URSI GASS 2020, Rome, Italy, 29 August - 5 September 2020

Causality Condition in Electromagnetics and Negative Consequences of Ill-Posed Source Models

Lotfollah Shafai*, Navid Rezazadeh, and Vladimir Okhmatovski
Department of Electrical and Computer Engineering
University of Manitoba, Winnipeg, MB, Canada, R3T 5V6

Abstract

The source in Applied Electromagnetics and Antennas is the important hardware, defining the causality condition, and solution results. In scattering problems, it is normally defined as a simple voltage or current source. This is acceptable if the interaction between the source and scatterer is negligible, which may not be the case when their separation distance is small. In antenna problems, this becomes more serious, as the source and antenna are directly connected and interact strongly. It is shown in this paper that, in such cases the source must be defined more accurately, and include its impedance characteristics. Otherwise, serious difficulties can be experienced in obtaining correct solutions, in particular, whenever internal resonances occur that cause rapid impedance variations.

1 Introduction

In canonical electromagnetic (EM) problems the excitation sources are normally made simple, to exclude their influence on the solution, and make the results governed entirely by the object's shape and boundary conditions. Thus, typical sources are simple voltage and current sources, as used by R. F. Harrington [1] (pp 95), and shown in figure 1.

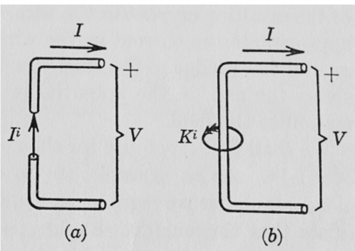


Figure 1. Circuit sources in terms of impressed currents, (a) current source, (b) voltage source, according to R.F. Harrington [1].

These simple sources exclude their finite internal impedances, and acceptable only if the interaction between the source and scattering object is negligible. This normally happens when the separation distance between the source and the scattering object is very large. In most cases, this implies that the simplified sources of figures 1, can be used with confidence, if the scattering object is at the far field of the source, provided no focusing effect is present.

The antenna problem is a special case of the scattering problem, where the source is directly connected to the antenna. In such a case, the combination of the source and antenna forms a single circuit problem, which will be governed by their impedances. Thus, the above source definitions are inappropriate for antennas problems, and their internal impedances must be included in the EM models, to provide correct EM solutions. The problem becomes more severe when internal resonances occur, within the antenna, where its input impedance charges rapidly and dramatically. For such cases, a correct source model must include the source impedance. These three source models, when connected to an antenna are shown in figure 2, (a) to (c). In this figure Vs and Is are the source voltage and source current, Zs and Za are the source and antenna impedances, and Va and Ia are the antenna voltage and current. The inclusion of the source impedance in figure 2(c) is necessary, to account for the losses , that incur due to the coupling currents. For scattering problems, with strong mutual coupling between the source and scattering object, a similar source model as figure 2(c) must also be used. Here, the reflected signal from the scatterer is received by the antenna, and undergoes losses in the source impedance Zs, and the remaining re-radiates back towards the scatterer, resulting in multiple scattering and multiple losses in Zs, which do not occur with the simplifies source models of figure 1. Thus, the source must include its internal impedance, to account for the losses, that incur due to the coupling currents.

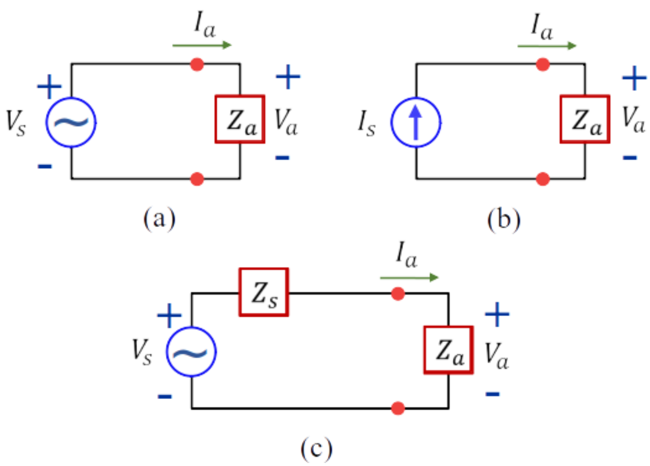


Figure 2. Circuit models for an antenna problem, (a) and (b) are models using idealized sources of figure 1, and (c) is the correct model that includes both source and antenna impedances.

2 Example of a Coated Slot Antenna

An example of applying the source models of figure 2, to a practical problem, is the case of slotted metallic structures to form slot antennas. These antennas have been used in many applications, where the host is a volumetric structure with metallic cover, to house the source components. Historically, to simplify obtaining the EM solution the excitation source was normally represented by a constant voltage Va across the slot, independent of geometry and other antenna parameters. Thus, the source was modeled as the case of figure 2(a), with no internal impedance. Such a solution is correct, if the slot is the only radiating source [1] (pp 301). However, if there are other sources, or the slot is coated with homogeneous materials or metamaterials that may cause resonances within the coating, such an idealized source can result in incorrect or false radiation properties [2], [3]. and the correct source model of figure 2(c) must be used.

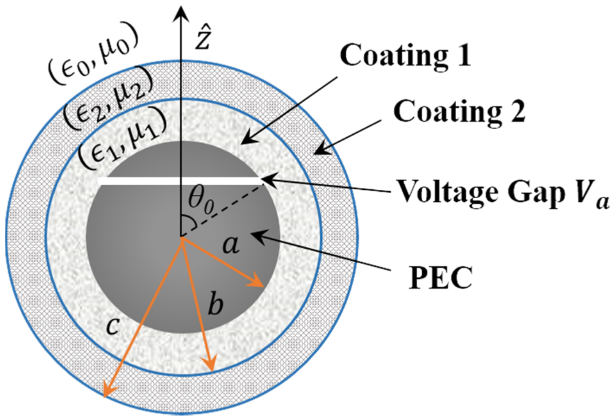


Figure 3. A spherical dipole antenna, coated with two homogeneous layers of material or metamaterial, and excited with a gap voltage Va.

To investigate such a problem, the case of a coated slotted sphere is selected as shown in figure 3. This geometry is essentially a spherical dipole antenna, with two layers of coatings, that could be real materials, metamaterials or their combinations. The spherical geometry is selected to allow for an exact analytic solution for the EM problem, without the need for any simplifying approximation, thereby allowing for accurate investigation of the source modelling effects.

For a uniform circumferential slot, the EM problem is straightforward, and can be obtained using a radial vector potential, in terms of the spherical wave functions. Also, for simplicity a delta function source is selected, as indicated in [1], (page 302). Because of this selected source geometry, the solution becomes independent of the azimuthal coordinate. The boundary conditions on the conductor surface of sphere, and at interfaces between the coatings, as well as between the second coating with free space, can be applied easily. Then, the orthogonality relationships, can be used to obtain a matrix equation for the unknown coefficients of the spherical wave functions. This results in a 5 x 5 matrix equation for each spherical mode, which can be solved, providing all necessary radiation parameters [2], such as the antenna input impedance, the actual slot excitation voltage Va, radiated power, and the antenna directivity. Note that, the antenna input impedance affects the slot excitation voltage, only for the source model of 2(c)

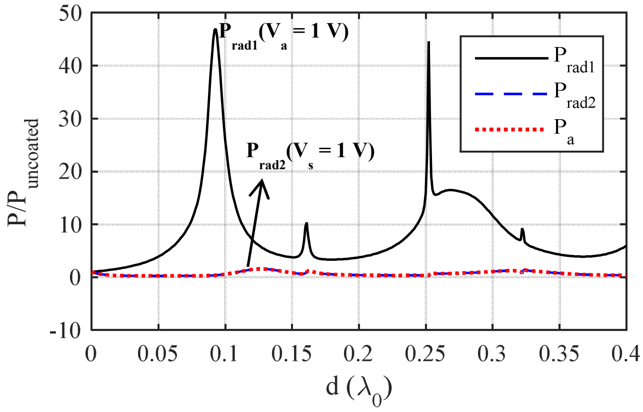
3 Difficulties in using ideal sources

For this problem, the use of an ideal source does not correctly represent the interaction between the source and the antenna, and can produce incorrect results. To show such a difficulty, here the variation of the radiated power with the coating thickness is provided in figure 4(a). The vertical axis is the radiated power normalized by that of the no coating case, and the horizontal axis is the coating thickness. The solid line represents, the normalized radiated power for the ideal source of figure 2(a), and shows four peaks. These peaks correspond to the resonance of modes 2, 3, 4 and 5 [4], inside the coating material. The other two plots are the radiated powers of the antenna, using the correct source model of 2(c). The dashed line is computed using the Poynting vector approach, i.e. the field vectors outside the antenna. Whereas, the dotted line is the power delivered to the antenna, i.e. the slot, using the circuit model of the antenna. This circuit power is interior to the antenna, and has not yet radiated. These two powers, i.e. the dashed and dotted results, are identical indicating that the EM problem has been solved correctly.

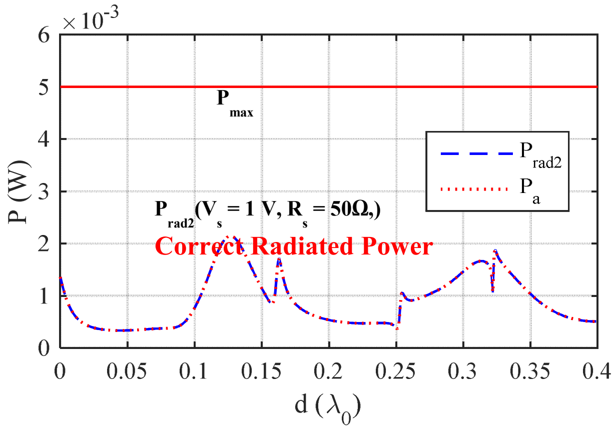
The resonance effects are mode dependent, and strong for even modes 2 and 4. This means that the antenna power is delivered mostly to these modes, which are calculated from the 5 x5 matrix equation for each mode, as indicated in the previous section. For odd 3 and 5 modes, the adjacent non-resonant modes are also excited, but with relatively weaker amplitudes. This power sharing reduces the resonance effect of these modes.

The amplitude and behavior of the radiated power, calculated using the source model 2(c), can be understood easily by considering the source and antenna impedances, Zs and Za, from which the maximum power transfer can also be calculated, when the antenna impedance is conjugate of the source impedance, and vice versa. The results are shown in figure 4(b), with the maximum radiated power shown in a solid Red line.

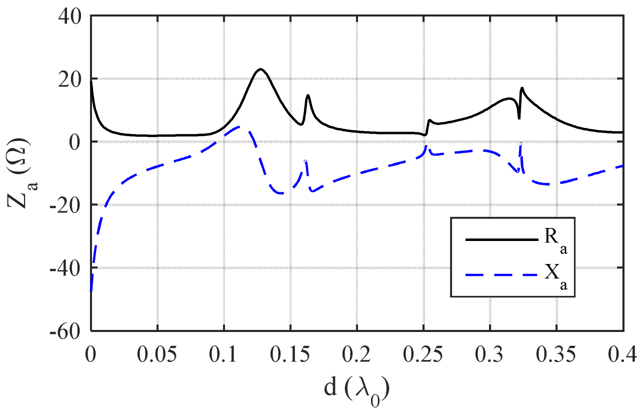
The calculated antenna input impedance Za is shown in figure 4(c), and from the source model 2(c), the slot excitation voltage Va can be calculated, which is shown in Figure 4(d). Comparing the results of 4(a) to 4(d), shows that at even mode resonances the antenna input impedance approaches zero. However, the source model 2(a), does not account for this fact and gives false enhanced power radiations. The source impedance in model 2(c) corrects



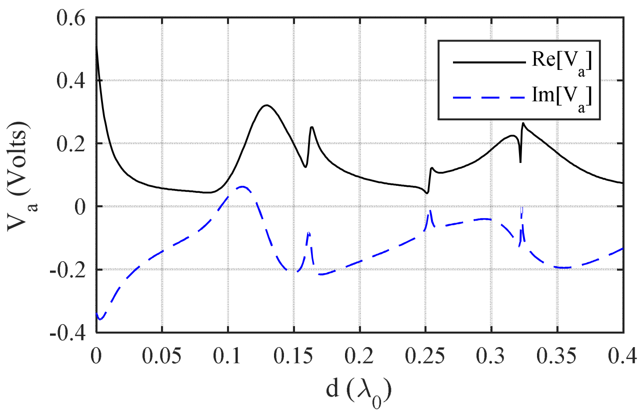
(a)



(b)



(c)



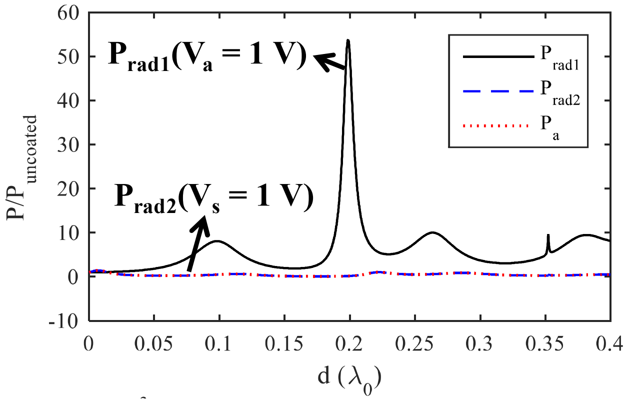
(d)

Figure 4. (a) Normalized radiated powers with source models 2(a) (solid) and 2(c) (dashed and dotted, calculated using the Poynting vector and the circuit models, respectively), The solid curve shows four peaks that correspond to the resonance modes 2, 3, 4 and 5 [3], inside the coating material. The impedance of the source model 2(c) is a 50ohm resistance, d = b – a, ka = 1.0, k is the propagation constant of free space, slot location 60deg, coating 1 permittivity=9 and permeability=1, coating 2 is air. (b) Dashed and dotted plots are the actual radiated powers of the antenna, calculated by the Poynting vector and the circuit model, using the source model 2(c), with an internal 50ohm resistance. (c) The antenna impedance Za. (d) The antenna excitation voltage Va, which in general is complex and approaches zero when the resonance of a single mode dominates

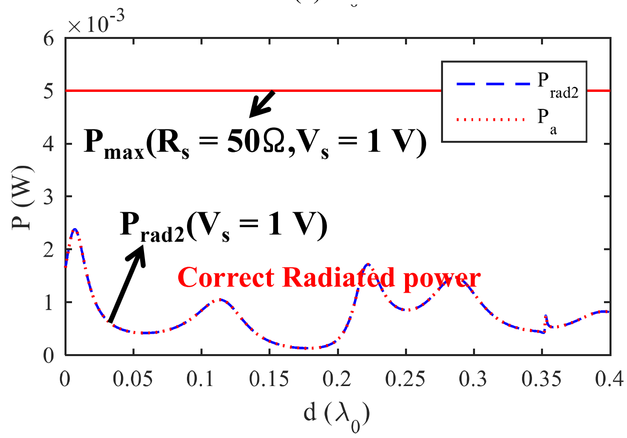
the problem and the radiated power remains finite and smooth throughout. It should also be noted that the same problem is encountered with metamaterial coating [5]. This problem is investigated next with both source models 2(a) and 2(c), and shown that the false enhanced power radiation disappears in using the correct source model 2(c).

4 Metamaterial Coating

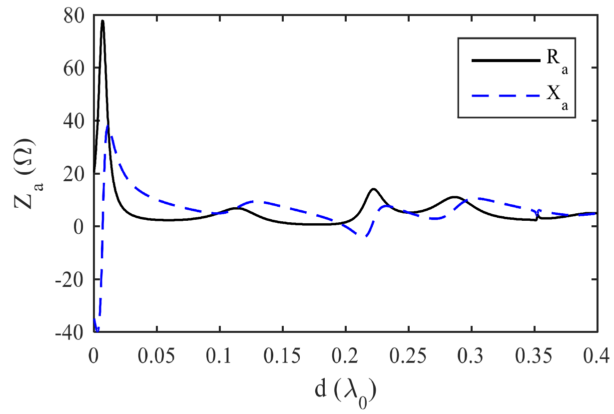
In this section, the results for the case of metamaterial coating is presented. So, the coting 1, is a double negative metamaterial, with a permittivity of -9, and permeability of -1, the same numbers as the previous case for a natural material coating. However, instead of using the same slot location, for this case the slot is placed at 90deg, which represents a symmetric spherical dipole antenna. Again, the problem is solved for the spherical mode coefficients, similar to the previous case, using the source models 2(a) and 2(c). The results are shown in Figure 5. The behavior of the results is very much the same as the previous case. For the source model 2(a), the normalized radiated power with coating shows one large peak at the mode resonance (mode 3), and a small one at mode 5. Even modes are eliminated because of the symmetric dipole feed. The radiated power, with the source model 2(c)) is finite and well behaved. Again, it is calculated by both Poynting vector and circuit models (shown with dashed and dotted lines), showing identical results, confirming the accuracy of the solution. The antenna input impedance Za results are shown in figure 5(c), and the computed slot excitation voltage Va in figure 5(d). At the mode resonance, where model 2(a) shows a peak in the radiated power, the antenna input impedance becomes almost zero, and the circuit model 2(c) makes the antenna excitation voltage Va almost zero. It is interesting to point out that, at this mode resonance, because of the reduced slot excitation, the radiated power, also reduces to almost zero, which is opposite to the result of source model 2(a).



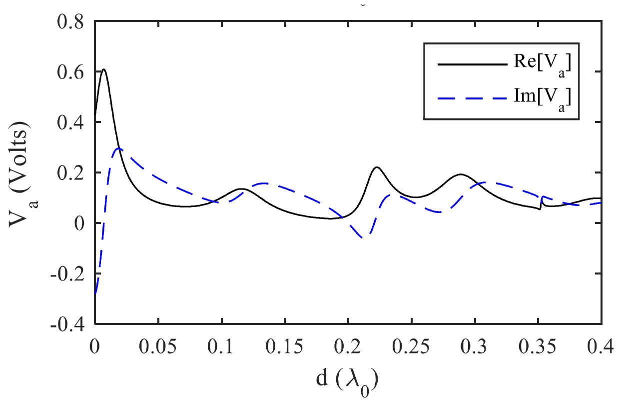
(a)



(b)



(c)



(d)

Figure 5. (a) Normalized radiated powers with source models 2(a) (solid) and 2(c) (dashed and dotted, calculated using the Poynting vector and the circuit models, respectively). The impedance of the source model 2(c) is a 50ohm resistance, d = b – a, ka = 1.0, k is the propagation constant in free space, slot location 90deg, coating 1 permittivity= -9 and permeability= -1, coating 2 is air. (b) Dashed and dotted plots are the actual radiated powers of the antenna, calculated by the Poynting vector and the circuit model, using the source model 2(c), with an internal 50ohm resistance. (c) The antenna impedance Za. (d) The antenna excitation voltage Va, which approaches zero when the resonance of a single mode dominates

5 conclusions

The causality condition for an EM-Antenna problem is viewed from the point of view of the excitation source. It is shown that, in general, the source must be defined properly, for both voltage and current sources, to include their internal impedances, i.e. the source impedance. Otherwise, incorrect, or false, results will be generated. The problem was investigated using a spherical dipole antenna, coated with either natural materials, or metamaterials. It was shown that excluding the source impedance can results in unrealistically excessive radiated powers, not present with properly defined sources with an internal impedance. The combination of natural material and metamaterial coatings have also been investigated, with similar results. They are excluded due to lack of available space.

6 References

1. R. F. Harrington, “Time Harmonic Electromagnetic Field”, McGraw-Hill, 1961.

2. L. Shafai, “Radiation from an Axial Slot Antenna Coated with a Homogeneous Material”, Canadian Journal of Physics, **50**,1972, pp. 3072 – 3077.

3. L. Shafai and R. K. Chugh, “Resonance Effects in Slotted Spherical Antennas Coated with Homogeneous Materials”, Canadian Journal of Physics, **51**, 1973, pp. 2341 – 2346.

4. L. Shafai and N. Rezazadeh, “On false Radiation Enhancement of Small Antennas Coated with Materials and Metamaterials”, IEEE ANTEM2018 conference, Waterloo, Canada, August 19 – 22, 2018.

5. R. W. Ziolkowski and A. D. Kipple, “Application of Double Negative Materials to increase the power Radiated by Electrically Small Antennas”, IEEE Trans. Antennas and Propagation, **51**, 10, October 2003, pp. 2626 – 2640, doi:10.1109/TAP.2003.817561.