# Loren(t)z

Giuseppe Pelosi, Stefano Selleri*
Department of Information Engineering
University of Florence, Via di Santa Marta, 3, Florence, Italy
(giuseppe.pelosi,stefano.selleri)@unifi.it

## Abstract

In Electromagnetics, important contributions were given by both Hendrik Antoon Lorentz and Ludvig Valentin Lorenz, yet the two, sharing a very similar family name, are sometimes confused and the more famous Lorentz is often credited also with the contributions by the less celebrated Lorenz. Jean Van Bladel pointed out this in a very short communication nearly 30 years ago, communication which we wish to remember and expand here.

## 1 Lorentz

Hendrik Antoon Lorentz (Arnhem, NL, July 18, 1853 – Haarlem, NL, February 4, 1928; Fig. 1a) is a well-known Dutch Physicist whose transformations, local time concept and moving body contraction hypothesis are at the basis of Einstein's special relativity theory. He shared with Pieter Zeeman (Zonnemaire, NL, May 25, 1865 - Amsterdam, NL, October 9, 1943) the Nobel Prize in Physics in 1902 "in recognition of the extraordinary service they rendered by their researches into the influence of magnetism upon radiation phenomena."

While most of this indeed dealt with electromagnetics and light propagation, he is well known to any student for his formulation of the Lorentz Force [1] (Fig. 2):

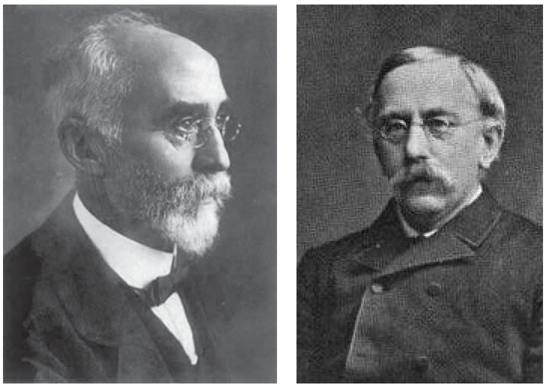
$$\mathbf{f} = q\mathbf{e} + q\mathbf{v} \times \mathbf{b} \tag{1}$$

While this is known from basic Physics course, students of more advanced electromagnetic courses may have come across "Lorentz potential" and "Lorentz gauge."

This on the other hand is a mistake which is found even in many textbooks, and which Jean Van Bladel discovered after having himself inaccurately credited Lorentz. In his own words "*It appears that the various authors of textbooks who sinned against historical accuracy – the undersigned being regretfully one of them – should amend their references in future printings of their books!*" [2]. And indeed the second edition of his book [3] is correct. Sadly, many widespread textbook on electromagnetics credits this gauge condition to Lorentz, among these, in no particular order, we cite [4, 5, 6].

## 2 Lorenz

The true father of the retarded potential and the gauge was Ludvig Valentin Lorenz (Helsingør, DK, January 18, 1829 - Frederiksberg, DK, June 9, 1891; Fig. 1b).



**(a)** **(b)**

**Figure 1.** (a) Hendrik Antoon Lorentz, on the left; (b) Ludvig Valentin Lorenz, on the right.

Ludvig Lorenz is far less known than Hendrik Lorentz. He was born in Helsingør, known worldwide for being the city were Shakespeare's Hamlet takes place, and studied at the Technical University in Copenhagen. He became professor at the Military Academy of that same city in 1876. He investigated the mathematical description for light propagation through a single homogeneous medium and described the passage of light between different media.

In [7] he writes the electric field, following Kirchhoff, as in Fig. 3, being $\Omega$ what we now call $\phi$, the scalar potential, and $\alpha$, $\beta$, $\gamma$ the Cartesian component of the vector potential **a** - vector notation still having to be introduced - and $k$ and $c$ constants. but then he defines a new *retarded* potential, taking into account explicitly the speed of light in defining the potential value at a distance from the source, as in Fig. 4. In this latter equation $\varepsilon'$ and $e'$ are the volumetric and surface charge densities, respectively, which we now call $\rho$ and $\rho_s$. He then defines a similar structure for a retarded vector potential.

VERSUCH EINER THEORIE

DER

ELECTRISCHEN UND OPTISCHEN ERSCHEINUNGEN
IN BEWEGTEN KÖRPERN

VON

H. A. LORENTZ,
PROFESSOR AN DER UNIVERSITÄT LEIDEN.

LEIDEN. — E. J. BRILL.
1895.

**Figure 2.** First page of the book by H.A. Lorentz, where the Lorentz force is derived.

$$
\left.
\begin{aligned}
u &= -2k\left(\frac{d\Omega}{dx} + \frac{4}{c^2}\frac{dU}{dt}\right),\\
v &= -2k\left(\frac{d\Omega}{dy} + \frac{4}{c^2}\frac{dV}{dt}\right),\\
w &= -2k\left(\frac{d\Omega}{dz} + \frac{4}{c^2}\frac{dW}{dt}\right);
\end{aligned}
\right\}\quad \cdot\ \cdot\ \cdot\ \cdot\ \cdot \quad (1)
$$

**Figure 3.** Equation (1) from [7]

$$\bar{\Omega} = \iiint \frac{dx'\,dy'\,dz'}{r}\, e'\left(t - \frac{r}{a}\right) + \int \frac{ds'}{r}\, e'\left(t - \frac{r}{a}\right)$$

**Figure 4.** Unnumbered equation end of page 289 from [7]

Weber's determination, $c = 284736$ miles, and therefore

$$\frac{c}{\sqrt{2}} = 201360,$$

a magnitude which remarkably agrees with the various determinations of the velocity of light; for they lie both above and

**Figure 5.** Unnumbered equation end of page 293 from [7]

Most important, in in Fig. 4 $a$ is the speed of light, and since he derives in his paper that Kirchhoff constant $c$ and his arbitrary velocity $a$ must be in the relation $c = \sqrt{2}a$, he correctly infers that the electrical perturbations he developed moves at a speed which is remarkably similar to the speed of light as it was known at the time (Fig. 5).

Lastly, he proves that (Fig. 6)

by partial integration and introduction of the designations $\alpha, \beta, \gamma$ we obtain

$$\frac{d\bar{\Omega}}{dt} = -2\left(\frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz}\right).$$

**Figure 6.** Unnumbered equation end of page 294 from [7]

Which, in modern notation, is

$$\nabla \cdot \mathbf{a} + \frac{1}{c_0}\frac{\partial \phi}{\partial t} = 0 \qquad (2)$$

That is Lorenz (and not Lorentz) gauge! And Lorentz was only 14 years old when Lorenz published [7].

As it is well known, the original ideas of Gauss, Weber and Neumann, developed by Kirchhoff in a first embryo of potentials [8], further developed into *retarded* by Lorenz, as shown before, were indeed fully developed shortly afterward by Helmholtz in a long series of papers [9, 10, 11]

## 3 Lorenz-Lorentz

Actually the two even discovered independently the same formula: a mathematical relationship between the refractive index and the density of a medium.

This was published by Lorenz in 1869 [12] (Fig. 7) and by Lorentz in 1878 [13, 14] and is therefore called the Lorentz–Lorenz equation. In modern notation:

$$\frac{n^2-1}{n^2+2} = \frac{4\pi}{3} N \alpha_m \qquad (3)$$

with $n$ the refraction index, $N$ the number of molecules per unit volume, and $\alpha_m$ the mean polarizability of the medium.

This is indeed very similar to the Clausius-Mossotti relation [15], where the relative permittivity $\varepsilon_r$ takes the place of the square refractive index.

Again, there is a sort of prevarication of the more famous on the less famous, since, due to publication priority, for such a formula the Lorenz-Lorentz name, as in the title of this section, would be more appropriate. Yet Lorentz-Lorenz is more widespread.

Funktion af Brydningsforhold og R

$$\frac{A^2-1}{A^2+2}v,$$

tes som tilnærmelsesvis konstant.

**Figure 7.** Lorenz version of the Lorenz-Lorentz equation; unnumbered equation middle of page 237 from [12]

## 4 Lorenz-Mie

Lorenz also developed a theory of light scattering, again publishing it in Danish so not so widely known, in 1890. Thanks to the translations of his works in French, in 1898 [16], this theory was available to a larger public. Yet, it was later independently rediscovered by Gustav Mie in 1908 [17], so it is sometimes referred to as Lorenz–Mie theory.

## 5 conclusions

It is understandable that a Nobel Prize active in electromagnetics might overshadow the contribution of a scientist with (almost) the same family name whose publications were in Danish and, later, translated only in French. This explains why this incorrect attribution is so widespread, especially in older textbooks.

Van Bladel was the first, to the best of the author's knowledge, nearly thirty years ago, to point out, in a dedicated article, the rightful attribution to Lorenz of the gauge, and, indeed, of the very idea of retarded potential itself.

This paper by Van Bladel, even if just one page long is, in the authors' opinion a remarkable contribution by him which ought to be celebrated in this special session.

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