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Finding the polarizability of radially anisotropic multilayer circular cylinder

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Introduction

Aims and Objective

1:
We have studied the electrostatic response of a Polarly Radially Anisotropic (PRA) multilayer circular cylinder. It consists of different components of the permittivity in radial and tangential directions for each layer

2:
We have considered the familiar model of quasi-static for obtaining the polarizability and effective permittivity of the PRA multilayer circular cylinder.

3:
We also have studied the behavior of polarizability of PRA multilayer circular cylinder, as a function of the number of anisotropic alternating layers by using a numerical approach

Let us consider a PRA multilayer circular cylinder with radius ak and alternative sequence of relative permittivity ε $_1$,

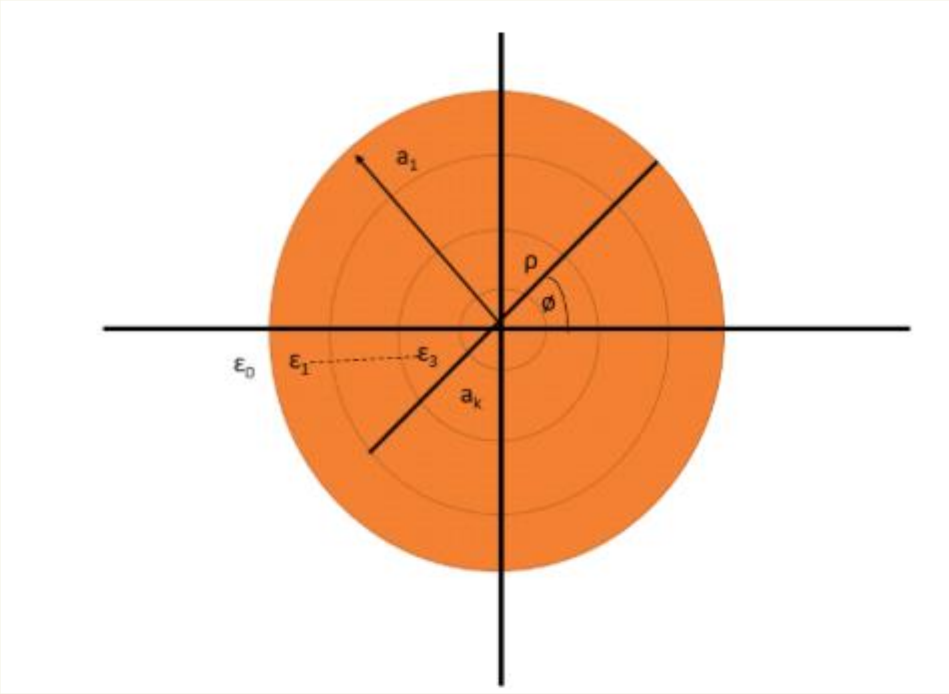
The cylinder may have an arbitrary axial permittivity component ε z, but since the axial electric field has not been excited, so this component has been omitted in our analysis. The radius of the outer cylinder is fixed and equal to a_1 and the internal radii may be written as

$$a_k = \frac{N-(k-1)}{N} a_1 \qquad (2)$$

The number of layers, N is arbitrary and $k = 1, 2, 3 \ldots\ldots N$.

Geometry

Formation:



Formation and Mathedology

To solve for the electric potential, the geometry has been excited by the electric field $\mathbf{E} = E\hat{\mathbf{x}}_0$ and Laplace's equation has been solved for cylindrical coordinates. The solution to the Laplace's equation in an arbitrary k_{th} subregion is as follows [9]

$$\Phi_k = B_k \rho^{\gamma} \cos\phi + C_k \rho^{-\gamma} \cos\phi \tag{3}$$

where the coefficient B_k represents the amplitude of the constant field component whereas the coefficient C_k describes the dipole-type contribution to the field. To solve the unknowns coefficients B_k and C_k, transmission-line method has been used [10]. The boundary conditions applied between any two adjacent subregions k and $k+1$ can be written as

$$\begin{pmatrix} B_k \\ C_k \end{pmatrix} = [P_k] \begin{pmatrix} B_{k+1} \\ C_{k+1} \end{pmatrix} \tag{4}$$

where

$$[P_k] = \frac{1}{2\gamma_k \varepsilon_k} \begin{pmatrix} P_{k11} & P_{k12} \\ P_{k21} & P_{k22} \end{pmatrix} \tag{5}$$

and:

$$\begin{aligned}
P_{k11} &= (\gamma_k \varepsilon_k + \gamma_{k+1} \varepsilon_{k+1})\, a_{k+1}^{\gamma_{k+1} - \gamma_k} \\
P_{k12} &= (\gamma_k \varepsilon_k - \gamma_{k+1} \varepsilon_{k+1})\, a_{k+1}^{-\gamma_{k+1} - \gamma_k} \\
P_{k21} &= (\gamma_k \varepsilon_k - \gamma_{k+1} \varepsilon_{k+1})\, a_{k+1}^{\gamma_{k+1} + \gamma_k} \\
P_{k22} &= (\gamma_k \varepsilon_k + \gamma_{k+1} \varepsilon_{k+1})\, a_{k+1}^{-\gamma_{k+1} + \gamma_k}
\end{aligned}$$

It is easily verifiable, when $\gamma_k = \gamma_{k+1} = 1$, the above expression is reduced into the case of isotropic multilayer circular cylinder.

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When we have considered all layers, we reached the following explicit relationship

$$\begin{pmatrix} B_0 \\ C_0 \end{pmatrix} = \prod_{k=0}^{N-1} [P_k] \begin{pmatrix} B_N \\ C_N \end{pmatrix} = [P] \begin{pmatrix} B_N \\ 0 \end{pmatrix} \tag{6}$$

with

$$[P] = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \tag{7}$$

As we know, that there is no reflected field component in the core region, i.e., $C_N = 0$. Here, the expression of polarizability is [9]

$$\alpha_P = \frac{2V\varepsilon_0}{a_1^2} \frac{P_{21}}{P_{11}} \tag{8}$$

where, ε_0 is the vacuum's permittivity and V represents the volume of the cylinder, i.e., $V = \pi r^2 h$. This also allows us to obtain the expression of an effective permittivity for multilayer PRA circular cylinder

$$\varepsilon_{eff} = \varepsilon_0 + \frac{\frac{\alpha_P}{V}}{1 - \frac{\alpha_P}{2\varepsilon_0 V}} \tag{9}$$

The polarizability of PRA circular cylinder in free space can be computed by taking $N = 1$; for this trivial case we have:

$$\alpha_P = 2V\varepsilon_0 \frac{(\gamma_0\varepsilon_0 - \gamma_1\varepsilon_1)}{(\gamma_0\varepsilon_0 + \gamma_1\varepsilon_1)} \tag{10}$$

For the case $N = 2$, the polarizability of two concentric PRA circular cylinders can be derived.

$$\alpha_P = 2V\varepsilon_0 \frac{(\gamma_0\varepsilon_0 - \gamma_1\varepsilon_1)(\gamma_1\varepsilon_1 + \gamma_2\varepsilon_2) + (\gamma_0\varepsilon_0 + \gamma_1\varepsilon_1)(\gamma_1\varepsilon_1 - \gamma_2\varepsilon_2)\left(\frac{a_2}{a_1}\right)^{2\nu_1\gamma}}{(\gamma_0\varepsilon_0 + \gamma_1\varepsilon_1)(\gamma_1\varepsilon_1 + \gamma_2\varepsilon_2) + (\gamma_0\varepsilon_0 - \gamma_1\varepsilon_1)(\gamma_1\varepsilon_1 - \gamma_2\varepsilon_2)\left(\frac{a_2}{a_1}\right)^{2\nu_1\gamma}} \tag{11}$$

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Numerical Results and Discussion

We have implemented above equation of polarizability of inhomogeneous multilayer cylinder as function of number of layers, using Matlab code.

We have fixed the following parameters

Radius of inner core = a_2=0.70;

Outer shell a_1=1;

Anisotropic ratio ={3,4}

Permittivity of two concentric layers={2,4}

In Fig. 2 we can see that for a variable number of layers, the value of the polarizability when the permittivity of the cover layer $\varepsilon_2 = 4$ has more weight as compared to the case when permittivity of cover layer is $\varepsilon_2 = 2$. Similarly, for the case of two layers, the same trend is observed for cover layers with permittivity $\varepsilon_2 = 4$ and $\varepsilon_2 = 2$ respectively. The polarizability of isotropic multilayer circular cylinder placed in free space, by inserting anisotropic ratios $\gamma_0 = \gamma_1 = \gamma_2 = 1$ is shown in Fig. 3.

During our numerical test, we have observed polarizability of isotropic multilayer circular cylinder and PRA multilayer circular cylinder with arbitrary number of layers by using two alternating values of anisotropic permittivity. We are hopeful that our improved approach will help researchers in developing applications. We will continue this model of PRA multilayer circular cylinder for cloaking application.

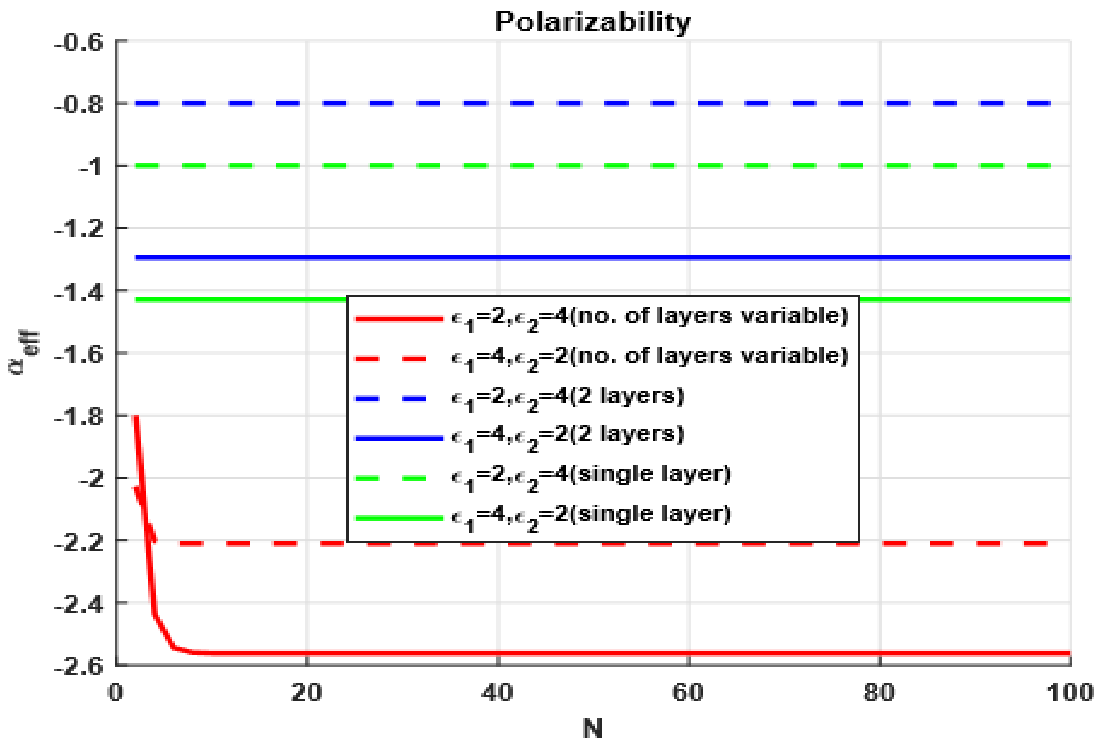

Figure 2. Normalized polarizability of PRA circular cylinder as a function of the number of layers, with the following parameters: ε_1=2; ε_2=4; γ_0=2, γ_1=3 and γ_2=4.

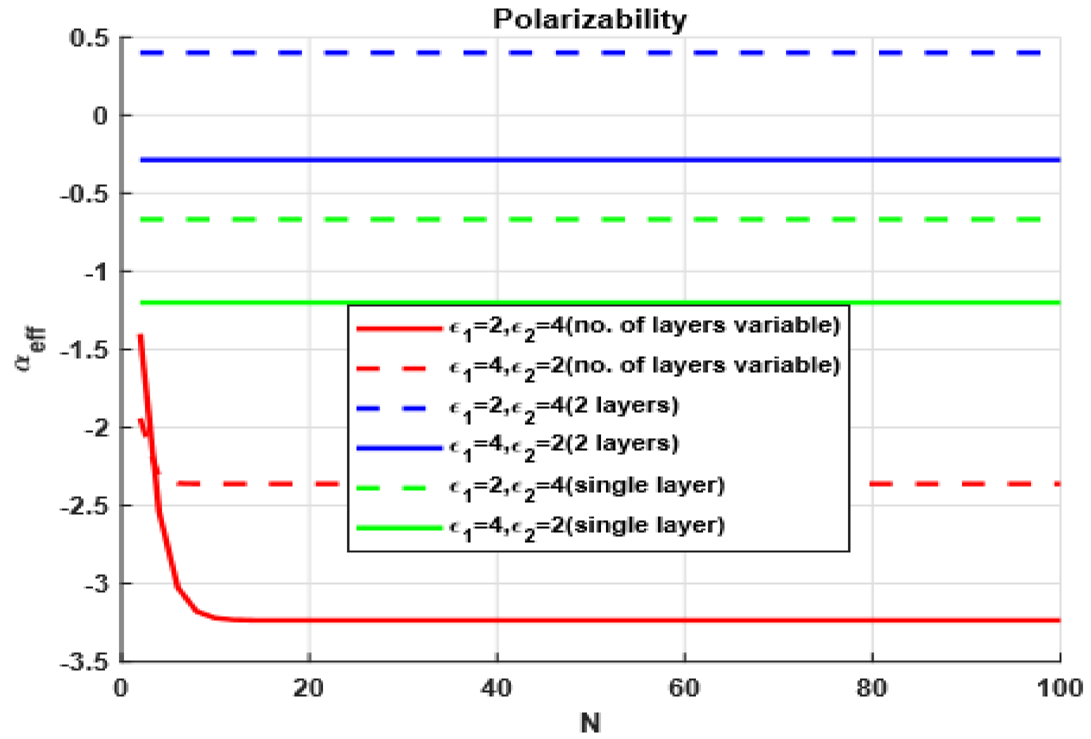

Figure 3. Normalized polarizability of an isotropic circular cylinder as a function of the number of layers, with the following parameters: ε_1=2; ε_2=4; $\gamma_0 = \gamma_1 = \gamma_2 = 1$.

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