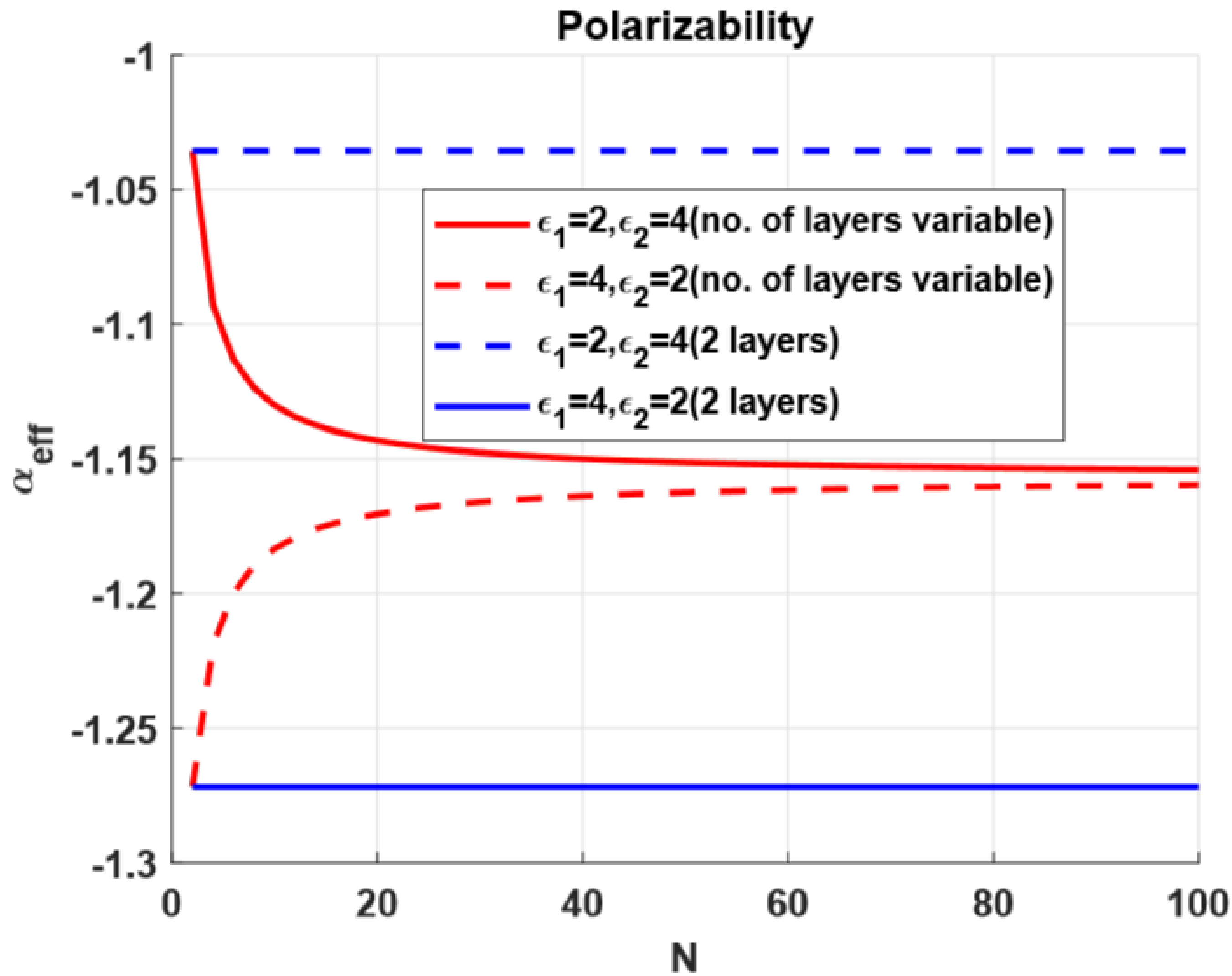

Figure 3. Normalized polarizability of an isotropic multilayer sphere as a function of the number of layers, with the following parameters: ε_1=2; ε_2=4; $\nu_1 = \nu_2 = 1$.

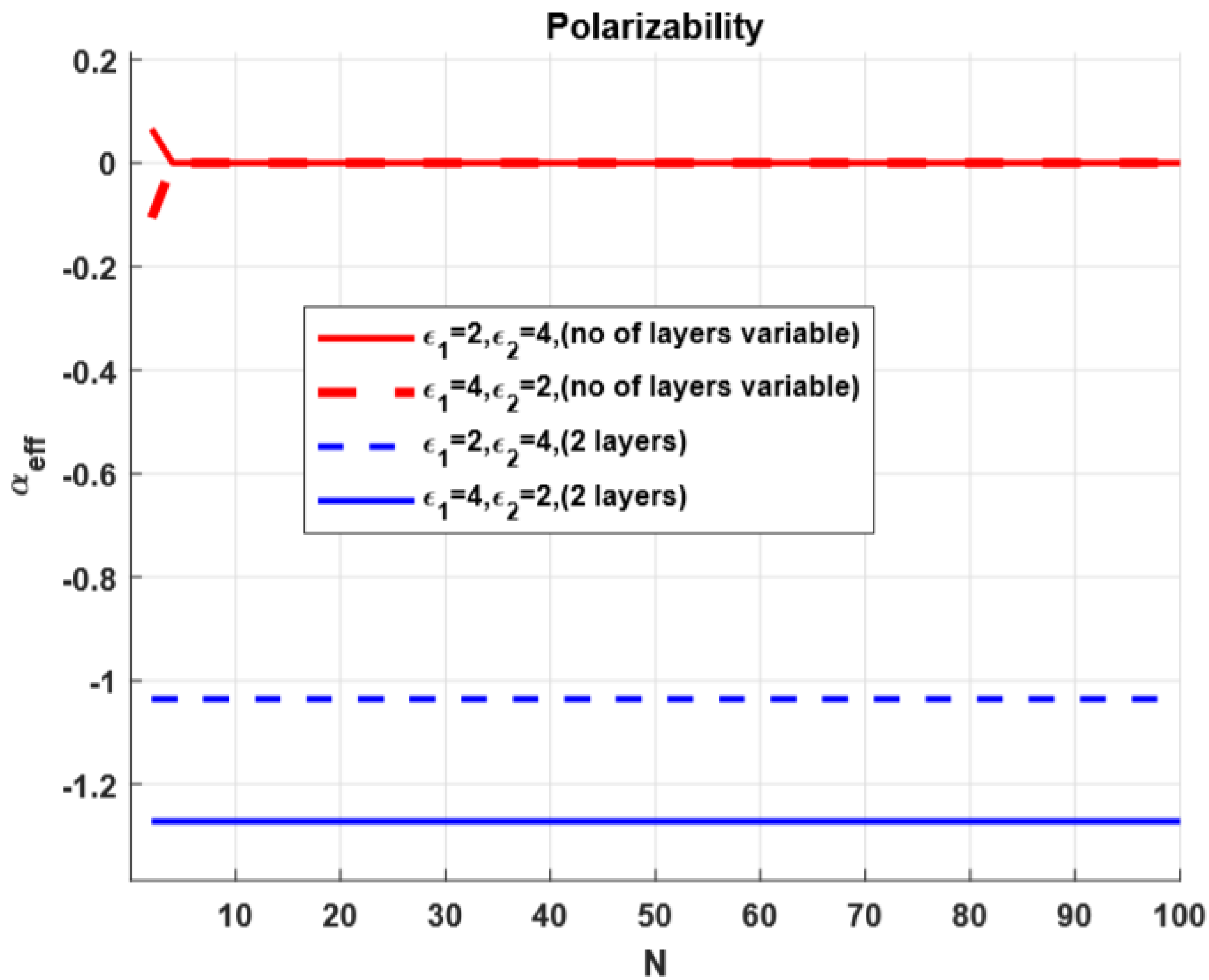

Figure 2. Normalized polarizability of alternative sequence of isotropic and SRA multilayer sphere as a function of the number of layers, with the following parameters: ε_1=2; ε_2=4; $v_1 = 1$ and $v_2 = 4$.

Conclusions

During our analysis, we have calculated a closed form of the polarizability of an inhomogeneous multilayer sphere having consecutive sequences of isotropic and SRA layers. The total number of layers is arbitrary and two alternating values of isotropic and anisotropic permittivity are employed. It has been detected that when layers of a sphere are alternatively isotropic and SRA, polarizability is zero for a higher number of layers. Whereas, when all consecutive layers are isotropic, polarizability is not zero rather it is near to zero. We will continue inspecting this model of inhomogeneous multilayer sphere for different applications.

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