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An Explicit Time Marching Scheme to Solve Surface Integral Equations for Acoustically Penetrable Scatterers

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Transient acoustic scattering from homogeneous penetrable objects can be numerically analyzed by solving time domain surface integral equations (TDSIEs) [1]. The schemes developed for solving TDSIEs have several advantages over those developed for directly solving the wave equation: (i) they discretize only the surface of the scatterer, (ii) they are more immune to numerical dispersion, and (iii) they do not need any (approximate) absorbing boundary conditions to truncate the physical domain. One of the well-known schemes developed for solving TDSIEs is the marching-on-in-time (MOT) scheme. The MOT scheme expands the (unknown) velocity potential induced on the scatterer surface using spatial and temporal basis functions. Inserting this expansion into the TDSIE and applying spatial testing at discrete times yield a system of equations. This system is solved by marching in time for the unknown expansion coefficients. Traditionally, the MOT scheme is "implicit", in other words, at a given time step, it calls for inversion of a matrix system for unknown coefficients associated with that step. Under low frequency excitations (for large time step sizes), this matrix becomes denser (even full) significantly increasing the computational cost of the implicit scheme.

To alleviate this bottleneck, an explicit MOT scheme is developed in this work. The velocity potential induced on the surface of the penetrable scatterer and its normal derivative are related to the incident velocity potential through a pair of coupled surface integral equations [1]. This TDSIE system is second-kind and is cast in the form of an ordinary differential equation (ODE) that relates the unknowns to their temporal derivatives. Unknowns are expanded in terms of high-order polynomials defined on curvilinear patches discretizing the scatterer surface [2]. Inserting these expansions into the ODE form of the TDSIEs and testing the resulting equation at the interpolation points yield a matrix system. This system is integrated in time using a predictor-corrector type multistep method to yield the unknown expansion coefficients [3]. This integration calls for temporal sampling, where piecewise (shifted) Lagrange polynomial interpolation functions [4] are used to enable the computation of retarded-time integrals present in the TDSIEs. Since the Gram matrix resulting from the spatial point testing is diagonal, the predictor-corrector type multistep method results in an explicit time marching where any matrix inversion is avoided. This makes the explicit solver faster than its implicit counterpart which calls for inversion of a dense matrix system at every time step under low frequency excitation. Numerical results, which demonstrate the accuracy and stability of the explicit solver, will be presented at the conference.

References

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