URSI GASS 2020, Rome, Italy, 29 August - 5 September 2020

Uncertainty Analysis for S_{11} Calibration of a Coaxial Line with Three Reference Materials without Short Termination

Kouji Shibata
Hachinohe Institute of Technology, Aomori, Japan, shibata@hi-tech.ac.jp

Abstract

The procedure for determination of measurement uncertainty in S_{11} calibration for a cut-off circular waveguide loaded coaxial line with three reference materials (pure water, methanol and air as reference materials) was examined. It was assumed that the S_{11} value of each calibrator deviated from the true value. The uncertainty of the measured S_{11} values with these three materials in the jig as unknown quantities was calculated for frequencies of 0.50, 1.5 and 3.0 GHz. This approach enabled quantitative evaluation of measurement uncertainty for S_{11} coaxial line calibration with the three reference materials.

1 Introduction

Shibata previously proposed a high-precision broadband dielectric measurement method for small amounts of certain liquids based on a reflection constant using a coaxial-feed-type open-ended cut-off circular waveguide [1]. In this work, application of the mode-matching (MM) technique enabled calculation of S_{11} for an analytical model more quickly than with other approaches. A method enabling S_{11} calibration for a cut-off circular waveguide loaded coaxial line using a vector network analyzer (VNA) with three reference materials without short termination was also proposed, and with effectiveness verified from electromagnetic (EM) analysis and actual S_{11} measurement [2]. In this study, calculation of measurement uncertainty at the time of S_{11} calibration for a cut-off waveguide loaded coaxial line with pure water, methanol and air as reference materials was examined. The uncertainty of the measured S_{11} with these unknown materials in the jig was then calculated for frequencies of 0.50, 1.5 and 3.0 GHz. This approach enabled quantitative evaluation of measurement uncertainty for S_{11} coaxial line calibration with three reference materials.

2 S_{11} calibration theory

This chapter first describes S_{11} calibration for a transmission line. In this approach, a reference material is used instead of load termination as a calibrator. S_{11} is also not determined with the coaxial tip short-circuited, but is measured with a second reference material inserted. Based on the above, S_{11} values determined with three reference materials in the jig as shown in Fig. 1 are used for S_{11} calibration in the coaxial line with this method [2]. The true values of complex permittivity for the three reference materials are first required, and the theoretical values of S_{11} at the front surface of the sample with the three termination conditions must also be calculated from equivalent circuit analysis assuming insertion of the three reference materials. Actual S_{11} values with these materials inserted is determined using VNA. The S_{11} value for the front surface of the sample with insertion of the unknown material is then calibrated via substitution of these values into the equation described later. The specific theory and procedure of S_{11} calibration with this method has been described in Ref. [2].

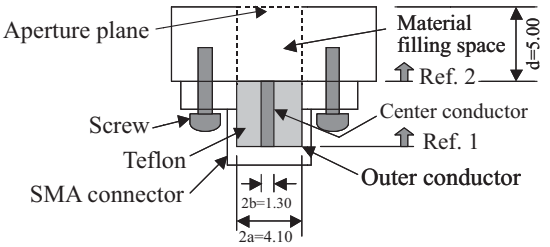

Figure 1. Jig cross section

3 Definition of measurement uncertainty

Uncertainty with respect to measured S_{11} values was examined after S_{11} coaxial line calibration using three reference materials. The equations used for S_{11} calibration as necessary for estimation of uncertainty are thus iterated. The reflection coefficient ρ_i at the SOL calibration plane (Ref. 1) is calculated using Eq. (1) in Ref. [2]. Moreover, the reflection coefficient Γ_{ii} at the front surface of the sample (Ref. 2) is also calculated using Eq. (1) in Ref. [2]. The measured reflection coefficient Γ_{corr} for Ref. 2 (Port 2) is then calculated from the measured reflection coefficient for Ref. 1 (Port 1) ρ_{meas} with an unknown material inserted, with the above relationship as follows:

$$\dot{\Gamma}_{corr} = \frac{a_2}{b_2} = \frac{\dot{\rho}_{meas} - \dot{E}_{DF}}{\dot{E}_{RF} + \dot{E}_{SF} \cdot \left(\dot{\rho}_{meas} - \dot{E}_{DF} \right)} \quad (1)$$

Uncertainty for the measured S_{11} after S_{11} calibration from the three reference materials is defined as:

$$u_c = \sqrt{\varepsilon_{t\_1}{}^2 \cdot u^2\left(\Gamma_1\right) + \varepsilon_{t\_2}{}^2 \cdot u^2\left(\Gamma_2\right) + \varepsilon_{t\_3}{}^2 \cdot u^2\left(\Gamma_3\right)} \quad (2)$$

Here, $u(\Gamma_i)$ represents uncertainty in each calibration condition. ε_{t_i} is measurement error determined from the propagation theory of uncertainty when the reflection coefficient differs from the true value with each reference material inserted, and the first letter t of the suffix represents the total error propagation for each condition. The next letter, i, expresses the individual insertions of the three reference materials. The value of ε_{t_i} is calculated for each reference material inserted as:

$$\varepsilon_{t\_i} = \frac{\partial \Gamma_{corr}}{\partial E_{DF}} \cdot \frac{\partial E_{DF}}{\partial \rho_i} \cdot \frac{\partial \rho_i}{\partial \Gamma_i} \cdot \varepsilon_i + \frac{\partial \Gamma_{corr}}{\partial E_{SF}} \cdot \frac{\partial E_{SF}}{\partial \rho_i} \cdot \frac{\partial \rho_i}{\partial \Gamma_i} \cdot \varepsilon_i + \frac{\partial \Gamma_{corr}}{\partial E_{RF}} \cdot \frac{\partial E_{RF}}{\partial \rho_i} \cdot \frac{\partial \rho_i}{\partial \Gamma_i} \cdot \varepsilon_{ii} \quad (3)$$

Here, Γ_{corr} is the measured reflection coefficient for Ref. 2 with an unknown material inserted. This is calculated by substituting E_{DF}, E_{RF}, E_{SF} as determined in the calibration procedure and ρ_{meas} for Ref. 1 into Eq. (1) with the unknown material inserted, and is measured via VNA after calibration. ε_i is the error defined as a complex number with respect to the reflection coefficient Γ_i for Ref. 2 based on the impedance standard for each known

reference material used in calibration for each condition. The three reference materials are distinguished with insertion depending on the subscripts i = 1, 2 and 3 for ε and Γ. ρ_i is the reflection coefficient for Ref. 1 with each reference material inserted. Accordingly, minor changes in ρ_i are calculated by substituting E_{DF}, E_{RF} and E_{SF} (as based on minor changes in the reflection coefficient Γ_i for Ref. 2 with the reference material inserted) and the S_{11} calibration value into Eq. (1) in Ref. [2]. The values necessary for calculation of measurement uncertainty via this procedure are then determined. In this way, complex permittivity ε_{rm} with the cut-off waveguide reflection method is first determined assuming the insertion of each reference material into the jig. Next, the reflection coefficient calculated using the MM technique based on ε_{rm} is defined as the true reflection coefficient Γ_{i_ideal} for Ref. 2 with the reference materials in the jig as calibration standards. Next, the measured values of the impedance standard with each reference material used for S_{11} calibration are defined as the reflection coefficient Γ_i. Here, Γ_i is assigned as the reflection coefficient determined by moving the reference plane to the front surface of the sample (Ref. 2) for S_{11} measured after normal SOL calibration. Errors associated with the calibration kit and observation plane movement are also included in the above Γ_i values. Thus, the measurement error ε_i in each calibration condition for substitution into Eq. (3) is defined from the difference between the measured known reflection coefficient Γ_i and the theoretical value Γ_{i_ideal} as:

$$\dot{\varepsilon}_i = \dot{\Gamma}_i - \dot{\Gamma}_{i\_ideal} \quad (4)$$

ε_i can be determined using Eq. (4) from the measured reflection coefficient in each calibration condition after calibration via SOL or other methods and its theoretical value Γ_{i_ideal} [3]. $u(\Gamma_i)$ in Eq. (2) is defined as the uncertainty of the calibration standard for each calibration condition. Accordingly, it is assumed that $u(\Gamma_i) \equiv \varepsilon_i$ from the results of Eq. (4). The uncertainty for the measured S_{11} value after calibration with the three reference materials is calculated by substituting the ε_i value of Eq. (4) and the measurement error ε_{t_i} in consideration of propagation calculated using Eq. (3) and $u(\Gamma_i)$ into Eq. (2). Execution of partial differentiation with each term is required for the iteration of Eq. (3). This partial derivative was calculated via numerical differentiation. The above derivation process is thus summarized. The uncertainty included in the measured S_{11} value after calibration of S_{11} for the coaxial line with the three liquid reference materials as determined using Eq. (2) is determined as follows:

1. Calculation of the measurement error ε_i for each calibration condition using Eq. (4)
2. Calculation of minor changes in Γ_{corr} using Eq. (1) from minor changes in E_{DF}, E_{SF} and E_{RF}
3. Calculation of minor changes in E_{DF}, E_{SF} and E_{RF} using Eqs.(1), (6) and (7) in Ref. [2] from minor changes in ρ_i
4. Calculation of minor changes in ρ_i using Eq. (1) in Ref. [2] from minor changes in Γ_i
5. Calculation of measurement error ε_{t_i} in consideration of the propagation theory of measurement uncertainty with the above values substituted into Eq. (3)
6. Definition of the uncertainty of calibration condition $u(\Gamma_i)$ based on results for the calculation of ε_i
7. Substitution of the measurement error ε_{t_i} in item 5 and $u(\Gamma_i)$ in item 6 into Eq. (2)

4 Budget sheet for measurement uncertainty

Next, the measurement error ε_i for each calibration condition was calculated using Eq. (4) to evaluate specific measurement uncertainty after S_{11} calibration using the three reference materials. For this purpose, the value calculated using the MM technique was substituted for the theoretical reflection coefficient Γ_{i_ideal} at the surface of the sample (Ref. 2) for each calibration standard. The measured value Γ_i of the reflection coefficient Γ_i was also determined using the following procedure: 1. The coaxial tip is calibrated using a short, open and loaded (SOL) calibration kit. 2. The jig is attached to the coaxial tip. 3. S_{11} on the SOL calibration surface is measured with each reference material. 4. The electrical length of the above measured value is moved to the front surface of the sample (Ref. 2) via comparison with the calculated value of S_{11} based on EM analysis as described in [4]. The results are shown in Table 1 (a). In this way, the value of the imaginary part of the input impedance of 1626.83 was determined from the value measured with the electrical length moved 22.43 mm from Port 1 to Port 2 of the load side with respect to the value of 1620.49 calculated using the MM technique. The calculated values based on electrical length offsetting thus showed close agreement with those calculated via EM analysis. Accordingly, the electrical length of the measured value on the SOL calibration plane (electrical length: 0.0 mm) was also offset to the sample front (Ref. 2) on the 22.43 mm load side with pure water and methanol in the jig. The reflection coefficients measured with each reference material in the jig after movement of the electrical length to the front of the sample (Ref. 2) based on another calibration method and the reflection coefficient calculated using the MM technique at frequencies of 0.50, 1.5 and 3.0 GHz are shown in Tables 1 (a), (b) and (c). Based on these considerations, the difference between measured and theoretical reflection coefficients at the front surface of the sample (Ref. 2) with each material in the jig was quantitatively evaluated. Tables 1 (a), (b) and (c) also show the calculated results for measurement error ε_i and uncertainty $u(\Gamma_1)$ based on Eq. (4) for each calibration standard with the reference material inserted. The measurement uncertainty of S_{11} with each sample after jig calibration is calculated from Eqs. (2), (3) and (4). Accordingly, measurement uncertainty $u(\Gamma_1)$ as listed in Table 1 is predominantly affected by the difference between the measured input impedance of the three reference materials at the front surface of the sample (Ref. 2) Z_1 as determined using another calibration method and the theoretical value Z_{ideal1} based on the MM technique. The measured input impedance at the front surface of the sample (Ref. 2) Z_1 based on another calibration method with three reference materials was determined in consideration of the difference from the theoretical value.

TABLE 1: Reflection coefficient, measurement error and uncertainty at the front surface of the sample (Ref. 2) with the reference material inserted

(a) Reference material 3 (air, 25°C)

Value	Frequency [GHz]		
	0.50	1.5	3.0
Measured input impedance with another calibration method Z	+454.21643 –j 10108.1895	+158.32234 –j 3417.36231	+78.99806213 –j 1626.8264160
Theoretical input impedance with the mode matching method Z_{ideal3}	0.00000000 –j 9734.5537	0.00000000 –j 3243.9670	0.00000000 –j 1620.4893
Measured reflection coefficient with another calibration method Γ	+0.99951 –j 0.00987	+0.99822 –j 0.02915	+0.99516 –j 0.06109
Theoretical reflection coefficient with the mode matching method Γ_{ideal3}	+0.99951 –j 0.00987	+0.99952 –j 0.003082	+0.99810 –j 0.06165
Measurement error ε_3	$-4.39493\cdot10^{-4}$ $+j\ 4.03999\cdot10^{-3}$	$-1.302205\cdot10^{-3}$ $+j\ 1.66514\cdot10^{-3}$	$-2.9414796\cdot10^{-3}$ $+j\ 5.657691\cdot10^{-3}$
Uncertainty $u(\Gamma_3)$	$-4.39493\cdot10^{-4}$ $+j\ 4.03999\cdot10^{-3}$	$-1.302205\cdot10^{-3}$ $+j\ 1.66514\cdot10^{-3}$	$-2.9414796\cdot10^{-3}$ $+j\ 5.657691\cdot10^{-3}$

(b) Reference material 1 (pure water, 25°C)

Value	Frequency [GHz]		
	0.50	1.5	3.0
Measured input impedance with another calibration method Z	+3.87963605 –j 160.536758	+3.77143216 –j 52.413837	+3.67059779 –j 24.64155197
Theoretical input impedance with the mode matching method Z_{ideal1}	+3.468596 –j 142.7852	+3.469369 –j 46.78456	+3.485887 –j 21.94313
Measured reflection coefficient with another calibration method Γ	+0.81210 –j 0.55985	–0.04637 –j 0.92956	–0.53884 –j 0.70652
Theoretical reflection coefficient with the mode matching method Γ_{ideal1}	+0.76999 –j 0.611422	–0.05927 –j 0.92684	–0.60030 –j 0.65654
Measurement error ε_1	$+4.21099\cdot10^{-2}$ $+j\ 5.43743\cdot10^{-2}$	$+1.05633\cdot10^{-1}$ $-j\ 2.72136\cdot10^{-3}$	$+7.08532\cdot10^{-2}$ $-j\ 5.96352\cdot10^{-2}$
Uncertainty $u(\Gamma_1)$	$+4.21099\cdot10^{-2}$ $+j\ 5.43743\cdot10^{-2}$	$+1.05633\cdot10^{-1}$ $-j\ 2.72136\cdot10^{-3}$	$+7.08532\cdot10^{-2}$ $-j\ 5.96352\cdot10^{-2}$

(c) Reference material 2 (methanol, 25°C)

Value	Frequency [GHz]		
	0.50	1.5	3.0
Measured input impedance with another calibration method Z	+41.77264786 –j 333.30685	+42.53910828 –j 112.779319	+40.68000031 –j 59.478950
Theoretical input impedance with the mode matching method Z_{ideal2}	+39.217224 –j 305.563	+39.771641 –j 104.3457	+38.84074783 –j 55.8549385
Measured reflection coefficient at the front surface of the sample (Ref. 2) with another calibration method Γ	+0.92321 –j 0.27888	+0.56519 –j 0.52991	+0.22895 –j 0.50575
Theoretical reflection coefficient with the mode matching method Γ_{ideal2}	+ 0.92321 –j 0.27888	+0.569189 –j 0.52927	+0.22895 –j 0.50575
Measurement error ε_2	$+1.12604\cdot10^{-2}$ $+j\ 2.26757\cdot10^{-2}$	$+\ 3.899482\cdot10^{-2}$ $+j\ 2.08132\cdot10^{-2}$	$+\ 3.568132\cdot10^{-2}$ $+j\ 1.450750\cdot10^{-3}$
Uncertainty $u(\Gamma_2)$	$+1.12604\cdot10^{-2}$ $+j\ 2.26757\cdot10^{-2}$	$+\ 3.899482\cdot10^{-2}$ $+j\ 2.08132\cdot10^{-2}$	$+\ 3.568132\cdot10^{-2}$ $+j\ 1.450750\cdot10^{-3}$

Measurement of S_{11} at the front surface of the sample (Ref. 2) was also performed using Eq. (1) with an unknown material inserted after calibration of the measurement system with the three reference materials using the proposed method for the purpose of uncertainty evaluation. Specifically, the reflection coefficient ρ_i at the SOL calibration plane (Ref. 1) with various unknown materials in the jig after coaxial line calibration with the three reference materials and no short termination were measured as the input impedance as shown in Table 2.

TABLE 2: Measured input impedance Z_{corr1} (25°C) for Ref. 1 with unknown materials inserted after calibration with three reference materials

Value	Frequency [GHz]		
	0.50	1.5	3.0
Unknown material 1 Pure water	+2.08833599 –j 112.792122	+2.24340391 –j 24.993719	+3.11580610 +j 11.8024073
Unknown material 2 Methanol	+13.43388748 –j 185.504227	+14.21072769 – j 54.650936	+15.5995226 –j 14.3712282
Unknown material 3 Ethanol	+38.06984711 –j 228.073166	+25.75860023 –j 85.072999	+14.06677723 –j 38.350910

5 Uncertainty measurement results

Consideration as outlined in this section was performed regarding the measured values of S_{11} with various unknown materials in the jig after coaxial line calibration with the three reference materials. Measurement uncertainty associated with the difference between the theoretical (true) value of each impedance standard used at the time of calibration and the value measured with another calibration method were then calculated using the above data. In this examination, measurement uncertainty was often reduced via sign cancellation in the iteration of Eq. (2) for real and imaginary parts as a complex number. Measurement uncertainty was also evaluated with separate calculation of real and imaginary parts with Eq. (2). The measurement error ε_{t_i} (determined from uncertainty propagation theory when the reflection coefficient with each reference material inserted differs from the true value at the time of S_{11} calibration) was first calculated via the procedure outlined in item 5 in Chapter 3. Measurement uncertainty u_c was then calculated using the proposed method via the procedures outlined in item 6 and item 7 in Chapter 3. The results for measured (corrected) S_{11} values and related uncertainty u_c with pure water in the jig after calibration as shown in Table 3 (a) show close agreement with theoretical values determined using the MM technique. This is attributed to the fact that the measured values shown in Table 2 were determined immediately after calibration. Measurement uncertainty values such as u_c = $5.3575\cdot10^{-3}$, $8.9520\ 10^{-3}$ were also determined at a frequency of 0.50 GHz with pure water in the jig as an unknown material. Accordingly, low measurement uncertainty values in the order of 10^{-3} were determined for both real and imaginary parts. This is attributed to the fact that the complex permittivity of pure water based on the Debye relaxation equation and the complex permittivity of methanol measured previously at the time of calibration are close to the true values. However, the value for the imaginary part regarding measurement uncertainty worsened at high frequencies, and the imaginary part in measurement uncertainty was large at 1.5 GHz. The results in both cases are attributed to the influence of the difference between the measured S_{11} value in Ref. 2 determined with another calibration method and the theoretical value determined using the MM technique with pure water and methanol in the jig for S_{11} calibration.

TABLE 3: Calculation of reflection coefficients at the front surface of the sample (Ref. 2) and uncertainty of measured values

(a) Pure water inserted as an unknown material (25°C)

Value		Frequency [GHz]		
		0.50	1.5	3.0
Measurement error with consideration of propagation	With pure water inserted ε_{t_1}	$+1.263206\cdot10^{-1}$ $+1.631226\cdot10^{-1}$	$+3.168929\cdot10^{-1}$ $-8.203589\cdot10^{-3}$	$+2.125576\cdot10^{-1}$ $-1.789049\cdot10^{-1}$
	With methanol inserted ε_{t_2}	$+5.666956\cdot10^{-2}$ $+5.342763\cdot10^{-2}$	$+1.5262449\cdot10^{-2}$ $-5.3343294\cdot10^{-2}$	$+8.282289\cdot10^{-2}$ $-1.546633\cdot10^{-1}$
	With air inserted ε_{t_3}	$-2.888015\cdot10^{-4}$ $+1.740144\cdot10^{-3}$	$+4.756431\cdot10^{-3}$ $+3.473369\cdot10^{-3}$	$+6.176862\cdot10^{-3}$ $+5.064244\cdot10^{-3}$
Theoretical reflection coefficient with the mode matching method Γ_{ideal}		+0.769990 –j 0.61422	–0.05927 –j 0.92684	–0.60030 –j 0.65654
Correction value for reflection coefficient after calibration Γ_{corr}		+0.769993 –j 0.614222	–0.0592675 –j 0.926837	–0.6096895 –j 0.646884
Measurement uncertainty u_c		$+3.5954\cdot10^{-3}$ $-1.3866\cdot10^{-2}$	$+3.4159\cdot10^{-2}$ $-1.46646\cdot10^{-3}$	$+4.97448\cdot10^{-3}$ $-2.58305\cdot10^{-2}$
Measurement uncertainty u_c		$+5.3575\cdot10^{-3}$ $+8.9520\cdot10^{-3}$	$+3.3999\cdot10^{-2}$ $+1.1105\cdot10^{-3}$	$+1.5348\cdot10^{-2}$ $+1.0671\cdot10^{-2}$

The measured reflection coefficient Γ_{corr} and measurement uncertainty u_c with methanol in the jig as an unknown material were determined after jig calibration with the three reference materials. The results (Table 3 (b)) indicate close agreement between measured and theoretical values based on the MM technique at 0.50 GHz. This is attributed to the fact that that the measured values in Table 2 were determined immediately after calibration. Measurement uncertainty values such as a u_c of $7.54175\cdot10^{-3}$ for the real part and $1.16654\cdot10^{-2}$ for the imaginary part were determined. Accordingly, measurement uncertainty for the real part in the order of 10^{-3} was determined under the above conditions. For the imaginary part, measurement uncertainty values in the order of 10^{-3} were also determined. However, the value for the imaginary part regarding measurement uncertainty worsened at high frequencies, and the real part in measurement uncertainty was large at 1.5 GHz. The results in both cases are attributed to the influence of the difference between the measured S_{11} value in Ref. 2 determined with another calibration method and the

theoretical value determined with the MM technique when pure water and methanol were in the jig for S_{11} calibration. In future work, S_{11} calibration for the measurement system is required with a reflection coefficient close to the true value of calibration standards with pure water and methanol in the jig.

(b) Methanol inserted as an unknown material (25℃)

Value		Frequency [GHz]		
		0.50	1.5	3.0
Measurement error with consideration of propagation	With pure water inserted ε_{r_1}	$+3.651538\cdot10^{-2}$ $+1.908025\cdot10^{-1}$	$+1.85449\cdot10^{-1}$ $+1.67213\cdot10^{-1}$	$+1.17004\cdot10^{-1}$ $-1.08094\cdot10^{-1}$
	With methanol inserted ε_{r_2}	$+2.489167\cdot10^{-2}$ $+6.901419\cdot10^{-2}$	$+1.16110\cdot10^{-1}$ $+5.23233\cdot10^{-2}$	$+4.33413\cdot10^{-2}$ $-9.07751\cdot10^{-2}$
	With air inserted ε_{r_3}	$-9.842897\cdot10^{-4}$ $+1.337849\cdot10^{-3}$	$+8.3259\cdot10^{-4}$ $+4.5640\cdot10^{-3}$	$+3.67183\cdot10^{-3}$ $+2.73697\cdot10^{-3}$
Theoretical reflection coefficient with the mode matching method Γ_{ideal}		$+0.92321$ $-j\ 0.2789$	$+0.56519$ $-j\ 0.52991$	$+0.22895$ $-j\ 0.50575$
Correction value for reflection coefficient after calibration Γ_{corr}		$+0.92321$ $-j\ 0.7888$	$+0.56519$ $-j\ 0.5299$	$+0.23071$ $-j\ 0.50574$
Measurement uncertainty u_c		$+7.541752\cdot10^{-3}$ $-1.166547\cdot10^{-2}$	$+2.03062\cdot10^{-2}$ $+1.76922\cdot10^{-2}$	$+2.16521\cdot10^{-3}$ $-1.49259\cdot10^{-2}$
Measurement uncertainty u_c		$+2.82833\cdot10^{-3}$ $+1.03705\cdot10^{-2}$	$+2.0106\cdot10^{-2}$ $+1.1803\cdot10^{-3}$	$+8.43312\cdot10^{-3}$ $+6.44757\cdot10^{-3}$

The measured reflection coefficient after calibration Γ_{corr} and uncertainty u_c were also determined with ethanol in the jig as an unknown material. The results (Table 3 (c)) show differences between measured values and those determined using the theoretical MM technique. This is attributed to the fact that the liquid (ethanol) evaluated as an unknown material differed from the reference materials (pure water and methanol) used at the time of calibration. However, at frequencies such as 0.50 GHz, measurement uncertainty u_c values of $1.56228\cdot10^{-3}$ for the real part and $1.04021\cdot10^{-2}$ for the imaginary part were determined. With these conditions, measurement uncertainty for the real part in the order of 10^{-3} was determined. For the imaginary part, measurement uncertainty values close to the order of 10^{-3} were also determined. Such low measurement uncertainty is considered to be related to the following:

1. S_{11} was calibrated using the same jig with estimation of the complex dielectric constant as an inverse problem of EM analysis using the cut-off circular waveguide reflection method.

2. The true value of the impedance standard was determined by substituting the complex permittivity value estimated using the above jig with calculation of input impedance using the MM technique (EM analysis).

The value of the imaginary part in terms of measurement uncertainty worsened at high frequencies. This is attributed to the influence of the difference between the measured S_{11} in Ref. 2 and the theoretical value determined using the MM technique at high frequencies with each reference material in the jig for S_{11} calibration. In future work, multiple re-measurements of S_{11} will be required with pure water, methanol and air in the jig as reference materials before and after calibration, including temporal changes with varying temperatures. Verification of measurement error and uncertainty relating to S_{11} at the front surface of the sample (Ref. 2) after calibration is also required based on repeated S_{11} measurement with an unknown material in the jig. Differences in measurement uncertainty relating to S_{11} calibration using the proposed method or SOL also need to be compared.

(c) Ethanol inserted as an unknown material (25℃)

Value		Frequency [GHz]		
		0.50	1.5	3.0
Measurement error with consideration of propagation	With pure water inserted ε_{r_1}	$+3.65154\cdot10^{-2}$ $+1.90802\cdot10^{-1}$	$+7.40524\cdot10^{-2}$ $+2.49456\cdot10^{-1}$	$-2.13305\cdot10^{-2}$ $-2.06499\cdot10^{-1}$
	With methanol inserted ε_{r_2}	$+2.48017\cdot10^{-2}$ $+6.90142\cdot10^{-2}$	$+7.48195\cdot10^{-2}$ $+1.09616\cdot10^{-1}$	$-6.08202\cdot10^{-2}$ $-1.16132\cdot10^{-1}$
	With air inserted ε_{r_3}	$-9.84289\cdot10^{-4}$ $+1.33785\cdot10^{-3}$	$-1.73949\cdot10^{-3}$ $+4.51087\cdot10^{-3}$	$+5.72372\cdot10^{-3}$ $-1.69148\cdot10^{-3}$
Theoretical reflection coefficient with the mode matching method Γ_{ideal}		$+0.9056046$ $-j\ 0.185193$	$+0.7299568$ $-j\ 0.238807$	$+0.6468458$ $-j\ 0.2755841$
Correction value for reflection coefficient after calibration Γ_{corr}		$+0.91691$ $-j\ 0.175422$	$+0.73792$ $-j\ 0.234101$	$+0.64955$ $+j\ 0.26323$
Measurement uncertainty u_c		$+8.92979\cdot10^{-3}$ $-1.10109\cdot10^{-2}$	$+8.44393\cdot10^{-3}$ $+2.67659\cdot10^{-2}$	$+1.3877\cdot10^{-2}$ $+1.3920\cdot10^{-2}$
Measurement uncertainty u_c		$+1.56228\cdot10^{-3}$ $+1.04021\cdot10^{-2}$	$+8.34874\cdot10^{-3}$ $+2.38934\cdot10^{-3}$	$+2.6446\cdot10^{-3}$ $+1.2316\cdot10^{-2}$

6 Conclusion

In this study, calculation of uncertainty in relation to measured S_{11} values was examined with respect to the S_{11} calibration method for a coaxial line. The approach excluded short termination, with the use of an arbitrary impedance standard or three reference materials for the purpose of dielectric measurement from S_{11} with liquids in a coaxial feed-type open-ended cut-off circular waveguide. Pure water, methanol and air were used as reference materials, and pure water, methanol and ethanol were the unknown materials. Uncertainty for measured S_{11} values with an unknown material in the jig after calibration was then calculated at the frequencies of 0.50, 1.5 and 3.0 GHz with the condition that the S_{11} values of these calibrators differed from the true values. With ethanol in the jig as an unknown material, measurement uncertainty values u_c of $1.56228\cdot10^{-3}$ for the real part and $1.04021\cdot10^{-2}$ for the imaginary part were determined at 0.50 GHz. Measurement uncertainty at the time of S_{11} coaxial line calibration with the three reference materials was thus examined quantitatively. The low measurement uncertainty observed is attributed to the following:

1. S_{11} was calibrated using the same jig in estimation of the dielectric constant as an inverse problem of EM analysis using the cut-off waveguide reflection method.

2. The true value of the impedance standard was determined by substituting the complex permittivity value estimated using the above jig with calculation of input impedance using the MM technique (EM analysis).

Future work will involve evaluation of other liquids and related temperature dependence along with extension of the method to research on liquids in the millimeter range. Round-robin testing at different institutions is necessary in association with standardization of the method.

7 References

1. K. Shibata, “Measurement of Complex Permittivity for Liquid Materials Using the Open-ended Cut-off Waveguide Reflection Method,” IEICE Trans. Electron., Vol. E93-C, No. 11, pp. 1,621 – 1,629, 2010-11.
2. K. Shibata, “S11 Calibration Method for a Coaxial Line with Three Reference Materials and No Short Termination Condition for Dielectric Measurement in Liquids,” Proc. of URSI AP-RASC 2019, New Delhi, India, 2019-3.
3. M. Rodriguez, “Tree-term error correction in one-port calibrations,” ANAMET Report 020, pp. 1 – 9, 1998-11.
4. K. Shibata, “Improvement in Liquid Permittivity Measurement Using the Cut-off Waveguide Reflection Method,” Proc. of 1st European Microw. Conference in Central Europe, EuMCE, Prague, Czech Republic, 2019-5.