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Electromagnetic interaction with a monodispersed system in sedimentation equilibrium

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Abstract

In this paper, a new model to explain the electromagnetic interaction with a monodispersed system in sedimentation equilibrium is presented. Our approach is based on the dispersive system as constituted by a large number of stratification each characterized a constant particle concentration. The interaction between the electromagnetic field and the stratified material is taken into consideration thanks to the T-matrix approach. The concentration of these media varies with continuity along the stratified direction. A host medium constitutes each layer, and it is also assumed to be identical for all layers in which some nano-spherical objects are immersed. Thanks to these hypotheses, the homogenization mixing formula has been taking into account. The reached results show the behavior of the stratified medium as a function of the density for the spherical hosted object and as a function of angular incident direction.

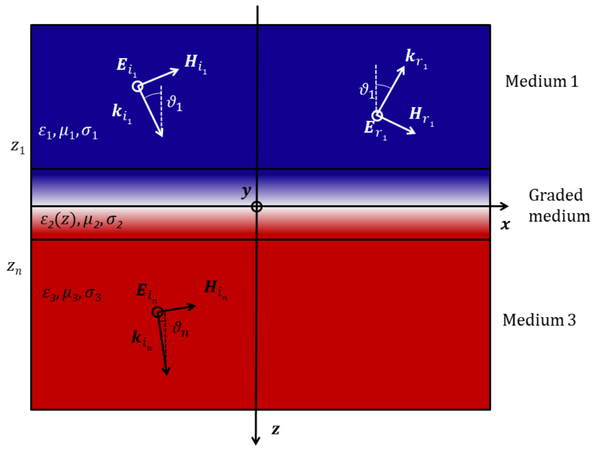

Figure 1. Depicted geometry of monodispersed systems in sedimentation equilibrium.

1 Introduction

Brownian particles reach a certain velocity under the action of an external field, this phenomenon is known as sedimentation. The differences in the particle concentration between the surface and the bottom of the dispersion system are due to sedimentation triggered by the gravity field of the earth. The concentration of the particles increases moving deeper into the dispersion system. The difference between various areas leads to a diffusion phenomenon from the bottom to the surface of the system. The equilibrium state, also named sedimentation equilibrium, is reached when the diffusion velocity equals the settling velocity [1, 2]. An electromagnetic model that takes into consideration all the electromagnetic interactions with a system in sedimentation equilibrium is introduced, the layers are assumed to be characterized by a gradual gradient of reflective index modeled as a stratified medium. The transmission through the stratified medium is determined by the important formalism of the T-Matrix. This matrix has been firstly introduced by Abelès, and in many optics and electromagnetic textbooks is also exposed [3]-[5]. Each layer is constituted by a mixture of dielectric media, i.e. a dielectric host containing a particular volume fraction of spherical objects, in order to simulate a colloidal system. Working with a wavelength of the incident field even more than ten times greater than the maximum radius of the spherical object, we can model each layer with the homogenization models, in particular with the Maxwell-Wagner formula [8, 10]. In this paper, the effects of the reflection coefficient on a layered medium characterized by a graded reflective index as a function of the incident angle and as a function of the volume fraction between the two phases constituting the dispersed system are shown.

2 Theoretical Approach

Fig. 1 illustrates the geometry of the problem. The first surface of the medium is impinged by the incident monochromatic radiation, characterized as a plane wave. The transition layer between the two phases, with a graded reflective index, is modeled with a stratified medium, whose thickness is at least 10 times lower than the incident wavelength. The stratification direction is indicated with z from layer 1 to layer $n+1$. The wave vector of the incident radiation creates an angle ϑ_j, and its projection on the interface creates an angle φ_j, both on the j-th interface. In the j-th layer, let us impose ε_j, μ_j, σ_j being the relative permittivity, relative permeability, and electrical conductivity respectively. We make the assumption that the layers are isotropic, homogeneous, linear, and generally dissipative and dispersive.

The stratified medium, as mentioned above, can be treated using the T-matrix approach. The formalism used to solve our problem will be explained in detail below. The following formula represents the incident elliptically-polarized plane wave [11, 12]:

$$\mathbf{E}_{i_1}(\mathbf{r}) = \left(E_{i_1}^{H}\,\vartheta_{0i} + E_{i_1}^{E}\,\varphi_{0i}\right) e^{i\mathbf{k}_i\cdot\mathbf{r}} \tag{1}$$

where:

$$\mathbf{k}_{i_1} = k_1\left(\sin\vartheta_{i_1}\cos\varphi_{i_1}\,\mathbf{x}_0 + \sin\vartheta_{i_1}\sin\varphi_{i_1}\,\mathbf{y}_0 + \cos\vartheta_{i_1}\,\mathbf{z}_0\right)$$
$$\vartheta_{0i} = \cos\vartheta_{i_1}\cos\varphi_{i_1}\,\mathbf{x}_0 + \cos\vartheta_{i_1}\sin\varphi_{i_1}\,\mathbf{y}_0 - \sin\vartheta_{i_1}\,\mathbf{z}_0$$
$$\varphi_{0i} = -\sin\varphi_{i_1}\,\mathbf{x}_0 + \cos\varphi_{i_1}\,\mathbf{y}_0$$

and with $\mathbf{x}_0$, $\mathbf{y}_0$, and $\mathbf{z}_0$ the Cartesian unit vectors. The reflected waves $\mathbf{E}_r(\mathbf{r})$ by the monodispersed system are given by:

$$\mathbf{E}_r(\mathbf{r}) = \left(R_E^H E_{i_1}^H\,\vartheta_{0r} + R_E^E E_{i_1}^E\,\varphi_{0r}\right) e^{i\mathbf{k}_r\cdot\mathbf{r}} \tag{2}$$

$$\tag{3}$$

where for a stratified medium for parallel (E) and perpendicular (H) polarization of the electric field R_E^H and R_E^E are the reflection coefficients, which characterized by the effective interface between layer 1 and layer $N+1$. Imposing the boundary conditions on each layer, the two expressions between the incident and reflected electric field (in the first medium) [12], and the transmitted (in the last medium):

$$\begin{pmatrix} E_{0i_{N+1}}^{E} \\ 0 \end{pmatrix} = \prod_{\ell=N}^{1}[M_\ell]\begin{pmatrix} E_{0i_1}^{E} \\ E_{0r_1}^{E} \end{pmatrix} = [M]\begin{pmatrix} E_{0i_1}^{E} \\ E_{0r_1}^{E} \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} E_{0i_{N+1}}^{H} \\ 0 \end{pmatrix} = \prod_{\ell=N}^{1}[N_\ell]\begin{pmatrix} E_{0i_1}^{H} \\ E_{0r_1}^{H} \end{pmatrix} = [N]\begin{pmatrix} E_{0i_1}^{H} \\ E_{0r_1}^{H} \end{pmatrix} \tag{5}$$

having indicated with the apes ^{E} and ^{H} the parallel and perpendicular polarization, respectively, with

$$[M_j] = \frac{1}{2\zeta_j}\begin{bmatrix} (\zeta_j+1)e^{i\phi_{j+1}} & (\zeta_j-1)e^{i\phi_{j+1}} \\ (\zeta_j-1)e^{-i\phi_{j+1}} & (\zeta_j+1)e^{-i\phi_{j+1}} \end{bmatrix} \tag{6}$$

$$[N_j] = \frac{1}{2\chi_j}\begin{bmatrix} (\chi_j+1)e^{i\phi_{j+1}} & (\chi_j-1)e^{i\phi_{j+1}} \\ (\chi_j-1)e^{-i\phi_{j+1}} & (\chi_j+1)e^{-i\phi_{j+1}} \end{bmatrix} \tag{7}$$

with

$$\zeta_j = \frac{\mu_j k_{i_{j+1_n}}}{\mu_{j+1} k_{i_{j_n}}} \tag{8}$$

$$\chi_j = \frac{\varepsilon_j k_{i_{j+1_n}}}{\varepsilon_{j+1} k_{i_{j_n}}} \tag{9}$$

The reported parameters, characterizing the behavior of the medium, can be manipulated to trigger particular comportments. Under specific hypotheses, each layer can be considered as an "effective medium" and the Maxwell-Wagner model can be used to determine its effective permittivity [8]. Moreover, when the wavelength is greater than the radius of the hosted spheres, the Eq.s (8) and (9) can be used combined with the Maxwell-Wagner mixture equation of the j-th layer, i.e.

$$\varepsilon_{eff_j} = \varepsilon_{h_j}\frac{2\varepsilon_{h_j} + \varepsilon_{s_j} - 2\phi(\varepsilon_{h_j} - \varepsilon_{s_j})}{2\varepsilon_{h_j} + \varepsilon_{s_j} + \phi(\varepsilon_{h_j} - \varepsilon_{s_j})} \tag{10}$$

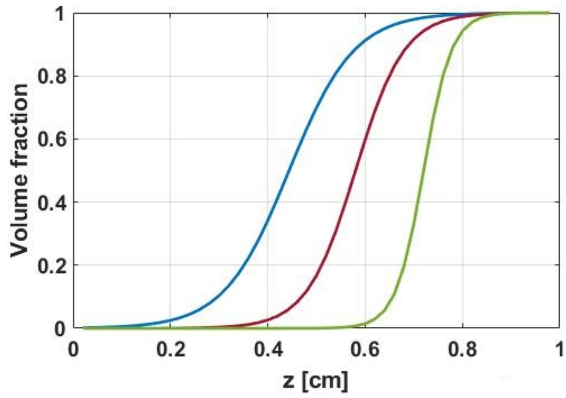

Figure 2. The trend of the volume fraction along the stratified direction of a monodispersed system in three different sedimentation equilibria. The following parameters are considered from [9] $\rho = 1.19$ g/cm^3 ,$\rho_F = 1.04$ g/cm^3, $L_g = 2.94$ mm.

3 Results and discussion

An elliptically polarized plane wave impinging on a complex structure is considered as a system. It is supposed to constituted by only two materials: a host material with $\varepsilon_h = 1$, $\mu_h = 1$ and $\sigma = 0$ S/m fictitious electromagnetic behavior and a particulate matter with $\varepsilon_h = 4$, $\mu_h = 1$, $\sigma = 0$ S/m, and particle radius 100 nm. We suppose that the concentration of particles varies according to the following law of the total force equilibrium [9]

$$\frac{\partial\mu^{id}}{\partial z} + \frac{\partial\mu^{ex}}{\partial z} = v(\rho - \rho_F)g \tag{11}$$
$$\frac{1}{\phi}\frac{\partial\phi}{\partial z} + \beta\frac{\partial\mu^{ex}}{\partial\phi}\frac{\partial\phi}{\partial z} = -\frac{1}{L_g}$$

where μ is the chemical potential composed of two contributions, one ideal contribution μ^{ex} and an excess contribution μ^{ex} of a particle type suspended in a continuum fluid, ρ is the particle mass density, v is the particle volume, g is the gravitational acceleration, and L_g is the gravitational length. Combining these equations with the eq. (11) we obtain a sigmoidal trend for the effective permittivity of the system along the stratified direction, z-axis, see Fig. 2. The trend of reflectivity in Figs. 3, 4 for both polarizations on a monodispersed system in different sedimentation equilibrium degrees are achieved. The results are reached thanks to all previous considerations, applying a circularly polarized plane wave of 1 V/m of intensity at wavelength $\lambda_0 = 1$ μm and varying the incident angle from perpendicular (0°) to parallel (90°) direction. How we can see from Figs. 3, 4 the TE polarization can be used to distinguish the state of sedimentation of a monodispersed system in an unambiguous way.

4 Conclusion

In conclusion, a new model to explain the electromagnetic interaction with a monodispersed system in sedimentation

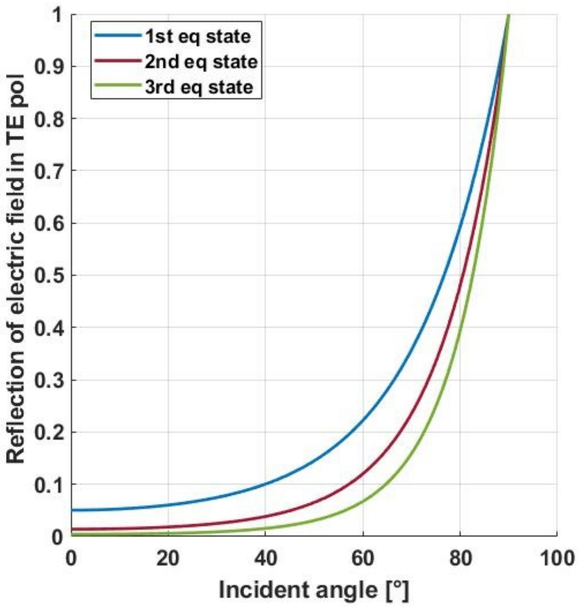

Figure 3. The trend of the reflectivity for TE-polarization on a monodispersed system as a function of the incident angle (between 0 and $\pi/$ rad) for three different sedimentation equilibrium states (see Fig. 2).

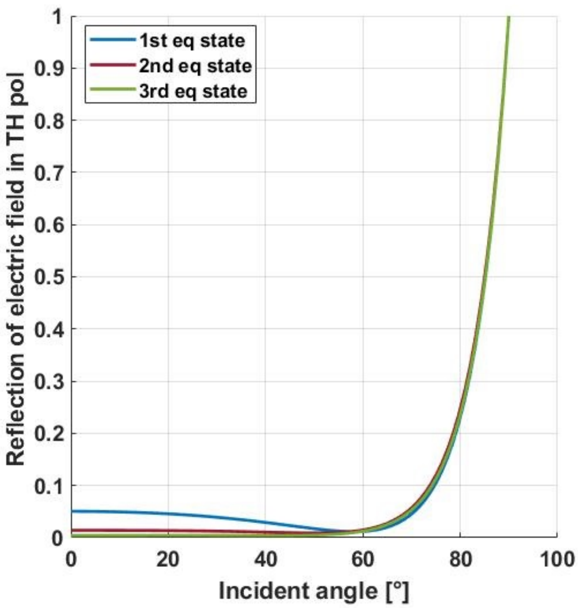

Figure 4. The trend of the reflectivity for TH-polarization on a monodispersed system as a function of the incident angle (between 0 and $\pi/$ rad) for three different sedimentation equilibrium states (see Fig. 2).

equilibrium has been introduced. From the above results, the model well represents a monodispersive system in sedimentation equilibrium. Furthermore, the system was confirmed to be very sensitive to the different electromagnetic polarization from which it is excited. In particular, the polarization TE was able to detect the sedimentation state of this complex system. This procedure could easily find applications in various areas, including biology and cultural heritage [13, 14].

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