High-frequency Lengthwise Diffraction by the Line Separating Soft and Hard Parts of the Surface

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URSI GASS 2020

Effects of diffraction by a discontinuity in the boundary condition

Studied in many papers • S.Hong, V.H.Weston, Radio Science, 1966, • N.C.Albertsen, P.L.Christiansen, SIAM J.Applied Math., 1978, • V.Philippov, Wave Motion, 1981, • M.Bérengier, G.A.Daigle, J. Acoust. Soc. Am., 1988, • I.Andronov, D.Bouche, N.Kirpichnikova, V.Philippov, Ann.Télécommun., 1997, • V.Philippov, N.Kirpichnikova, S.Kirpichnikov, J.Math.Sci., 2000.

But the direction of the wave is usually orthogonal to the line where the boundary condition changes.

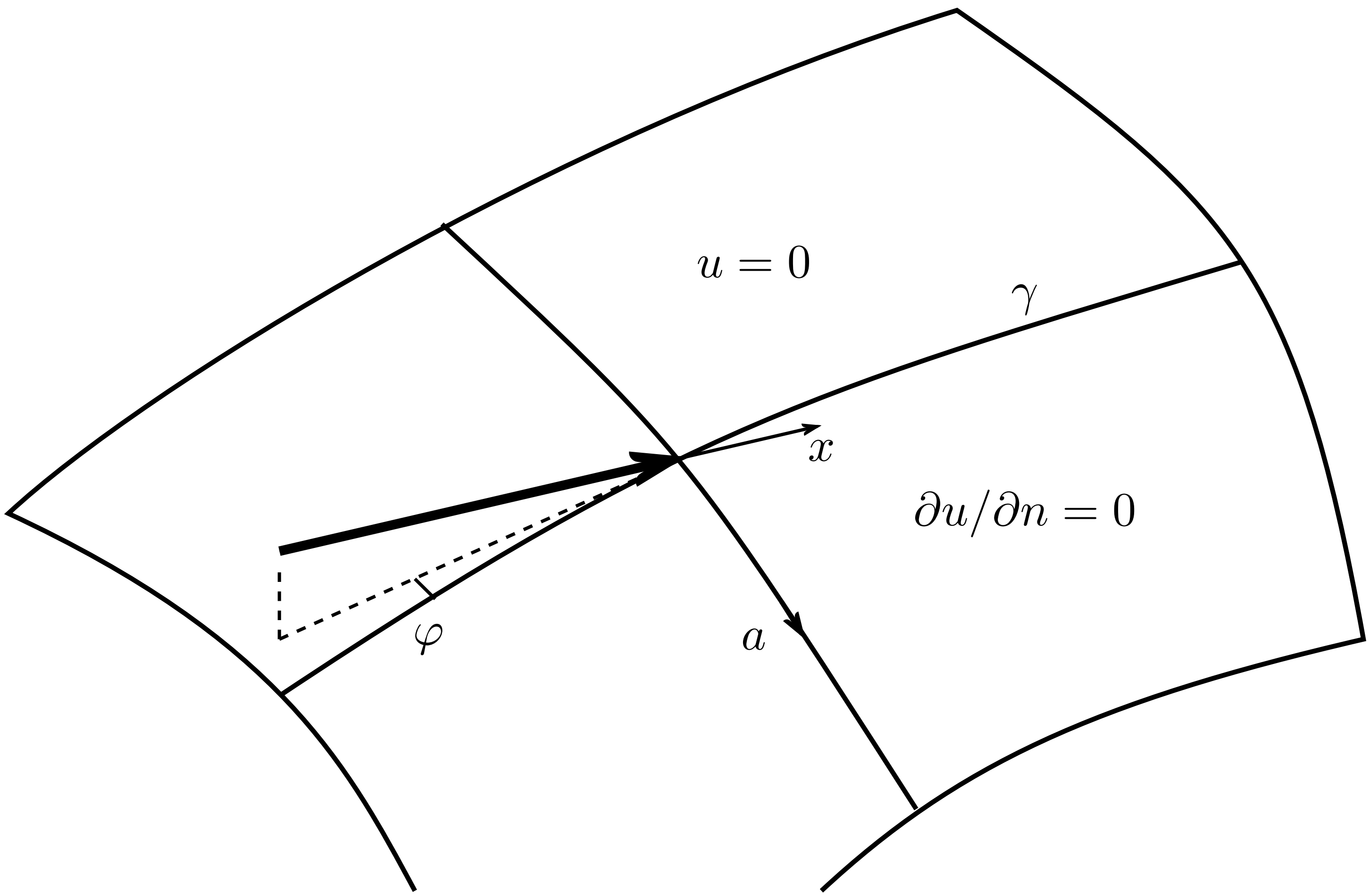
First attempt to consider the case when these directions are almost parallel.

Model problem (scalar waves, soft-hard surface)

Specific effect is that creeping waves have transverse structure with a rapid change of the amplitude.

New test problem.

Geometry of the problem



High-frequency approximation. Large parameter $m = (k\rho/2)^{1/3}$.

Semi-geodesic coordinates (s, a, n)

Light-shadow boundary is $s = s_0(a)$, curve γ is $a = a_0(s)$.

Assumptions:

- Angle φ is small, that is $\psi \equiv m a_0'(0)$ is finite, $|\psi| < \psi_0 = O(1)$.
- Curve γ is close to a geodesics, $\rho a_0''(s) = o(1)$.

Anzats

$$u = e^{iks} \sum_{\ell=0}^{\infty} m^{-\ell} U_\ell(\sigma, \alpha, \nu),$$

where

$$\sigma = m\frac{s}{\rho(0)}, \quad \nu = 2m^2\frac{n}{\rho(0)}, \quad \alpha = 2m^2\frac{a}{\rho(0)} + 2\psi\sigma$$

The boundary value problem for the Helmholtz equation is reduced to the parabolic equation

$$i\frac{\partial U_0}{\partial \sigma} + \frac{\partial^2 U_0}{\partial \nu^2} + \frac{\partial^2 U_0}{\partial \alpha^2} + 2i\psi\frac{\partial U_0}{\partial \alpha} + \nu U_0 = 0$$

with the boundary conditions

$$U_0|_{n=0} = 0, \quad \alpha < 0; \qquad \left.\frac{\partial U_0}{\partial n}\right|_{n=0} = 0, \quad \alpha > 0.$$

Fourier transforms with respect to σ and α result in the Riemann problem on the real axis of dual to α variable. This allows explicit solution to be found

$$U_0 = \frac{e^{i\psi^2\sigma}}{\sqrt{\pi}} \int\limits_{-\infty}^{+\infty} e^{i\xi\sigma} \Big\{ v(\xi + \psi^2 - \nu) - \frac{v'(\xi+\psi^2)}{w_1'(\xi+\psi^2)} w_1(\xi + \psi^2 - \nu)$$

$$+ \frac{i}{2\pi} \frac{e^{-i\alpha\psi}}{w_1'(\xi+\psi^2)} \int\limits_{-\infty}^{+\infty} e^{-i\alpha\mu} \frac{G^{(+)}(\mu,\xi)}{G^{(+)}(-\psi,\xi)} \frac{w_1(\xi+\mu^2-\nu)}{w_1(\xi+\mu^2)} \frac{d\mu}{\mu+\psi-i0} \Big\} d\xi$$

where $v()$ and $w_1()$ are the Airy functions in Fock notations and $G^{(\pm)}$ are analytic functions in the upper/lower half-plane of μ which perform multiplicative factorization of the symbol

$$G(\xi,\mu) = \frac{w_1(\xi+\mu^2)}{w_1'(\xi+\mu^2)}.$$

This factorization is based on the representation of Airy functions as infinite products (Weierstrass factorization theorem)

$$w_1(z) = w_1(0)\,\mathrm{exp}\left(\frac{w_1'(0)}{w_1(0)}z\right)\prod_{n=1}^{\infty}\left[\left(1-\frac{z}{t_n}\right)\mathrm{exp}\left(\frac{z}{t_n}\right)\right],$$

$$w_1'(z) = w_1'(0)\prod_{n=1}^{\infty}\left[\left(1-\frac{z}{t_n'}\right)\mathrm{exp}\left(\frac{z}{t_n'}\right)\right],$$

where t_n and t_n' are the zeros of $w_1(z)$ and $w_1'(z)$ and have the asymptotics

$$t_n \sim e^{\pi i/3}\left[\frac{3}{2}\pi\left(n-\frac{1}{4}\right)\right]^{\frac{2}{3}}, \quad t_n' \sim e^{\pi i/3}\left[\frac{3}{2}\pi\left(n-\frac{3}{4}\right)\right]^{\frac{2}{3}} \text{ as } n \to \infty.$$

This yields

$$G = \frac{w_1(\xi)}{w_1'(\xi)}G^{(+)}(\xi,\mu)G^{(-)}(\xi,\mu), \quad G^{(\pm)}(\xi,\mu) = \prod_{n=1}^{\infty}\frac{1\pm\frac{\mu}{\sqrt{t_n-\xi}}}{1\pm\frac{\mu}{\sqrt{t_n'-\xi}}}.$$

Numerical results

$$U_0 = \frac{1}{\sqrt{\pi}} \int\limits_{-\infty}^{+\infty} e^{i\lambda\sigma} \Big\{ v(\lambda-\nu) - \frac{v'(\lambda)}{w_1'(\lambda)} w_1(\lambda-\nu) + \frac{i}{2\pi} \frac{e^{-i\alpha\psi}}{w_1'(\lambda)} \times$$

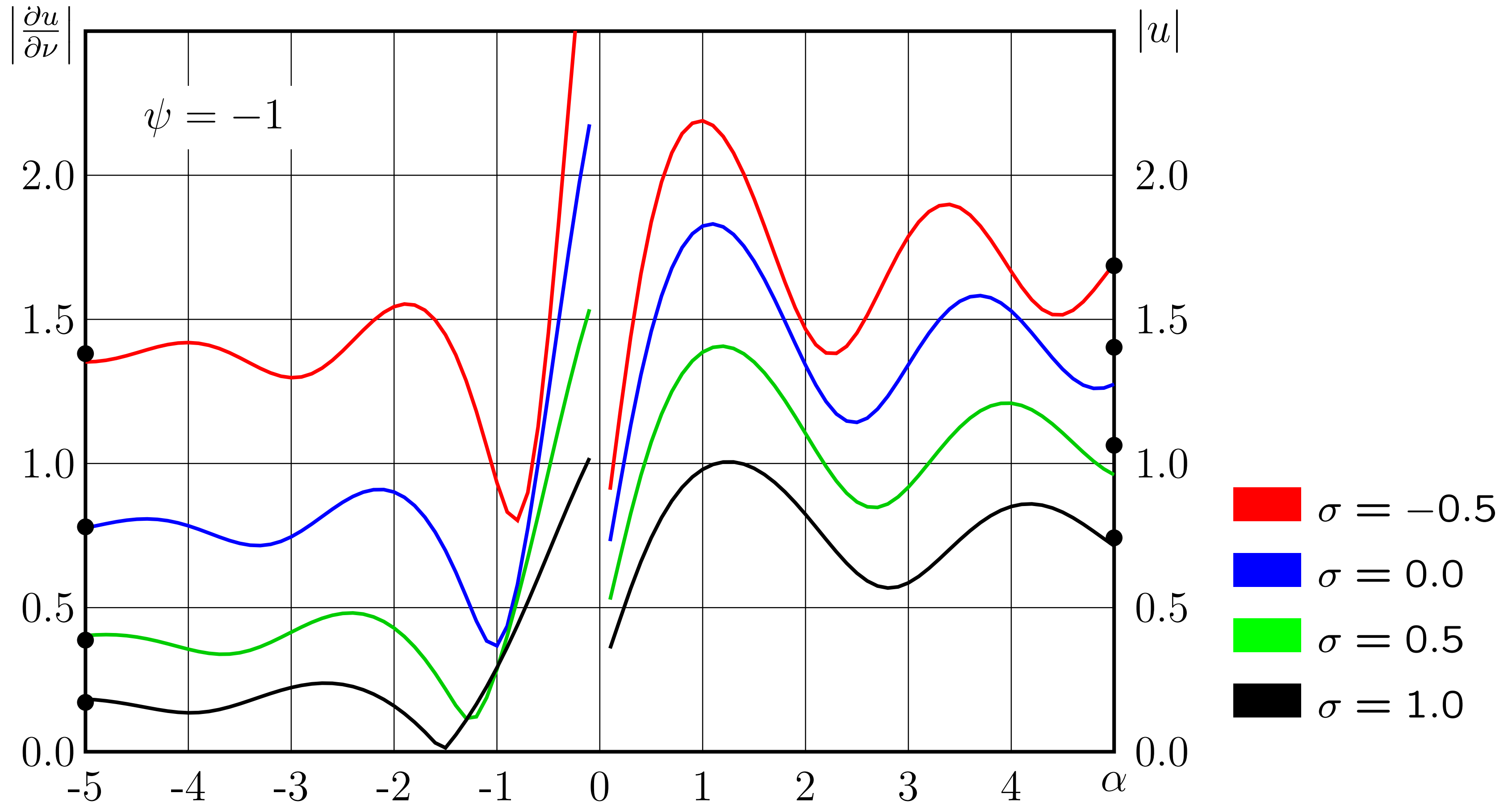
$$\times \int\limits_{-\infty}^{+\infty} e^{-i\alpha\mu} \frac{G^{(+)}(\mu, \lambda-\psi^2)}{G^{(-)}(\psi, \lambda-\psi^2)} \frac{w_1(\lambda-\psi^2+\mu^2-\nu)}{w_1(\lambda-\psi^2+\mu^2)} \frac{d\mu}{\mu+\psi-i0} \Big\} d\lambda$$

The inner integral (with respect to μ) can be reduced to the infinite sum of residues, though convergence is as

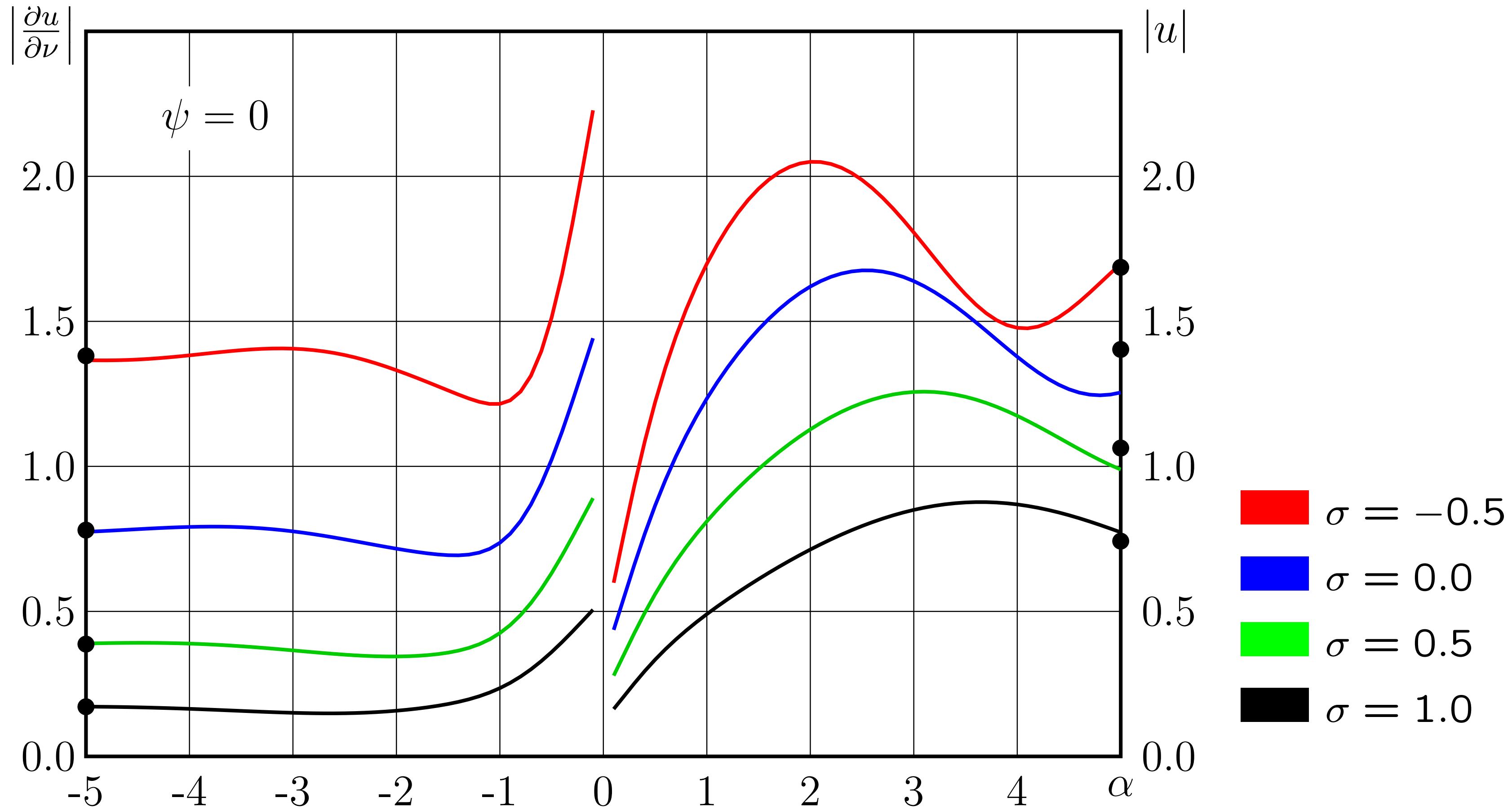
$$\sum_n \exp(-Cn^{1/3}\alpha) n^{-4/3},$$

which does not allow to get accurate results for small $|\alpha|$.
The outer integral (with respect to λ) is computed as usually with the negative part of the integration path shifted to the ray $\arg(\lambda) = \frac{2\pi}{3}$.

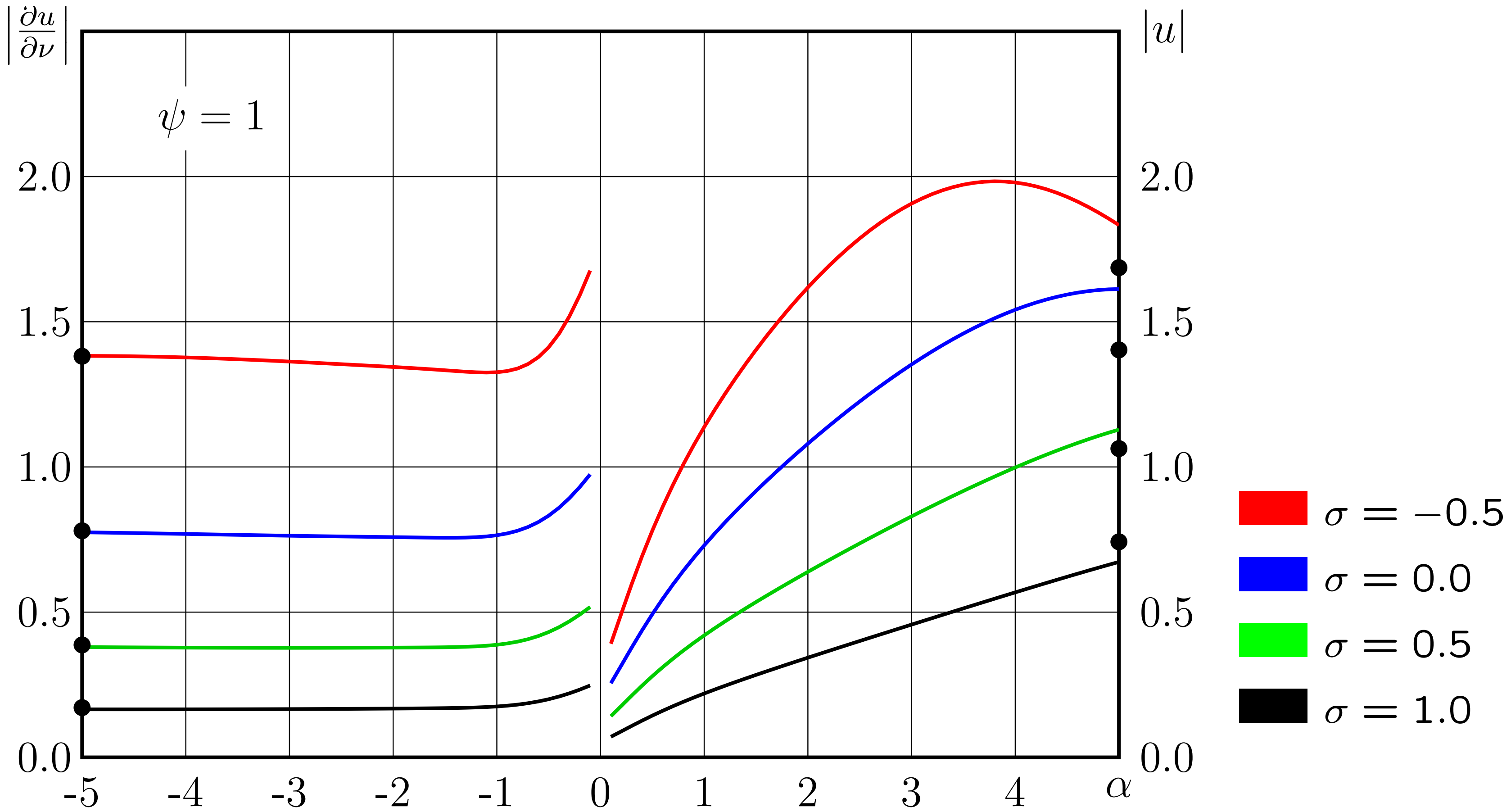
Numerical results



Numerical results



Numerical results



Asymptotic representation of the field for $\sigma \to +\infty$ is formed by the contributions of the singularities:

- the poles $\xi = t_q - \psi^2$ if $\psi > 0$,
- the poles $\xi = t'_q - \psi^2$ if $\psi < 0$,
- the branch-points $\xi = t_q$,
- the branch-points $\xi = t'_q$.

The effect of creeping wave interaction with line γ, where the boundary condition changes, can be described as follows: the incident creeping wave with the index p is reflected and forms a series of creeping waves with all indices, it is also transmitted and forms a series of creeping waves of the other type. The process is accompanied by the formation of specific creeping head waves (contributions of the branch-points) which have an additional decay $\sigma^{-3/2}$.

Conclusion

The leading order approximation for the field is derived and studied both numerically and asymptotically for $\sigma \to +\infty$.

Further goals:

- Uniform asymptotic analysis for both σ and α large.
- Generalization to the impedance case.
- Generalization to the case of electromagnetic waves.

Thank you!