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Phaseless near-field techniques from a random starting point

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Abstract

In this paper we address the problem of reconstructing the far field radiated by focusing sources from the knowledge of square amplitude samples of the radiated field on more measurement lines in near zone. We show that it is possible to estimate the angular sector where the far field is significantly different from zero. This allows to adopt an efficient procedure for the reconstruction of the far field without the use of prior information about it. Such procedure exploits the quadratic inversion approach for phase retrieval and it consists of two steps. In the first step, it recovers only those samples of the radiation pattern significantly different from zero. Later, it reconstructs all the samples of the radiation pattern starting from an initial guess which is equal to the solution of the first step, where the radiation pattern is significantly different from zero, and zero otherwise. Numerical results show the feasibility of the technique, and its efficiency in terms of data required for convergence.

1 Introduction

Near field to far field methods are usually exploited to reconstruct the radiation pattern of an antenna. Such techniques results very useful since they allow to estimate the far field starting from a set of measurements that can be performed in restricted domains like lab environments. However, in many cases it is not possible to perform accurate phase measurements hence it arises the need to develop phaseless near field to far field techniques [1, 2]. A typical way to reconstruct the far field pattern from phaseless data is to tackle a phase retrieval problem by using the quadratic inversion method [3]. Such technique is based on the minimization of a quartic functional and therefore it may contain trap points, like local minima and saddle points with null gradient. However, the problem of traps can be overcome by enlarging the dimension of data space by an increasing of the number of scanning [4]. In order to reduce the number of scanning surface, in this paper we develop a procedure for the reconstruction of the far field from phaseless near field data that can be used for focusing sources.

2 Mathematical formulation

Let us consider a magnetic current $\underline{J}_m(x) = J_m(x)\,\hat{y}$ supported over the interval $[-a;a]$ of the x axis and directed along the y axis that is the axis of invariance (see fig. 1).

Such current radiates an electric field $\underline{E}(x,z)$ in an homogeneous medium with wavenumber β. Denote by $E(x,z)$ the x component of the electric field, and suppose to observe $|E(x,z)|^2$ over P lines located at the distances $z_1, z_2, ..., z_P$ which extend along the x axis over the intervals $[-X_{o1}, X_{o1}], [-X_{o2}, X_{o2}], ..., [-X_{oP}, X_{oP}]$ respectively.
The problem we address in this paper is the reconstruction of the radiation pattern starting from the knowledge of the square amplitude samples of the electric field E on the observation domains described above.

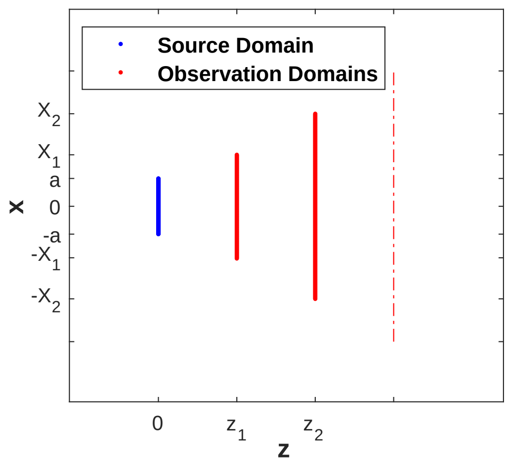

Figure 1. Geometry of the problem

In order to formulate the problem from the mathematical point of view, now we derive a suitable field expansion. For the considered $2D$ geometry, $E(x,z)$ can be expressed as

$$E(x,z) = \int_{-a}^{a} \frac{j\beta z}{4R} H_1^2(\beta R)\, J_m(x')\, dx' \quad (1)$$

where H_1^2 is the Hankel function of second kind and first order, and $R(x',x,z) = \sqrt{(x-x')^2 + z^2}$.
In far zone, the x component of the electric field can be simplified as follows

$$E_\infty(r,u) = -\frac{1}{2}\sqrt{\beta}\, e^{j\frac{\pi}{4}} \sqrt{1-\left(\frac{u}{\beta}\right)^2}\, \frac{e^{-j\beta r}}{\sqrt{2\pi r}} I(u) \quad (2)$$

where $r = \sqrt{x^2+z^2}$, $u = \beta\, sin\,\theta$ with θ indicating the angle with the x axis, $I(u)$ is the *magnetic radiation integral* and given by $I(u) = \int_{-a}^{a} e^{jux'} J_m(x')\, dx'$.
The Fourier transform relation between the magnetic current J_m and the magnetic radiation integral I suggests the

Fourier harmonics as a possible set of functions expanding the magnetic current J_m. Here, we assume that only a finite number of harmonics are sufficient to represent the magnetic current, hence $J_m(x') = \sum_{n=-N_0}^{N_0} I(u_n)\, e^{-j\frac{n\pi}{a}x'}$.
By substituting the truncated Fourier expansion of J_m into (1), we obtain the following representation of the x component of the radiated field

$$E(x,z) = \sum_{n=-N_0}^{N_0} I(u_n)\, \psi_n(x,z) \qquad (3)$$

where the functions $\psi_n(x,z) = \frac{j\beta z}{4}\int_{-a}^{a} H_1^2(\beta R)\, e^{-j\frac{n\pi}{a}x'}dx'$ are called *pseudosampling functions*. Note that the latter can be regarded as the Fourier Transform of the restriction to the set $[-a,a]$ of $H_1^2(\beta R)$. For this reason they can be efficiently computed from the numerical point of view by a FFT algorithm.
The introduction of the field expansion (3) allows us to formulate the problem of the reconstruction of the radiation pattern from the square amplitude samples of the radiated field. In fact, it is possible to express the square amplitude functions of the radiated field on the measurements lines as

$$\left|E(x,z_p)\right|^2 = \left|\sum_{n=-N_0}^{N_0} I(u_n)\, \psi_n(x,z_p)\right|^2 \quad p = 1,...,P \qquad (4)$$

Hence, by discretizing (4), we obtain a set of P subsystems in the unknown $\underline{x} = [I(u_{-N_0}),....,I(u_{N_0})]^T \in C^{2N_0+1}$ which can be expressed as

$$|\underline{\underline{L}}_1\underline{x}|^2 = \underline{M}_1^2 \quad |\underline{\underline{L}}_2\underline{x}|^2 = \underline{M}_2^2 \quad \dots \quad |\underline{\underline{L}}_P\underline{x}|^2 = \underline{M}_P^2 \qquad (5)$$

where $\forall p = 1,\dots,P$

1) $|\underline{\underline{L}}_p\underline{x}|^2 = \left(\underline{\underline{L}}_p\underline{x}\right) \odot \left(\underline{\underline{L}}_p\underline{x}\right)^*$ with $\odot$ indicating the Hadamard product and $\left(\underline{\underline{L}}_p\underline{x}\right)^*$ denoting the conjugate of $(\underline{\underline{L}}_p\underline{x})$;

2) $\underline{\underline{L}}_p \in C^{M_p\times(2N_0+1)}$ is a matrix whose mth column is given by $[\psi_{-N_0}(x_m,z_p)\,,\psi_{-N_0+1}(x_m,z_p)\,,\dots,\psi_{N_0}(x_m,z_p)]$;

3) $\underline{M}_p^2 \in R^{M_p}$ is the vector defined as $\underline{M}_p^2 = \left[\,|E(x_1,z_p)|^2, |E(x_2,z_p)|^2, ..., |E(x_{M_p},z_p)|^2\,\right]$.

By defining $\underline{\underline{L}} = [\underline{\underline{L}}_1,\dots,\underline{\underline{L}}_P]^T$ and $\underline{M}^2 = [\underline{M}_1^2,\dots,\underline{M}_P^2]^T$ it is possible to rewrite the set of P subsystems (5) under

$$|\underline{\underline{L}}\underline{x}|^2 = \underline{M}^2 \qquad (6)$$

The problem of finding $\underline{x}$ from the phaseless data $\underline{M}^2$ falls into the realm of phase retrieval.

3 The quadratic inversion method

A solution of such problem can be found by using the quadratic inversion method [3], which consists in the minimization of the quartic functional

$$\Phi(\underline{x}) = \left|\left|\,|\underline{\underline{L}}\underline{x}|^2 - \underline{M}^2\,\right|\right|^2 \qquad (7)$$

Since the functional is not quadratic, it may suffer of the traps problem. However, as shown in [4], the presence of traps is related to the essential dimension of data. The latter can be evaluated by introducing a linear representation of the data, and by computing the number of significant singular values of the linear operator which represents the data. In order to obtain a linear representation of the data, let us rewrite the square amplitude samples of the radiated field on the p th measurement line in the form

$$\underline{M}_p^2 = \left|\sum_{n=-N_0}^{+N_0} I(u_n)\,\underline{\psi}_{pn}\right|^2 = \sum_{n=-N_0}^{+N_0}\sum_{l=-N_0}^{+N_0} I(u_n)I(u_l)^*\left(\underline{\psi}_n \odot \underline{\psi}_l^*\right) \qquad (8)$$

where $\underline{\psi}_{pn}$ is the $n+N_0+1$ th column of the matrix $\underline{\underline{L}}_p$.
By introducing a mapping from each couple (n,l) into a different integer q, it is possible to recast (8) under

$$\underline{M}_p^2 = \sum_{q=1}^{(2N_0+1)^2} X_q\underline{\Psi}_q \quad \text{where} \quad \begin{array}{c} X_q = I(u_n)\,I(u_l)^* \\ \underline{\Psi}_q = \underline{\psi}_n \odot \left(\underline{\psi}_l\right)^* \end{array} \qquad (9)$$

The equation (9) can be rewritten as $\mathscr{A}_p(\underline{X}) = \underline{M}_p^2$ where $\mathscr{A}_p$ is the linear operator which maps the unknown vector $\underline{X} = [X_1,X_2,...,X_{(2N_0+1)^2}]^T$ into the data $\underline{M}_p^2$ collected on the p th measurement line. By introducing the linear operator $\mathscr{A}(\underline{X}) = [\mathscr{A}_1(\underline{X}),\dots,\mathscr{A}_P(\underline{X})]^T$, it is possible to represent the data by the linear representation

$$\mathscr{A}(\underline{X}) = \underline{M}^2 \qquad (10)$$

Then, the essential dimension of the data space can be evaluated by computing the singular values decomposition of $\mathscr{A}$, and by counting the singular values of $\mathscr{A}$ upon a threshold dictated by the noise level.
According to [4], local minima disappear if the essential dimension of data is high enough. Despite this, in same cases the number of data that ensures the lack of traps can be quite high. With the aim to reduce the number of data, in this paper we restrict the attention to the reconstruction of the far field radiated by the wide class of focusing sources. In fact, for these sources it is possible to use an efficient procedure for phase retrieval. Such procedure allows to reduce the number of data required for convergence since it exploits some information about the radiation pattern that can be deduced directly from the phaseless data.

4 The estimation of the field direction

In order to use the phase retrieval technique to be described in section V, we need to know the angular sector where the radiation pattern is significantly different from zero or, equivalently, (since $u = \beta\, sin\theta$) the set $[u_{min}, u_{max}]$ where $I(u)$ is significantly different from zero. Such information can be deduced directly from the phaseless data as follow. The magnetic radiation integral $I_m(u)$ is related to the field at $z = z_p$ by the equation

$$I(u) = e^{jwz_p}\int_{-\infty}^{+\infty} E(x,z_p)\, e^{jux}dx \qquad (11)$$

Consequently, the Fourier transform of $|E(x,z_p)|^2$ is

$$\int_{-\infty}^{+\infty} |E(x,z_p)|^2 e^{jux} dx = \left(I(u)\, e^{-jwz_p}\right) * \left(I(-u)\, e^{-jwz_p}\right)^* \tag{12}$$

By virtue of (12), we can state that the support of the amplitude spectrum of $|E(x,z_p)|^2$ has an extension that is twice than the support extension of $I(u)\, e^{-jwz_p}$. Hence, by computing the Fourier transform of $|E(x,z_p)|^2$ we can derive the support extension of the function $I(u)\, e^{-jwz_p}$, and consequently, we can have also a rough idea of the support extension of $I(u)$. However, since $|E(x,z_p)|^2$ is a real valued function, its spectrum is a Hermitian function. As a consequence, the amplitude spectrum of $|E(x,z_p)|^2$ is always an even function centered at $u=0$. For this reason, the amplitude spectrum of $|E(x,z_p)|^2$ allows to know only the width of the set $[u_1,u_2]$ where $I(u)$ is different from zero but it does not permits to know the set $[u_1,u_2]$. In other words, the amplitude spectrum of $|E(x,z_p)|^2$ contains only the information about the size of angular sector where the radiation pattern is relevant but it does not allow to compute such an angular sector. Nevertheless, a rough estimation of the main beam direction in far zone can be obtained by observing where $|E(x,z_P)|^2$ attains its maximum, say for $x = x_{P\,max}$. Hence, we can assume that the direction of main beam in far zone is approximately given by

$$\theta_{max} \approx arctg\,(x_{P\,max}/z_P) \tag{13}$$

Consequently, $|I(u)|$ attains its maximum at $u_{max} = \beta \sin\theta_{max}$.

5 Description of the procedure

Once the set $[u_{min}, u_{max}]$ on which $I(u)$ is relevant has been estimated, it is possible to use the following efficient procedure for phase retrieval. The procedure consists of two steps. In the first step, since some far field samples have low amplitude, the size of the unknown vector is reduced by neglecting the quasi-zero components of $\underline{x}$. Hence, the new unknown vector $\underline{x}_{rel} \in C^{N_1}$ contains only the relevant components of $\underline{x} \in C^N$ with $N = 2N_0+1$. Consequently, the problem to be solved in the first step can be expressed as

$$|\underline{\underline{L}}_{rel}\, \underline{x}_{rel}|^2 = \underline{M}^2 \tag{14}$$

where $\underline{\underline{L}}_{rel}$ can be obtained by deleting the columns of $\underline{\underline{L}}$ corresponding to the zero or quasi zero components of the vector $\underline{x}$. The solution of the quadratic system can be found by minimizing the functional

$$\Phi_{rel}\,(\underline{x}) = \left|\left|\, |\underline{\underline{L}}_{rel}\, \underline{x}_{rel}|^2 - \underline{M}^2 \,\right|\right|^2 \tag{15}$$

The advantage of minimizing the quartic functional (15) instead of (7) is that in the first case the number of unknowns is lower (by definition $N_1 < N$), and, consequently, also the ratio between the essential dimension of data and number of unknowns is more favorable. For this reason, it is easier to ensure the absence of traps in (15) than in (7). Once that a global minimum of the functional (15) has been found, the second step of the procedure is to minimize the functional (7) starting from an initial guess vector $\underline{x}^{(0)}$ whose main components are $\underline{x}_{rel}$ and zero the other ones. In such case, even if the functional may exhibit trap points, the starting point is in the attraction region; therefore the minimization scheme reaches the global minimum of the functional.

6 Number of data required for convergence

In this section, with reference to a particular focusing source, we will estimate the minimum value of the ratio between the essential dimension of data (M) and the number of real unknown (N_r) which allows to reconstruct the actual solution of the problem by the above described procedure. In particular, we want to show how the ratio M/N_r increases with N_r. As a test case, we consider the reconstruction of the magnetic radiation integral $I(u)$ (shown in fig. 2) corresponding to

$$J_m(x) = cos\left(\frac{\pi}{2a}x\right)\, e^{j\,u_{max}\,x} \text{ with} \tag{16}$$

with $u_{max} = 2$, and $x \in [-a,a]$.

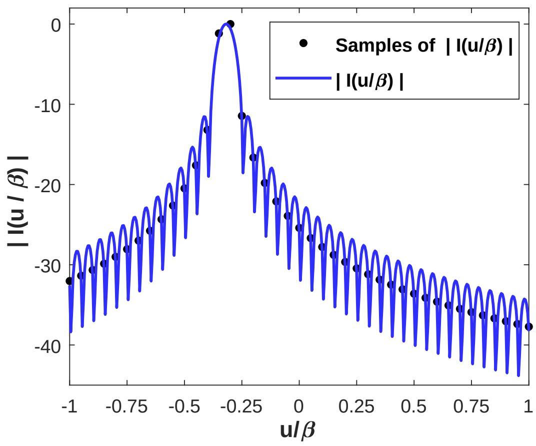

Figure 2. Radiation integral, and samples of the radiation integral to be reconstructed for a source of extension 20λ.

The square amplitude samples of radiated field are sampled with a uniform sampling step $\Delta x = \lambda/4$. The extension of the measurement lines along the x axis is such that the number of significant values of $\mathscr{A}_1$, and $\mathscr{A}_2$ differ for only 1 unit from the case in which the measurements lines are unbounded [5]. The behavior of the square amplitude functions of the radiated field on the measurement lines for a source of extension $2a = 20\lambda$ is shown in figure 3. In such case, the maximum point of $|E(x,z_2)|^2$ is located at $x_{2\,max} = -1.86\lambda$. Since $z_2 = 6\lambda$, from the equation (13) it follows that $\tilde{\theta}_{max} = -0.301\, rad$, hence $\tilde{u}_{max}/\beta = sin\,\tilde{\theta}_{max} = -0.296$. Such value is very close to the actual value of $u_{max}/\beta = -0.318\, rad$.

Since N_r is related to the dimension of the source by the equation $N_r = 2N = 8a/\lambda$, in order to establish how the ratio M/N_r that ensures the convergence changes with respect to N_r we perform a set of simulations for each source of dimension $2a/\lambda$ belonging to the set $[5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 70, 80, 90, 100]$.

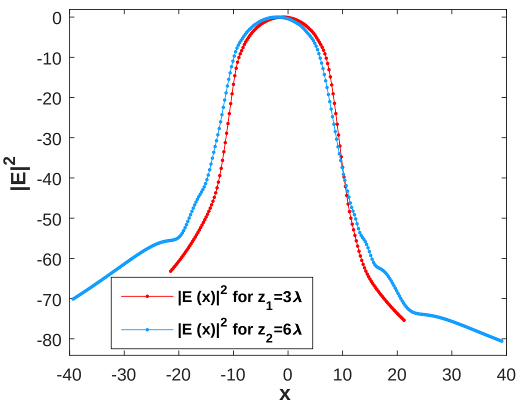

Figure 3. Square amplitude functions of the electric field in dB for $z_1 = 3\lambda$, and $z_2 = 6\lambda$.

In particular for each value of the size source, we perform a first set of 50 simulations where we search for a number of real unknowns $N_{r1} < N_r$. If the minimization scheme reconstructs the unknown sequence in all the 50 test cases, we assume that the ratio between the essential dimension of data and the number of real unknowns is high enough to ensure the absence of the traps in the functional, and so we pass to the second step of the procedure. Otherwise, we enlarge the ratio dimension of data - unknowns by increasing the number of observation circles and, we repeat the set of 50 simulations until we found a configuration whose ratio dimension of data - unknowns guarantees the recovery of the unknown vector in all the test cases. In all the simulations the initial guess is chosen at *random* according to a complex uniform distribution whose components have both the real and the imaginary part supported on the set $[-500, 500]$. From this numerical analysis, we find that:
1) in all the considered case 2 measurement lines suffice to ensure the convergence;
2) the ratio between the essential dimension of data and the number of real unknowns varies according to the diagram in fig. 4a.
As it can be seen from fig. 4a, the ratio M/N_r that allows to reach the solution, changes very slow with N_r. In fig. 4b the value of the ratio between the essential dimension of data and the number of unknowns in the first step and in the second step of the procedure is compared. As it can be noted from the figure, in the first step of the procedure such ratio is higher or equal to that of the second step. This behavior makes the problem of finding a global minimum of (15) easier than finding a global minimum of (7).

7 Conclusion

In this paper we address the problem of reconstructing the far field from phaseless near field data. To this end, we introduce a phaseless near field to far field method that allows to reconstruct the radiation pattern of focusing sources starting from a random initial guess. The main advantage of the such technique is that it requires a low ratio between the essential dimension of the data and the number of unknowns to attain the actual solution of the problem.

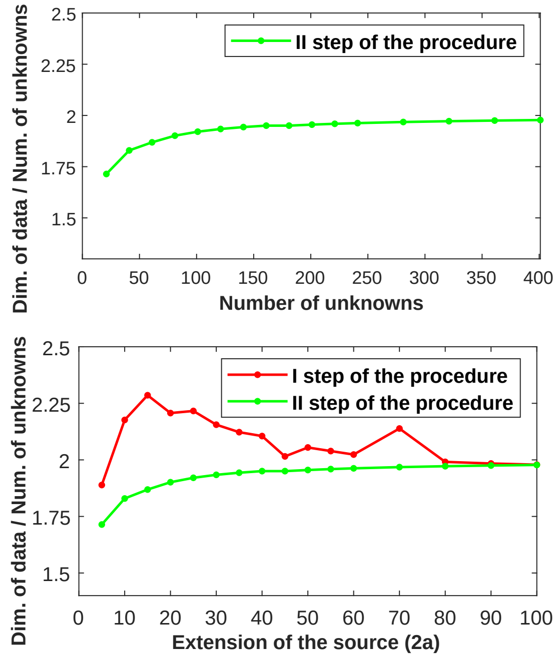

Figure 4. a) Ratio M/N_r in the second step of procedure. **b)** Comparison of the ratio between the essential dimension of the data and the number of real unknowns in the first and in second step of the procedure. The essential dimension of data is computed by counting the normalized singular values of the linear operator $\mathscr{A}$ upon the value $10^{-1.5}$.

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