Applications of the Random Coupling Model for stacked printed circuit boards.

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The Random Coupling Model (RCM) [1] is a statistical circuit model that can be applied to chaotic systems. The interactions between the ports are modelled by the impedance $\underline{\underline{Z}}^{\mathrm{cav}}$ in a chaotic system. Two behaviors are combined in the RCM, defined by two impedances $\underline{\underline{Z}}^{\mathrm{rad}}$ and $\underline{\underline{\xi}}$. The first one accounts for the system without the boundary conditions that make it reverberant. The second one, included in $\underline{\underline{\xi}} = -\frac{\mathrm{j}}{\pi}\underline{\underline{W}}\,[\,\underline{\underline{\lambda}} - \mathrm{j}\alpha\underline{\underline{1}}]^{-1}\underline{\underline{W}}^{T}$, models the reverberant part of the system. The random matrix $\underline{\underline{\lambda}}$ includes the eigenmode spectrum of the system and $\underline{\underline{W}}$ models the couplings between eigenmodes and ports. Finally, the losses are included in α. Thus, $\underline{\underline{Z}}^{\mathrm{cav}}$ is obtained by [2]:

$$\underline{\underline{Z}}^{\mathrm{cav}} = \mathrm{j}\Im\left\{\underline{\underline{Z}}^{\mathrm{rad}}\right\} + \left[\Re\left\{\underline{\underline{Z}}^{\mathrm{rad}}\right\}\right]^{1/2}\underline{\underline{\xi}}\left[\Re\left\{\underline{\underline{Z}}^{\mathrm{rad}}\right\}\right]^{1/2} \tag{1}$$

MONTE-CARLO iterations, where random matrices $\underline{\underline{\xi}}$ are generated, allow to compute statistical quantities. Then, currents or voltages at ports may be estimated. In order to compare RCM simulations with experimental data, a setup has been designed and made. It is composed of a computer chassis mockup that offers a reverberant environment, and a mode stirrer allowing to create a chaotic environment by setting a new set of boundary conditions in the system at each position. Four printed circuit boards (PCB) embedding microstrip transmission lines (TL), are placed in the mockup. Then, a VNA is used to measure couplings between the first TL (at the bottom of the stack where 10 dBm is injected) and three other TL. Figure 1 depicts cumulative distribution functions (CDF) of the magnitude of measured $|I^{M}_{2\cdots4}|$ and simulated $|I^{\mathrm{RCM}}_{2\cdots4}|$ currents. The subscript identifies the PCB in the stack.

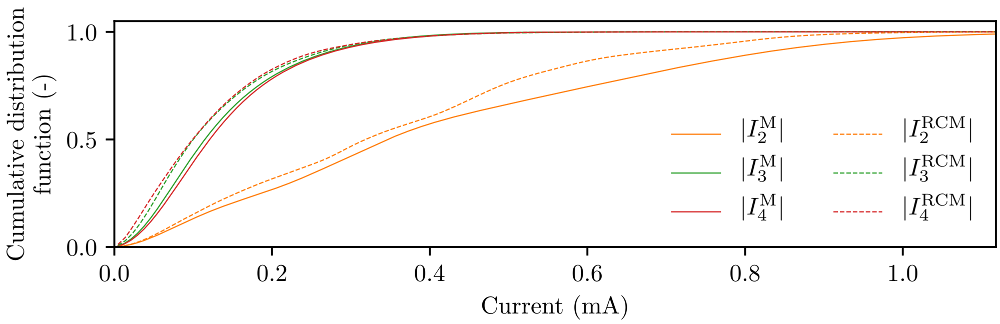

Figure 1. Comparison of CDF of induced current magnitudes computed from measurements and from simulation.

We observe that the measured and simulated curves are close to each other and that two behaviors, depending if the PCB are in view ($|I_2^{M/\mathrm{RCM}}|$) or not ($|I_3^{M/\mathrm{RCM}}|$ and $|I_4^{M/\mathrm{RCM}}|$), may be identified. We conclude that the RCM is able to statistically simulate a chaotic system without the need to entirely describe it. Moreover, less than 4 min are needed to compute these reliable results by means of an optimized software. This method could be applied at the earliest stages of the design and development of an equipment, in order to mitigate unwanted induced currents.

References

[1] G. Gradoni, J.-H. Yeh, B. Xiao, T. M. Antonsen, S. M. Anlage, and E. Ott, "Predicting the statistics of wave transport through chaotic cavities by the random coupling model: A review and recent progress," *Wave Motion*, vol. 51, no. 4, pp. 606 – 621, 2014.

[2] X. Zheng, T. M. Antonsen, and E. Ott, "Statistics of impedance and scattering matrices of chaotic microwave cavities with multiple ports," *Electromagnetics*, vol. 26, no. 1, pp. 37–55, 2006.