

## Excitation of an Electromagnetic Field in a Large Nerve Fiber by an Array of Electric-Dipole Filaments

Vasiliy A. Es'kin, Alexander V. Kudrin, and Anna A. Popova  
University of Nizhny Novgorod, 23 Gagarin Ave., Nizhny Novgorod 603950, Russia

### Abstract

Excitation of an electromagnetic field with the desired spatial structure in a relatively large nerve fiber consisting of tightly packaged axons by an array of filamentary electric-dipole sources is studied. The procedure of finding the dipole moments of the filamentary sources located around the nerve fiber is proposed to ensure the required field pattern inside the nerve. Numerical results demonstrating the possibility of excitation of the desired field in the considered fiber are reported.

### 1 Introduction

In recent years, non-contact methods for stimulation of nerve fibers by the electromagnetic radiation of the infrared and optical frequency ranges have attracted enhanced interest (see, e.g., [1, 2] and references therein). These methods have no such drawbacks, which appear when stimulating the nerve tissue by contact electrodes, as simultaneous excitation of several closely located neurons by a single electrode and an immune response of the organism to the direct contact of electrodes with tissues.

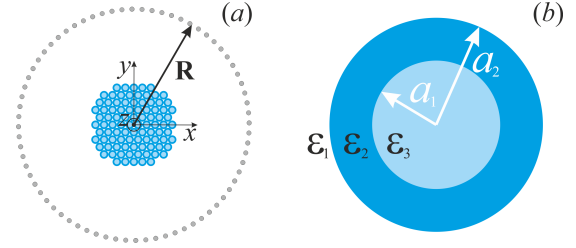
In this work, we consider the features of excitation of an electromagnetic field with the desired spatial distribution by an array of filamentary sources. It is assumed that the electromagnetic radiation is excited by electric-dipole filaments located at the same distance from the nerve-fiber axis. The emphasis is placed on the possibility of formation of the desired field distribution inside the nerve by appropriately choosing the electric dipole moments of the filaments.

### 2 Formulation of the Problem and Basic Equations

Consider a system of filamentary sources aligned with the  $z$ -axis of a Cartesian coordinate system  $(x, y, z)$  and located on a cylindrical surface of radius  $R$  [see Fig. 1(a)] in the region  $|z| < d$ , where  $d$  is the half-length of the sources. Let us introduce an individual cylindrical coordinate system  $(\rho_l, \phi_l, z_l)$  for each source, where  $l$  is the source number. The electric current density of the  $l$ th source in its cylindrical coordinate system can be written in the form

$$\mathbf{j}_l^{(\Sigma)} = \frac{\delta(\rho_l)}{2\pi\rho_l} \sum_{s=1}^S \mathbf{j}_{l,s}^{(0)} \exp(-ik_0 p_s z_l) [U(z+d) - U(z-d)], \quad (1)$$

where  $k_0 = \omega/c$  is the free-space wave number ( $c$  is the speed of light in free space and  $\omega$  is the angular fre-



**Figure 1.** Geometry of the problem (a) and the structure of an axon (b).

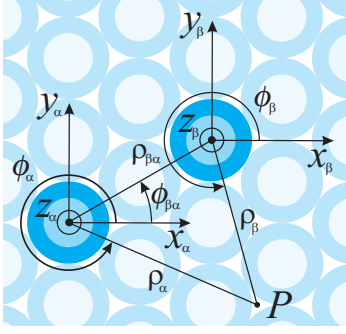
quency),  $p_s$  denotes the normalized (to  $k_0$ ) propagation constants of the current waves traveling along the filament,  $S$  is the number of such current-wave terms in the filament,  $\delta$  is the Dirac function,  $U$  is the Heaviside function, and  $\mathbf{j}_{l,s}^{(0)} = i\omega \mathcal{P}_{l,s}(\mathbf{x}_0 \cos \theta_l + \mathbf{y}_0 \sin \theta_l)$ . Here,  $\mathcal{P}_{l,s}$  is the electric dipole moment per unit length of the filament, which corresponds to a current wave with the propagation constant  $p_s$ , and  $\theta_l$  is the angle specifying the dipole moment direction. Hereafter, we omit the time factor  $\exp(i\omega t)$  and take into account that  $z_l = z$ . Such electric-dipole filaments can be realized as arrays of nanoantennas operated in the infrared or optical frequency ranges [3]. Note that the Gaussian system of units is used in this work.

We assume that a nerve fiber, which is placed in the inner region of the described system, consists of tightly packaged identical myelinated axons that are parallel to the  $z$  axis. We employ the model of infinitely long, longitudinally homogeneous axons, which is considered in [2]. The outer and inner regions of each axon are filled predominantly with water and have the dielectric permittivities  $\epsilon_1$  and  $\epsilon_3$ , respectively [see Fig. 1(b)]. Each axon has a myelin sheath with the dielectric permittivity  $\epsilon_2$  between cylindrical surfaces with the radii  $a_1$  and  $a_2$ . Myelin is assumed to be a continuous medium.

As is known, if a certain value of the electric field in the regions of location of some axons in the nerve fiber is exceeded, then these axons turn out to be stimulated. This fact stipulates the purpose of this work, which is aimed at determining the dipole moments of sources that provide the given field distribution inside the nerve.

The sources (1) can be represented as

$$\mathbf{j}_l^{(\Sigma)} = (2\pi)^{-1} k_0 \int_{-\infty}^{\infty} \mathbf{j}_l(\rho_l, p) \exp(-ik_0 pz) dp, \quad (2)$$



**Figure 2.** Observation point  $P$  with cylindrical coordinates  $(\rho_\alpha, \phi_\alpha)$  and  $(\rho_\beta, \phi_\beta)$  in the coordinate systems related to the  $\alpha$ th and  $\beta$ th axons, respectively.

where

$$\mathbf{j}_l(\rho_l, p) = \frac{\delta(\rho_l)}{\pi \rho_l} \sum_{s=1}^S \mathbf{j}_{l,s}^{(0)} \frac{\sin[k_0(p-p_s)d]}{k_0(p-p_s)}. \quad (3)$$

At first, we obtain the longitudinal components of the electric and magnetic fields excited by the current (2) in a homogeneous medium with the dielectric permittivity  $\epsilon_1$ . These components are written in the cylindrical coordinate system related to the  $l$ th source as follows:

$$\begin{aligned} E_{z,l} &= \frac{k_0^2}{2\pi} \int_{-\infty}^{\infty} p \frac{q_l}{\epsilon_1} C_l^{(e)} H_1^{(2)}(k_0 q_l \rho_l) \cos(\phi_l - \theta_l) e^{-ik_0 p z} dp, \\ H_{z,l} &= \frac{k_0^2}{2\pi} \int_{-\infty}^{\infty} q_l C_l^{(e)} H_1^{(2)}(k_0 q_l \rho_l) \sin(\phi_l - \theta_l) e^{-ik_0 p z} dp. \end{aligned} \quad (4)$$

Here,  $C_l^{(e)} = 2\pi \sum_{s=1}^S \mathcal{P}_{l,s} \sin[k_0(p-p_s)d] / (p-p_s)$ ,  $q_1 = (\epsilon_1 - p^2)^{1/2}$ , and  $H_m^{(k)}$  is the Hankel function of the  $k$ th kind of order  $m$  ( $k = 1, 2$ ).

To find the total field in the system considered, we proceed to the analysis of the scattering of the electromagnetic radiation by a nerve fiber. For convenience of the subsequent mathematical calculations, we introduce auxiliary coordinate systems related to individual axons (see Fig. 2). The radial and azimuthal coordinates of the observation point in the coordinate systems of different axons, which have, for example, the numbers  $\alpha$  and  $\beta$ , will be denoted  $(\rho_\alpha, \phi_\alpha)$  and  $(\rho_\beta, \phi_\beta)$ , respectively.

The total field in the coordinate system of the  $\alpha$ th axon can be expressed via the longitudinal components, which are represented in terms of azimuthal harmonics as

$$\begin{bmatrix} E_{z;\alpha}(\mathbf{r}_\alpha) \\ H_{z;\alpha}(\mathbf{r}_\alpha) \end{bmatrix} = \sum_{m=-\infty}^{\infty} \frac{k_0}{2\pi} \int_{-\infty}^{\infty} \begin{bmatrix} E_{z;\alpha,m}(\rho_\alpha, p) \\ H_{z;\alpha,m}(\rho_\alpha, p) \end{bmatrix} e^{-im\phi_\alpha - ik_0 p z} dp, \quad (5)$$

where  $m$  is the azimuthal index ( $m = 0, \pm 1, \pm 2, \dots$ ). The functions  $E_{z;\alpha,m}(\rho_\alpha, p)$  and  $H_{z;\alpha,m}(\rho_\alpha, p)$  satisfy the equations

$$\hat{L}_{\alpha,m} E_{z;\alpha,m} = 0, \quad \hat{L}_{\alpha,m} H_{z;\alpha,m} = 0, \quad (6)$$

where  $\hat{L}_{\alpha,m} = \frac{\partial^2}{\partial \rho_\alpha^2} + \frac{1}{\rho_\alpha} \frac{\partial}{\partial \rho_\alpha} - \frac{m^2}{\rho_\alpha^2} + k_0^2 q_k^2$ ,  $k = 1, 2, 3$ , and  $q_k^2 = \epsilon_k - p^2$  denotes the transverse wave number in the  $k$ th region of the axon. The radial and azimuthal components of the electric and magnetic fields can be expressed via the longitudinal components (5).

Azimuthal harmonics of the electric and magnetic fields inside the  $\alpha$ th axon are described by the expressions

$$\begin{aligned} \begin{bmatrix} E_{z;\alpha,m} \\ H_{z;\alpha,m} \end{bmatrix} &= \begin{bmatrix} B_{\alpha,m}^{(1)} \\ B_{\alpha,m}^{(2)} \end{bmatrix} q_3 J_m(k_0 q_3 \rho_\alpha), \quad \rho_\alpha \leq a_1, \\ \begin{bmatrix} E_{z;\alpha,m} \\ H_{z;\alpha,m} \end{bmatrix} &= \sum_{k=1}^2 \begin{bmatrix} C_{\alpha,m}^{(k)} \\ \tilde{C}_{\alpha,m}^{(k)} \end{bmatrix} q_2 H_m^{(k)}(k_0 q_2 \rho_\alpha), \quad a_1 \leq \rho_\alpha \leq a_2, \end{aligned} \quad (7)$$

where  $B_{\alpha,m}^{(1,2)}$ ,  $C_{\alpha,m}^{(1,2)}$ , and  $\tilde{C}_{\alpha,m}^{(1,2)}$  are the amplitude coefficients corresponding to the  $m$ th azimuthal harmonic and  $J_m$  is the Bessel function of the first kind of order  $m$ .

In the outer medium outside the axons, the total field (5) is determined by several components, including the field scattered by the  $\alpha$ th axon, the field scattered by other axons with the numbers  $\beta \neq \alpha$ , and the field of filamentary sources. Azimuthal harmonics of the electric and magnetic fields scattered by the  $\alpha$ th axon are represented in its coordinate system in the form

$$\begin{bmatrix} E_{z;\alpha,m}^{(sc)} \\ H_{z;\alpha,m}^{(sc)} \end{bmatrix} = \begin{bmatrix} D_{\alpha,m}^{(1)} \\ D_{\alpha,m}^{(2)} \end{bmatrix} q_1 H_m^{(2)}(k_0 q_1 \rho_\alpha), \quad (8)$$

where  $D_{\alpha,m}^{(1)}$  and  $D_{\alpha,m}^{(2)}$  are the scattering coefficients corresponding to the azimuthal index  $m$ . To find the scattering coefficients, the total field in the outer medium should be written in a coordinate system of the  $\alpha$ th axon. To this end, we use Graf's addition theorem for cylindrical functions, according to which the azimuthal harmonic of the field at some observation point  $P$  (see Fig. 2), which is written in the coordinate system of the  $\beta$ th axon, can be represented in the coordinate system of the  $\alpha$ th axon in the following way (see in [2, 4]):

$$\begin{aligned} &H_n^{(2)}(k_0 q_1 \rho_\beta) e^{-in\phi_\beta} \\ &= \sum_{m=-\infty}^{\infty} J_m(k_0 q_1 \rho_\alpha) H_{m-n}^{(2)}(k_0 q_1 \rho_\beta) e^{i(m-n)\phi_{\beta\alpha} - im\phi_\alpha}. \end{aligned} \quad (9)$$

Here,  $\rho_{\beta\alpha}$  is the distance between the axes of the axons with the numbers  $\alpha$  and  $\beta$ , and  $\phi_{\beta\alpha}$  is the azimuthal angle of the axis of the  $\beta$ th axon in the coordinate system of the  $\alpha$ th axon. In (9), it is assumed that the condition  $\rho_\alpha < \rho_{\beta\alpha}$  is fulfilled. In what follows, this condition is ensured by the fact that this formula is used only to represent the incident field at the outer boundary of the axon. As a result, azimuthal harmonics of the total electric and magnetic fields outside the axons of a nerve fiber in the coordinate system of the  $\alpha$ th axon take the form

$$E_{z;\alpha,m} = E_{z;\alpha,m}^{(sc)} + E_{z;\alpha,m}^{(ex)}, \quad H_{z;\alpha,m} = H_{z;\alpha,m}^{(sc)} + H_{z;\alpha,m}^{(ex)}, \quad (10)$$

where

$$E_{z;\alpha,m}^{(ex)} = q_1 \mathcal{E}_{\alpha,m} J_m(k_0 q_1 \rho_\alpha), \quad H_{z;\alpha,m}^{(ex)} = q_1 \mathcal{H}_{\alpha,m} J_m(k_0 q_1 \rho_\alpha).$$

Here,

$$\begin{aligned} \begin{bmatrix} \mathcal{E}_{\alpha,m} \\ \mathcal{H}_{\alpha,m} \end{bmatrix} &= \sum_{\beta \neq \alpha} \sum_{n=-\infty}^{\infty} \begin{bmatrix} D_{\beta,n}^{(1)} \\ D_{\beta,n}^{(2)} \end{bmatrix} H_{m-n}^{(2)}(k_0 q_1 \rho_{\beta\alpha}) e^{i(m-n)\phi_{\beta\alpha}} \\ &+ \sum_{l=1}^{N_s} \begin{bmatrix} E_l^{(0)} \\ H_l^{(0)} \end{bmatrix} \left\{ H_{m-1}^{(2)}(k_0 q_1 \rho_{l\alpha}) e^{i(m-1)\phi_{l\alpha} + i\theta_l} \right. \\ &\left. - \begin{bmatrix} 1 \\ -1 \end{bmatrix} H_{m+1}^{(2)}(k_0 q_1 \rho_{l\alpha}) e^{i(m+1)\phi_{l\alpha} - i\theta_l} \right\}, \end{aligned} \quad (11)$$

where  $E_l^{(0)} = k_0 p C_l^{(e)} / (2\epsilon_1)$ ,  $H_l^{(0)} = ik_0 C_l^{(e)} / 2$ ,  $N_a$  is the number of the axons in the nerve fiber, and  $N_s$  is the number of the filamentary sources.

The coefficients  $B_{\alpha,m}^{(1,2)}$ ,  $C_{\alpha,m}^{(1,2)}$ ,  $\tilde{C}_{\alpha,m}^{(1,2)}$ , and  $D_{\alpha,m}^{(1,2)}$  are found from boundary conditions for the tangential field components on the inner and outer surfaces of the myelin sheath of each axon. Using the technique based on the scattering matrix method [4] and the boundary conditions on the surfaces  $\rho_\alpha = a_1$  and  $\rho_\alpha = a_2$  of the myelin sheath of the  $\alpha$ th axon, we can obtain a system of equations for the scattering coefficients  $D_{\alpha,m}^{(1)}$  and  $D_{\alpha,m}^{(2)}$  in the form

$$\begin{aligned} D_{\alpha,m}^{(1)} &= S_m^{ee} \mathcal{E}_{\alpha,m} + S_m^{eh} \mathcal{H}_{\alpha,m}, \\ D_{\alpha,m}^{(2)} &= S_m^{he} \mathcal{E}_{\alpha,m} + S_m^{hh} \mathcal{H}_{\alpha,m}, \end{aligned} \quad (12)$$

where  $S_m^{ee}$ ,  $S_m^{eh}$ ,  $S_m^{he}$ , and  $S_m^{hh}$  are the elements of the scattering matrix of a single cylindrical scatterer. To find these elements, we consider the system of equations, which is obtained from the boundary conditions on the surfaces  $\rho_\alpha = a_1$  and  $\rho_\alpha = a_2$  of a single scatterer:

$$\begin{pmatrix} \hat{J}_m & 0 & -\hat{H}_{m,1}^{(1)} & -\hat{H}_{m,1}^{(2)} & 0 & 0 & 0 & 0 \\ 0 & \hat{J}_m & 0 & 0 & -\hat{H}_{m,1}^{(1)} & -\hat{H}_{m,1}^{(2)} & 0 & 0 \\ -\tilde{J}_m & \tilde{J}_m & \tilde{H}_{m,1}^{(1)} & \tilde{H}_{m,1}^{(2)} & -\hat{H}_{m,1}^{(1)} & -\hat{H}_{m,1}^{(2)} & 0 & 0 \\ -\epsilon_3 \hat{J}_m & -\tilde{J}_m & \epsilon_2 \hat{H}_{m,1}^{(1)} & \epsilon_2 \hat{H}_{m,1}^{(2)} & \tilde{H}_{m,1}^{(1)} & \tilde{H}_{m,1}^{(2)} & 0 & 0 \\ 0 & 0 & -\hat{H}_{m,2}^{(1)} & -\hat{H}_{m,2}^{(2)} & 0 & 0 & \hat{H}_m^{(2)} & 0 \\ 0 & 0 & 0 & 0 & -\hat{H}_{m,2}^{(1)} & -\hat{H}_{m,2}^{(2)} & 0 & \hat{H}_m^{(2)} \\ 0 & 0 & \tilde{H}_{m,2}^{(1)} & \tilde{H}_{m,2}^{(2)} & -\hat{H}_{m,2}^{(1)} & -\hat{H}_{m,2}^{(2)} & -\tilde{H}_m^{(2)} & \hat{H}_m^{(2)} \\ 0 & 0 & \epsilon_2 \hat{H}_{m,2}^{(1)} & \epsilon_2 \hat{H}_{m,2}^{(2)} & \tilde{H}_{m,2}^{(1)} & \tilde{H}_{m,2}^{(2)} & -\epsilon_1 \hat{H}_m^{(2)} & -\tilde{H}_m^{(2)} \end{pmatrix} \times \begin{pmatrix} \mathcal{B}_m^{(1)} \\ \mathcal{B}_m^{(2)} \\ \mathcal{E}_m^{(1)} \\ \mathcal{E}_m^{(2)} \\ \tilde{\mathcal{E}}_m^{(1)} \\ \tilde{\mathcal{E}}_m^{(2)} \\ \mathcal{D}_m^{(1)} \\ \mathcal{D}_m^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -q_1 J_m(Q_1) \\ 0 \\ \frac{pm}{Q_1} J_m(Q_1) \\ i\epsilon_1 J_m'(Q_1) \end{pmatrix} \mathcal{A}_m^{(1)} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -q_1 J_m(Q_1) \\ -iJ_m'(Q_1) \\ \frac{pm}{Q_1} J_m(Q_1) \end{pmatrix} \mathcal{A}_m^{(2)}, \quad (13)$$

where

$$\begin{aligned} \hat{J}_m &= q_3 J_m(Q_3), \quad \hat{H}_{m,\gamma}^{(k)} = q_2 H_m^{(k)}(\tilde{Q}_\gamma), \quad \hat{H}_m^{(2)} = q_1 H_m^{(2)}(Q_1), \\ \tilde{J}_m &= \frac{pm}{Q_3} J_m(Q_3), \quad \tilde{H}_{m,\gamma}^{(k)} = \frac{pm}{Q_\gamma} H_m^{(k)}(\tilde{Q}_\gamma), \quad \tilde{H}_m^{(2)} = \frac{pm}{Q_1} H_m^{(2)}(Q_1), \\ \hat{J}_m &= iJ_m'(Q_3), \quad \hat{H}_{m,\gamma}^{(k)} = iH_m^{(k)' }(\tilde{Q}_\gamma), \quad \hat{H}_m^{(2)} = iH_m^{(2)' } (Q_1), \\ Q_1 &= k_0 q_1 a_2, \quad \tilde{Q}_\gamma = k_0 q_2 a_\gamma, \quad Q_3 = k_0 q_3 a_1, \quad \gamma = 1, 2. \end{aligned} \quad (14)$$

In (14), the prime indicates the derivative with respect to the argument. Upon finding the field coefficients  $\mathcal{B}_{\alpha,m}^{(1,2)}$ ,  $\mathcal{E}_{\alpha,m}^{(1,2)}$ ,  $\tilde{\mathcal{E}}_{\alpha,m}^{(1,2)}$ , and  $\mathcal{D}_{\alpha,m}^{(1,2)}$  from system (13), the quantities  $S_m^{ee}$ ,  $S_m^{eh}$ ,  $S_m^{he}$ , and  $S_m^{hh}$  are represented as  $S_m^{ee} = \mathcal{D}_m^{(1)}$  and  $S_m^{he} = \mathcal{D}_m^{(2)}$  under the conditions  $\mathcal{A}_m^{(1)} = 1$  and  $\mathcal{A}_m^{(2)} = 0$ , and as  $S_m^{eh} = \mathcal{D}_m^{(1)}$  and  $S_m^{hh} = \mathcal{D}_m^{(2)}$  under the conditions  $\mathcal{A}_m^{(1)} = 0$  and  $\mathcal{A}_m^{(2)} = 1$ .

Application of (12) for each axon and each azimuthal index  $m = 0, \pm 1, \dots, \pm M$  yields a system of  $N_a(4M + 2)$  inho-

mogeneous linear equations for the coefficients  $D_{\alpha,m}^{(1)}$  and  $D_{\alpha,m}^{(2)}$ , where  $M$  is the absolute value of the number of the highest azimuthal harmonic taken into account (the summation over  $n$  in (11) is now performed from  $-M$  to  $M$ ). The choice of a finite number of azimuthal harmonics is determined by the required accuracy of numerical calculations. Upon solving the system of equations described above, we find the scattering coefficients  $D_{\alpha,m}^{(1)}$  and  $D_{\alpha,m}^{(2)}$ . Using these coefficients and the system of equations (13), it is then easy to find the field coefficients in the body and the myelin sheath of each axon by putting  $\mathcal{A}_m^{(1)} = \mathcal{E}_{\alpha,m}$  and  $\mathcal{A}_m^{(2)} = \mathcal{H}_{\alpha,m}$ .

### 3 Determining the Dipole Moments of the Filamentary Sources

The problem of finding the amplitudes of dipole moments of the given sources is reduced to an optimization procedure. Such a procedure requires considerable computational resources. Instead of optimization, we can use a simpler and less resource-consuming method for determining the amplitudes of the sources, which is as follows. Let the desired distribution of the longitudinal electric-field component be  $E_z^{(d)}$  in the region  $\rho < R$ . Consider an auxiliary magnetic field  $H_z^{(d)}$  that has the same spatial distribution as that of the desired electric field. We expand this magnetic field in terms of the eigenwaves of a homogeneous medium with the permittivity  $\epsilon_1$  in the cross section  $z = 0$  as

$$H_z^{(d)} = \int_0^\infty H_z^{(r)}(\rho, \phi, q) dq, \quad (15)$$

where

$$\begin{aligned} H_z^{(r)}(\rho, \phi, q) &= \sum_{m=-M_e}^{M_e} a_m(q) q J_m(k_0 q \rho) e^{-im\phi}, \\ a_m(q) &= \frac{k_0^2}{2\pi} \int_0^{2\pi} d\phi \int_0^\infty H_z^{(d)} J_m(k_0 q \tilde{\rho}) e^{im\phi} \tilde{\rho} d\tilde{\rho}. \end{aligned} \quad (16)$$

Here,  $M_e$  is the absolute value of the number of the highest azimuthal harmonic taken into account in this representation, and  $a_m$  is the expansion coefficient. Then we replace (15) by the quadrature formula

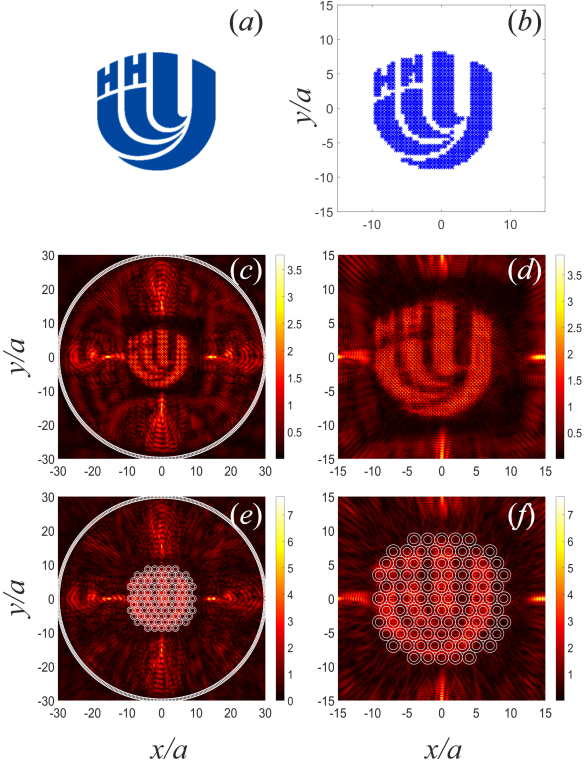
$$H_z^{(d)} = \sum_{s=1}^{S_{\max}} A_s H_z^{(r)}(\rho, \phi, q_s), \quad (17)$$

where  $S_{\max}$  is the maximum value taken by the index of  $s$ , which is determined by the specified accuracy of numerical calculations, and the quantity  $A_s$  is determined by the used numerical integration method. Consider the field  $H_z$  which has the distribution described by the function  $A_s H_z^{(r)}(\rho, \phi, q_s)$  in the region  $\rho < R$  and is zero in the region  $\rho > R$ . The surface electric current exciting this field is found from the boundary conditions on the surface  $\rho = R$  as follows:

$$\mathbf{i}(\phi, q_s) = \phi_0 c A_s H_z^{(r)}(R, \phi, q_s) / (4\pi), \quad (18)$$

Then we take the quantities  $p_s$  in (1) equal to  $(\epsilon_1 - q_s^2)^{1/2}$  and adopt that the quantity  $\mathcal{P}_{l,s}$  for the  $l$ th source is specified with the help of (18) as

$$\mathcal{P}_{l,s} = c A_s H_z^{(r)}(R, \phi_{l0}, q_s) \Delta L / (4\pi\omega), \quad (19)$$



**Figure 3.** The desired distribution of the longitudinal electric field (a) and its pattern specified in discrete form (b). Normalized magnitude of the longitudinal electric field in the plane  $z = 0$  (c-f).

where  $\phi_{l0}$  is the azimuthal coordinate of the  $l$ th source in the coordinate system  $(\rho, \phi, z)$ ,  $\Delta L$  is the distance between the nearest sources, and  $S = S_{\max}$ . We take the angle  $\theta_l$ , which specifies the dipole moment direction of the  $l$ th source, equal to  $\phi_{l0}$  for maximization of the excited electric field. The total field, which is excited by the found sources, turns out to be close in structure to the desired distributions of both the magnetic and electric longitudinal components.

## 4 Numerical Results

Numerical calculations were performed for the nerve fiber consisting of 92 tightly packaged axons. The inner and outer radii of the myelin sheath were taken equal to  $a_1 = 1.5 \mu\text{m}$  and  $a_2 = 2.5 \mu\text{m}$ , respectively. The array of filamentary sources consisted of 497 elements located on a cylindrical surface of radius  $R = 30a_2$ . Each source operated at a frequency which corresponded to the wavelength  $\lambda_0 = 2010 \text{ nm}$  and has the half-length  $d = 2 \times 10^4 \lambda_0$ . The distance  $\Delta L$  between the nearest sources was equal to  $\lambda_0/2$ . We used the quadrature method based on Simpson's rules of numerical integration and took the values  $M_e = 295$  and  $q_S = 1.8$  for calculation of dipole moments (19). The maximum absolute value of the azimuthal index used in the calculations of the scattered fields was limited by  $M = 15$ . The dielectric permittivities of water and myelin were calculated as in [2] for  $\lambda_0 = 2010 \text{ nm}$  and the relatively small dielectric permittivity of the myelin-based proteins:  $\epsilon_1 = \epsilon_3 = 1.68 - i0.0026$  and  $\epsilon_2 = 1.97 - i0.0008$ .

Figure 3(a) shows the desired distribution of the longitudinal electric field, which represents the logo of University of Nizhny Novgorod. This distribution is used to specify the required field pattern in discrete form, as is shown in Fig. 3(b). The desired field is specified as follows  $E_z^{(d)} = \sum_{i=1}^{644} \delta(x - x_i) \delta(y - y_i)$ . The coordinates  $(x_i, y_i)$  are marked by the asterisks in Fig. 3(b). Figure 3(c) shows the quantity  $|E_z|$  when the field is excited by the found electric-dipole filaments in a homogeneous medium with the dielectric permittivity  $\epsilon_1$ . Figure 3(e) shows  $|E_z|$  in the presence of the nerve fiber. Figures 3(d) and 3(f) present the enlarged central parts of Figs. 3(c) and 3(e), respectively. The circumferences near the centers of the figures show the inner and outer boundaries of the axons. The circumferences around the sources located at the distance  $30a_2$  from the fiber axis indicate the strong-field regions in which the magnitude of the field cannot be shown using the scale adopted for the figure. As is evident from the figure, it is indeed possible to create the desired pattern of the field inside the nerve fiber by appropriately choosing the dipole moments of the filamentary sources.

## 5 Conclusion

In this paper, we have solved the problem of excitation of an electromagnetic field with the specified spatial distribution by a set of electric-dipole filaments in the presence of a nerve fiber. It has been demonstrated that the field of such sources with their dipole moments chosen appropriately can have a spatial structure that provides a selective impact on specified regions of the nerve fiber. The results obtained can be useful in developing methods and tools for high-precision non-contact stimulation of nerve fibers and selective action on the compound of different (e.g., normal and cancer) tissues.

## 6 Acknowledgements

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