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Elliptically inhomogeneous plane wave impinging on an infinite number of parallel cylinders

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Abstract

This paper introduces the interaction between an ensemble of cylinders and an inhomogeneous plane wave, which is determined through a rigorous theoretical approach. The scattered electromagnetic field by an indefinite number of infinite circular cylinders is analyzed through an application of the generalized Vector Cylinder Harmonics (VCH) expansion. The scenario described above is represented by an exact mathematical model that considers the so-called complex-angle formalism reaching a superposition of VCH and the Foldy-Lax Multiple Scattering Equations (FLMSE) to take into account the multi-scattering process between the cylinders. The validation of the method was performed with a comparison between the numerical results based on the Finite Element Method (FEM) and a homemade Matlab code.

1 Introduction

In the last decades, researchers focused their attention on solving Maxwell's equations to determine the field scattered by complex scenario such as spheres, cylinders, and axially symmetric objects [1, 2, 3, 4, 5]. In the literature, ensembles of different configurations of scattering objects are also found [6, 7, 8]. Moreover, different physical and chemical characteristics of materials constituting the scatterer have been analyzed to discover and learn the various physical and analytical behaviors [9, 10, 11, 12]. In this paper, an accurate method for showing an elliptically inhomogeneous plane wave polarized as an expansion of VCH is presented. Furthermore, the so-called T-matrix approach [13, 14] is used to address the multi-scattering process. The FLMSEs [15, 16] is applied to impose the continuity of the tangential components of the electromagnetic fields on the surface of each scatterer. The general representation of an electromagnetic wave as an inhomogeneous wave has attracted a lot of interest from the researchers. The present study shows the potential of using the complex-angle [17] formulation to represent the incident field as VCHs superposition. One of the main advantages is the great simplicity in the elaboration. Also, this approach will be generalized for the scattering from a cylinder immersed in a lossy medium. The paper provides a contribution with numerical comparisons for different developments in cylindrical vector waves. In addition, the study will focus on the analysis of an ensemble of infinite lossy cylinders immersed in a lossy medium and its results. The multi-paradigm numerical computing environment Matlab was used for the implementation of the various formulations, while the proper model was simulated by COMSOL Multiphysics, a commercial software based on the FEM.

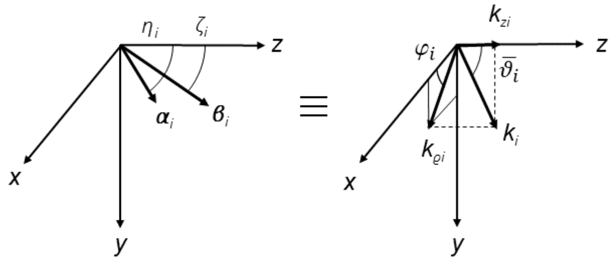

Figure 1. The left figure represents the complex wave vector of an inhomogeneous plane wave with the phase and attenuation vectors. The right figure represents the same vectors with the complex-angle formulation.

2 Methods

From literature, two formalisms are known to be used for representing an inhomogeneous wave propagating in a lossy medium. The former and also the one with best characteristics is the formalism known as Adler-Chu-Fano formulation; its propagation vector has a complex nature with $k_i = \beta_i + i\alpha_i$ represented by the phase and attenuation vectors, $\beta_i, \alpha_i \in \Re$, respectively. The latter, once again, has a complex propagation vector represented by the superposition of real and imaginary parts $k_i = k_\mathbf{R} + ik_\mathbf{I}$, which forms a complex angle with an axis of the Cartesian reference system $\bar{\vartheta}_i = \vartheta_\mathbf{R} + i\vartheta_\mathbf{I}$ [17], see Fig. 1. The symbol $\bar{\vartheta}$ was used to highlight the complex nature of the angle. This study demonstrates that using a superposition of basic cylindrical waves to represent the field through the use of the complex-angle formalism can be expressed with relative simplicity. The following wave, in which the vectors α_i and β_i are forming the angles ζ_i and η_i with the z-axis is also placed on the same plane passing through the z-axis, and they are creating a real angle φ with the x-axis (see Fig. 1). In the case described above, the two formalisms have the following relations [17]:

$$\vartheta_{\mathbf{R}} = \arccos \frac{k_{\mathbf{R}}\beta \cos \xi + k_{\mathbf{I}}\alpha \cos \eta}{\sqrt{k_{\mathbf{R}}^2\beta^2 - k_{\mathbf{I}}^2\alpha^2 + 2(k_{\mathbf{R}}k_{\mathbf{I}})^2}} \tag{1}$$

$$\vartheta_{\mathbf{I}} = \frac{1}{2}\operatorname{atanh}\left(\frac{2\beta\alpha}{k^2}\right) \tag{2}$$

where η and ξ are the angles that the vectors α and β, respectively, form with the z-axis. The plane having $\varphi = 0$ was considered for simplicity of study, even if the subsequent considerations can be easily applied to each plane with $\varphi \neq 0$.

Let us consider a simple inhomogeneous plane wave, using the formalism presented for the first time by Frezza et al. [18, 19]. Any obliquely polarized elliptical field, with respect to the surface of a cylinder, can be represented as a linear combination of two components, one vertical and one horizontal, each multiplied by its polarization coefficient (E_{vi} and E_{hi}, respectively):

$$\mathbf{E}(\mathbf{r}) = \left[E_{vi}\mathbf{v}_0(\bar{\vartheta}_i,\varphi_i) + E_{hi}\mathbf{h}_0(\bar{\vartheta}_i,\varphi_i)\right]e^{i\mathbf{k}\cdot\mathbf{r}} = \\ = \sum_{m=-\infty}^{+\infty}\left[a_m\mathbf{M}_m(k^*\mathbf{r}) + b_m\mathbf{N}_m(k^*\mathbf{r})\right] \tag{3}$$

imposing the following definitions [18]:

$$a_m = \frac{E_{hi}}{k_\rho}(-i)^{m-1}e^{-im\varphi_i} \tag{4}$$

$$b_m = -\frac{E_{vi}}{k_\rho}(-i)^m e^{-im\varphi_i} \tag{5}$$

$$\mathbf{k}_i = k^*\left(\sin\bar{\vartheta}_i\cos\varphi_i\mathbf{x}_0 + \sin\bar{\vartheta}_i\varphi_i\mathbf{y}_0 + \cos\bar{\vartheta}_i\mathbf{z}_0\right) \tag{6}$$

$$\mathbf{M}_m = (im\frac{Z_m(k_\rho\rho)}{\rho}\rho_0 - k_\rho\frac{\partial Z_m(k_\rho\rho)}{\partial\rho}\varphi_0)e^{im\varphi}e^{ik_z z - i\omega t} \tag{7}$$

$$\mathbf{N}_m = (i\frac{k_z k_\rho}{k}\frac{\partial Z_m(k_\rho\rho)}{\partial\rho}\rho_0 - \frac{mk_z}{k}\frac{Z_m(k_\rho\rho)}{\rho}\varphi_0 + \\ + \frac{k_\rho^2}{k}Z_m(k_\rho\rho)\mathbf{z}_0)e^{im\varphi}e^{ik_z z - i\omega t} \tag{8}$$

having indicated with k^* the complex conjugate of the wavenumber k. Considering several parallel cylinders in free space, their scattering with the defined incident field is analyzed, as shown in Fig. 2.

An arbitrary number L of dielectric cylinders, with relative permittivities ε_j, with $j = 1,\ldots,N$, infinite length, and radii r_j in a free-space filled by a lossy medium, in general dissipative, with relative permittivity ε_e, relative permeability μ_e, and electric conductivity σ_e are considered. The incident field, as usual, is an elliptically polarized inhomogeneous plane wave. In order to apply the Foldy-Lax Multiple scattering equations, the external field on the surface of the q-th cylinder, also called the exciting field, needs to be taken into consideration. The exiting field is the superposition of the incident field and all the scattered fields by the cylinders:

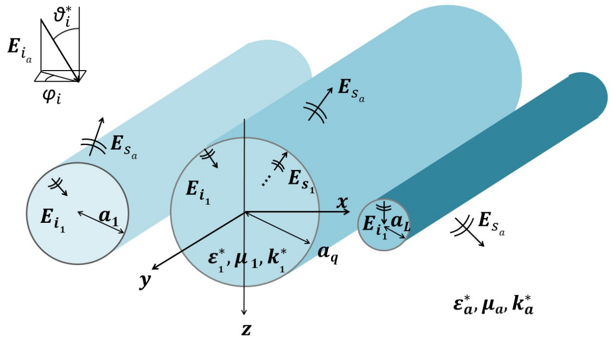

Figure 2. Depiction of the problem.

$$\mathbf{E}_{ex}^q = \mathbf{E}_i + \sum_{\substack{p=1 \\ p\neq q}}^{L}\mathbf{E}_s^p. \tag{9}$$

The incident field can be expressed as a function of vector cylindrical harmonics centered on the q-th cylinder [18]:

$$\mathbf{E}_i(k\rho_q) = [E_{v0}v + E_{h0}h]\,e^{i\mathbf{k}_0\cdot\rho_q}e^{i\mathbf{k}_0\cdot\rho_{0q}} = \\ = \sum_{m=-\infty}^{+\infty}\left[a_m\mathbf{M}_m^{(1)}(\rho-\rho_q) + b_m\mathbf{N}_m^{(1)}(\rho_{0q})\right]e^{i\mathbf{k}_0\cdot\rho_q} \tag{10}$$

with $\rho_{0q} = |\rho - \rho_q|$. The exiting field of the q-th cylinder is:

$$\mathbf{E}_{ex}^q(k\rho_q) = \sum_{m=-\infty}^{+\infty}\left[w_m^q\mathbf{M}_m^{(1)}(\rho_{0q}) + v_m^q\mathbf{N}_m^{(1)}(\rho_{0q})\right] \tag{11}$$

while the scattered electric field from $p \neq q$-th cylinder is:

$$\mathbf{E}_s^p(\rho_p) = \sum_{m'=-\infty}^{+\infty}\Big[T_{m'}^M w_{m'}^p\mathbf{M}_m^{(3)}(\rho_{0p}) + \\ + T_{m'}^N v_{m'}^p\mathbf{N}_m^{(3)}(\rho_{0p})\Big] \tag{12}$$

having indicated with $T_{m'}^M$ and $T_{m'}^N$ the scattering coefficients in dielectric cylinder case, i.e. the T-matrix coefficients are [15, 16]:

$$T_m^M = -\frac{J_m'(k_{i\rho}a)}{H'^{(1)}_m(k_{i\rho}a)} \tag{13}$$

$$T_m^N = -\frac{J_m(k_{i\rho}a)}{H_m^{(1)}(k_{i\rho}a)}. \tag{14}$$

Applying the Addition theorem on the VCHs function, we obtain:

$$M_{m'}^{(3)}(\rho_{0q}) = \sum_m H_{m-m'}^{(1)}(k\rho_{qp})e^{-i(m-m')\varphi_{pq}}M_m^{(1)}(\rho_{0q}) \tag{15}$$

$$N_{m'}^{(3)}(\rho_{0q}) = \sum_m H_{m-m'}^{(1)}(k\rho_{qp})e^{-i(m-m')\varphi_{pq}}N_m^{(1)}(\rho_{0q}) \tag{16}$$

$$M_{m'}^{(1)}(\rho_{0q}) = \sum_m J_{m-m'}^{(1)}(k\rho_{qp})e^{-i(m-m')\varphi_{pq}}M_m^{(1)}(\rho_{0q}) \tag{17}$$

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Replacing all fields inside the FLMSEs and using the orthogonal properties of the VCHs, the following linear system is obtained:

$$w_m^q = \tilde{a}_m + \sum_{m'=-\infty}^{+\infty} \sum_{\substack{p=1 \\ p\neq q}} A_{mm'} T_{m'}^M w_{m'}^p \tag{19}$$

$$v_m^q = \tilde{b}_m + \sum_{m'=-\infty}^{+\infty} \sum_{\substack{p=1 \\ p\neq q}} A_{mm'} T_{m'}^N v_{m'}^p. \tag{20}$$

At this point, the linear system can be solved and the coefficients w_m^q and v_m^q determined. Being the scattered field by the q-th cylinder writable as a superposition of VCHs, as:

$$\mathbf{E}_s^q = \sum_{m=-\infty}^{+\infty} \left[e_m^q \mathbf{M}_m^{(3)}(k, \rho_{0q}) + f_m^q \mathbf{N}_m^{(3)}(k, \rho_{0q}) \right] \tag{21}$$

the coefficients of the q-th cylinder can be written as follows:

$$e_m^q = T_m^M w_m^q \tag{22}$$

$$f_m^q = T_m^N v_m^q \tag{23}$$

3 Results and Conclusions

A comparison as a result of the validation process was performed both on the determined formulation and on a canonical case of electromagnetic scattering. In this research paper, three infinite lossy dielectric cylinders have been taken in consideration. In Figs. 3 and 4, the results obtained with Matlab and Comsol in the case of $k_e = 1 - i$ 1/m for the environment and $k_c = 2 - 0.5i$ 1/m for all cylinders (arbitrary parameters) are shown. In particular, the figures have shown the absolute value of the x and y components of the scattered electric field (the z component is null). As noted, the results are perfectly compatible with one another. Subsequently (see Fig. 5), the most general case of an inhomogeneous plane wave with a complex value for ϑ was shown. In particular, the fictitious parameters of the previous case with the complex angle $\vartheta = \pi/6(1+0.5i)$ rad and $\varphi = \pi/4$ rad were considered.

In Fig. 5, the contribution of the complex angle is highlighted by the direction of the incident wave forming. Moreover, in the same direction an attenuation effect is also present.

In summary, an accurate method to obtain an expansion of an inhomogeneous elliptically polarized plane wave in terms of vectorial cylinder harmonics to solve the multi-scattering by an ensemble of cylinders is presented. The determination of the expansion coefficients and the application of the so-called Foldy-Lay equations for the use of the complex-angle formalism contribute to the determination of the solution to the electromagnetic problem. A light and elegant formalism was achieved with this approach. The procedure was validated with some numerical results as well as with comparisons through simulations in the COMSOL environment.

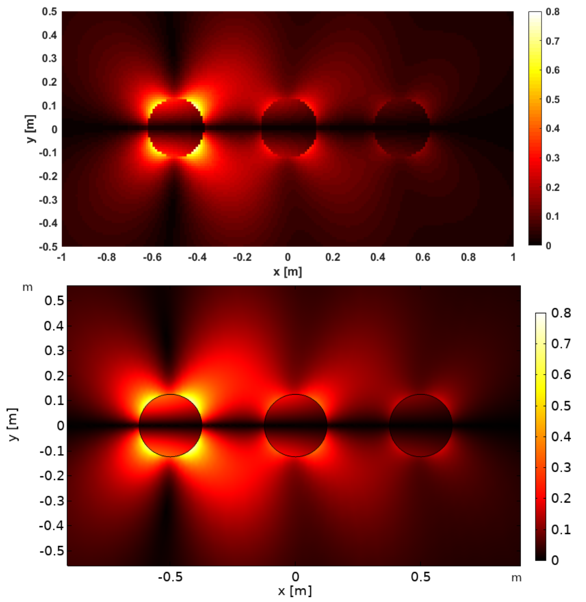

Figure 3. Comparison of the spatial distribution (on xy-plane) of the absolute value of the x-component of the scattered electric field $|E_{s_x}|$, obtained with Matlab (on top) and with Comsol (on bottom).

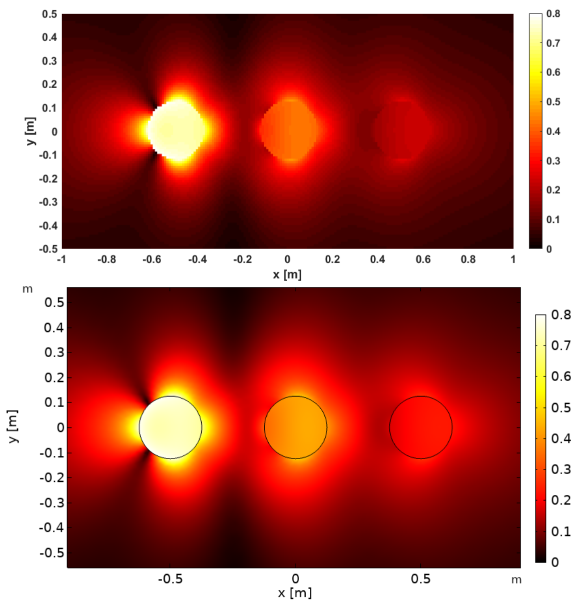

Figure 4. Comparison of the spatial distribution (on xy-plane) of the absolute value of the y-component of the scattered electric field $|E_{s_y}|$, obtained with Matlab (on top) and with Comsol (on bottom).

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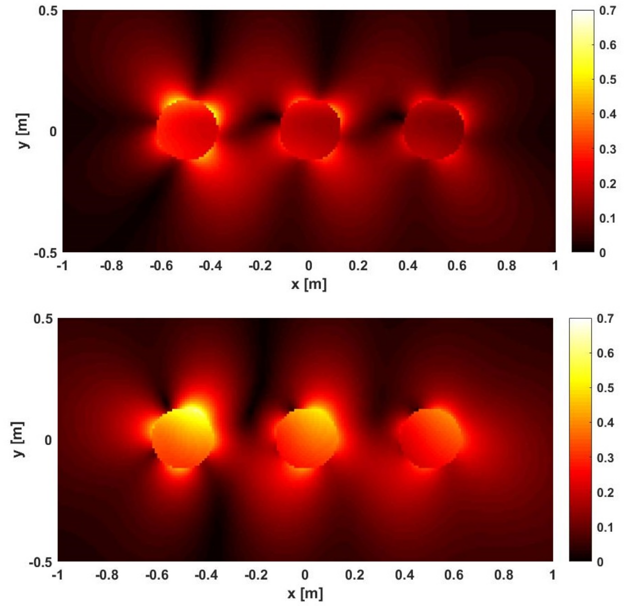

Figure 5. Spatial distribution of the absolute value (on *xy*-plane) for the *x* (on top) and *y*-components (on bottom) of the scattered electric field $|E_{s_x}|$ and $|E_{s_y}|$ obtained with Matlab code.

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