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Resonant Inductive WPT Link with Multiple Transmitters

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Abstract

This paper analyzes the case of a resonant inductive wireless power transfer link using multiple transmitters. The optimal analytical solution is obtained by solving a generalized eigenvalue problem. The general case of a link using N transmitters with any coupling is solved and discussed.

1 Introduction

Near-field Wireless Power Transfer (WPT) using resonant inductive schemes is receiving an increasingly growing interest being an effective technological solution for wireless recharging electronic devices [1–3]. Among the various configurations that have been investigated in the literature of particular interest is the one using multiple transmitters and/or receivers [4–8].

In particular, the use of multiple transmitters suitably distributed in space can be exploited for obtaining a nearly constant efficiency even for a moving receiver [5, 7].

With regard to the link optimization, in practical applications the interest is on how to maximize either the power transfer efficiency or the power delivered to the load.

In [9] the maximum efficiency solution for a link with multiple transmitters is formulated as a convex optimization problem.

In [3, 8, 10] the specific case of a three–port network using either two transmitters and a single receiver or a single transmitter and two receivers is analyzed. In particular, in [8] the analytical expressions of the optimal load/loads for both the maximum power and the maximum efficiency solution are derived.

In this paper, the general case of a link using N transmitters is analyzed and the optimal configuration for maximizing the efficiency is calculated. By modeling the link as an $(N+1)$–ports network described by its impedance matrix, a generalized eigenvalue problem is obtained. With respect to [8], where only the load impedance was optimized, the general solution including the optimal expressions of the input currents is calculated.

2 Analyzed Problem

The problem analyzed in this paper is illustrated in Fig. 1. The case of a resonant inductive WPT link using N transmitters (TXs) and having a single receiver (RX) is considered.

The WPT system is represented as an $(N+1)$–port strictly passive reciprocal network, described in terms of an impedance matrix $\mathbf{Z}$. The impedance matrix of the $(N+1)$–network is:

$$\mathbf{Z} = \left[\begin{array}{c|c} Z_\mathrm{o} & \mathbf{Z}_\mathrm{c}^\mathsf{T} \\ \hline \mathbf{Z}_\mathrm{c} & \mathbf{Z}_\mathrm{i} \end{array}\right] = \left[\begin{array}{c|ccc} R_0 & j\omega M_{01} & \cdots & j\omega M_{0N} \\ \hline j\omega M_{01} & R_1 & \cdots & j\omega M_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ j\omega M_{0N} & j\omega M_{1N} & \cdots & R_N \end{array}\right]. \quad (1)$$

By introducing the following normalization matrix:

$$\mathbf{n} = \begin{bmatrix} \frac{1}{\sqrt{\omega L_0}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\sqrt{\omega L_N}} \end{bmatrix}, \quad (2)$$

it is possible to obtain the normalized impedance matrix of the network:

$$\mathbf{z} = \mathbf{nZn} = \left[\begin{array}{c|c} z_\mathrm{o} & \mathbf{z}_\mathrm{c}^\mathsf{T} \\ \hline \mathbf{z}_\mathrm{c} & \mathbf{z}_\mathrm{i} \end{array}\right] = \left[\begin{array}{c|ccc} \frac{1}{Q_0} & jk_{01} & \cdots & jk_{0N} \\ \hline jk_{01} & \frac{1}{Q_1} & \cdots & jk_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ jk_{0N} & jk_{1N} & \cdots & \frac{1}{Q_N} \end{array}\right]. \quad (3)$$

In (3), Q_n are the quality factors of the coupled resonators and k_{nm} are the coupling factors:

$$Q_n = \frac{\omega_n L_n}{R_n} \qquad (n = 0, \dots, N), \quad (4)$$

$$k_{nm} = \frac{M_{nm}}{\sqrt{L_n L_m}} \qquad (n, m = 0, \dots, N; n \neq m). \quad (5)$$

For the average output (P_o) and input (P_i) power the following expressions can be derived:

$$P_\mathrm{o} = -\frac{1}{4}(v_\mathrm{o}^* i_\mathrm{o} + i_\mathrm{o}^* v_\mathrm{o}) = -\frac{1}{4}\left[i_\mathrm{o}^*(z_\mathrm{o} + z_\mathrm{o}^*)\, i_\mathrm{o} + i_\mathrm{o}^* \mathbf{z}_\mathrm{c}^\mathsf{T} \mathbf{i}_\mathrm{i} + \mathbf{i}_\mathrm{i}^\mathsf{H} \mathbf{z}_\mathrm{c}^* i_\mathrm{o}\right], \quad (6)$$

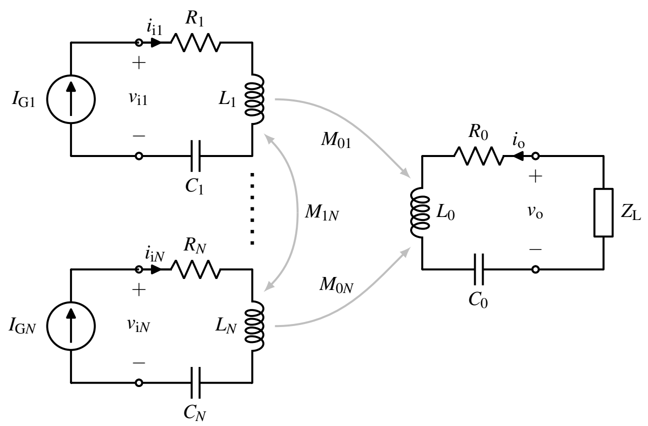

Figure 1. Inductively-coupled WPT link with N transmitters.

$$P_i = \frac{1}{4}\left(\mathbf{v}_i^H \mathbf{i}_i + \mathbf{i}_i^H \mathbf{v}_i\right) = \frac{1}{4}\left[\mathbf{i}_i^H\left(\mathbf{z}_i + \mathbf{z}_i^H\right)\mathbf{i}_i + \mathbf{i}_i^H \mathbf{z}_c i_o + i_o^* \mathbf{z}_c^H \mathbf{i}_i\right]. \tag{7}$$

The previous expressions can be written as

$$P_o = -\frac{1}{4}\mathbf{i}^H \mathbf{A}\mathbf{i} \qquad P_i = \frac{1}{4}\mathbf{i}^H \mathbf{B}\mathbf{i} \tag{8}$$

where

$$\mathbf{A} = \left[\begin{array}{c|c} z_o + z_o^* & \mathbf{z}_c^T \\ \hline \mathbf{z}_c^* & \mathbf{0} \end{array}\right] \qquad \mathbf{B} = \left[\begin{array}{c|c} 0 & \mathbf{z}_c^H \\ \hline \mathbf{z}_c & \mathbf{z}_i + \mathbf{z}_i^H \end{array}\right]. \tag{9}$$

Accordingly, for the power transfer efficiency the following expression can be obtained:

$$\eta = \frac{P_o}{P_i} = -\frac{\mathbf{i}^H \mathbf{A}\mathbf{i}}{\mathbf{i}^H \mathbf{B}\mathbf{i}}. \tag{10}$$

The efficiency is a function of the normalized port currents, whose stationary points can be determined by requiring $\delta\eta = 0$. This implies

$$-\mathbf{A}\mathbf{i} = \eta \mathbf{B}\mathbf{i} \tag{11}$$

which can be recognized as a generalized eigenvalue problem. The eigenvalues can be determined by requiring that determinant of $\mathbf{A} + \eta\mathbf{B}$ vanishes. By a trivial but tedious calculation, the following expression can be obtained for the maximum efficiency of the network:

$$\eta_{MAX} = \frac{\alpha - 1}{\alpha + 1} \tag{12}$$

where the parameter α is defined as:

$$\alpha = \sqrt{1 + Q_0 \sum_{n=1}^{N} k_{0n}^2 Q_n}. \tag{13}$$

The corresponding eigenvector can be obtained from (11) by choosing an arbitrary value, acting as a normalization constant, for one of the port currents. By choosing, for instance, an arbitrary value for i_o the normalized currents at the input ports can be derived as

$$\mathbf{i}_{in} = j\frac{k_{0n} Q_n}{\alpha - 1} i_o \qquad (n = 1, \ldots, N). \tag{14}$$

The normalized port voltage can then be obtained by means of the normalized impedance matrix as

$$v_o = -\frac{\alpha}{Q_0} i_o \tag{15}$$

and

$$\mathbf{v}_{in} = -\frac{1}{\alpha - 1}\left[\sum_{\substack{m=1 \\ m\neq n}}^{N} k_{0m} k_{nm} Q_m - j\alpha k_{0n}\right] i_o \quad (n = 1, \ldots, N). \tag{16}$$

From the normalized voltage and current at the output port it is possible to derive the optimal normalized load impedance

$$z_L = -\frac{v_o}{i_o} = \frac{\alpha}{Q_0}. \tag{17}$$

The unnormalized expression is:

$$Z_L = \omega_0 L_0 z_L = R_0 \alpha. \tag{18}$$

Accordingly, in order to obtain the maximum efficiency given by (12), the output port has to be terminated on the load expressed in (18), provided that the network is powered by input currents satisfying the relation given in (14). In this regard, it is worth observing that from (14) it is evident that the optimal input currents are in phase but, depending on the couplings, they have in general a different amplitude.

3 Numerical verification

In order to validate the theory presented in the previous section a numerical example has been considered and analyzed through circuital simulations. For the sake of comparison, the analyzed example is the same analyzed in [8] using two transmitters and a single receiver. The parameters of the resonators are (see Table IV of [8]): $L_i = 4.59\,\mu\text{H}$, $C_i = 120\,\text{pF}$, $Q_i = 270$, $i = 1, 2, 3$. As per the couplings, the values corresponding to cases A and B of Table IV of [8] have been considered. In particular, case A is the case where the RX is equally coupled with the TXs while the TXs are uncoupled: $k_{01} = k_{02} = 0.15$, $k_{12} = 0$. Case B is the case where the couplings between the TXs and the RX are different and the TXs are uncoupled: $k_{01} = 0.15$, $k_{02} = 0.1$, $k_{12} = 0$.

3.1 Case A: transmitters equally coupled with the receiver

In this case, according to the theory developed in [8] focusing on the optimization of the load Z_L for maximizing the efficiency, the optimal load is purely resistive and it is given by $Z_L = 41.5\,\Omega$; the efficiency corresponding to the optimal load is of about 0.97. As per the theory developed in this paper, by using (17) the value that can be obtained for the optimal load is in a perfect agreement with that reported in [8], i.e. $Z_L = 41.5\,\Omega$ corresponding to $\eta = 0.97$. Additionally, by using (14) it can be easily verified that the optimal input currents have the same amplitude and the same phase. By introducing the parameter n defined as the ratio between the amplitudes of the input currents and the parameter δ defined as the difference between the phases of the input currents, i.e.:

$$n = \frac{|i_{i1}|}{|i_{i2}|} = 1, \tag{19}$$

$$\delta = \arg(i_{i2}) - \arg(i_{i1}). \tag{20}$$

For case A the optimal configuration corresponds to $n = 1$ and $\delta = 0$. The results obtained from circuital simulations are reported in Fig. 2. It can be seen that the results confirm that the value of the load maximizing the efficiency is that expected from the theory. By comparing the results illustrated in Fig. 2 with those reported in [8], it can be concluded that for the case of TXs equally coupled with the RX the approach proposed in [8], which is based on the simple optimization of the load, provides the same optimal results corresponding the general approach presented in this paper.

3.2 Case B: transmitters differently coupled with the receiver

In this case the two TXs, while being still uncoupled as for case A, have a different coupling with the RX: $k_{01} = 0.15$, $k_{02} = 0.1$. By using the theory presented in [8], the optimal load is $Z_L = 338.4\,\Omega$ and corresponds to a maximum of the efficiency of 0.67. By using the theory presented in this paper, a different result is obtained. In particular, from (17) and (14), the following values can be obtained: $Z_L = 35.3\,\Omega$, $n = 1.5$, and $\delta = 0$. The optimal configuration corresponds to a maximum of the efficiency of about 0.96. The results obtained for the efficiency as function of the load when the network is powered by the optimal input currents are reported in Fig. 3 and confirm the theory.

By comparing the results illustrated in Fig. 3 and those reported in [8], it is evident that in this case the use of the optimal input currents to power the network plays a key role in obtaining the best efficiency for the link. This is highlighted by the results summarized in Figs. 4 and 5 where

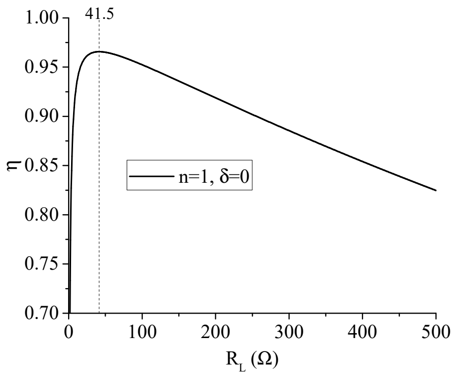

Figure 2. Results obtained for case A: efficiency as function of R_L.

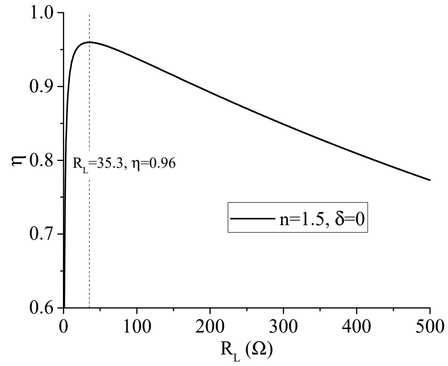

Figure 3. Results obtained for case B: efficiency as function of R_L.

the behaviour of the efficiency as function of the parameters n and δ is reported for $R_L = 35.3\,\Omega$. For completeness, Fig. 6 shows the results obtained for the efficiency as function of the load when two voltages generators are used at the input ports. Data obtained from circuital simulations for different values of the parameter m defined as the ratio between the amplitudes of the input voltages (i.e., $m = |V_{G1}|/|V_{G2}|$) are reported. The results reported in [8], where two voltage generators with unitary amplitude were used, are those corresponding to $m = 1$. From the figure it can be seen that better results for the efficiency can be obtained by suitably choosing the ratio of the amplitudes of the input voltages.

4 Conclusions

In this paper the case of a resonant inductive WPT link using multiple transmitters has been analyzed. By solving a generalized eigenvalue problem, the general solution for a network with any number of transmitters and any combination of couplings has been reported and discussed. The expression of the maximum efficiency of the link described as an $N+1$–port has been derived. It has been shown that the maximum efficiency can be obtained by suitably choosing the load impedance provided that the network is powered

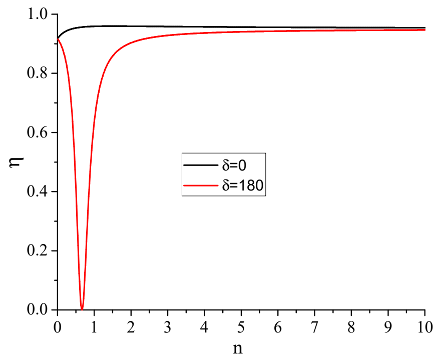

Figure 4. Results obtained for case B by varying the ratio n of the amplitude of the input currents (i.e., $n = |i_{i1}|/|i_{i2}|$); the results obtained for the case of in-phase (i.e., δ =0°) and out-phase (i.e., δ =180°) currents are reported. The reported data have been obtained for $R_L = 35.3\,\Omega$.

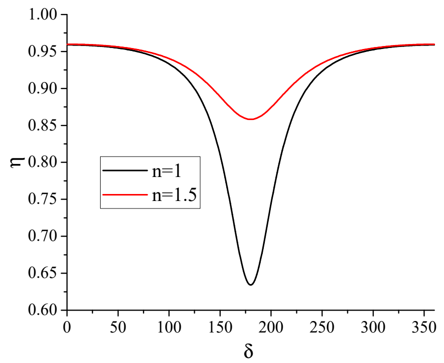

Figure 5. Results obtained for case B by varying the phase difference δ of the input currents for $R_L = 35.3\,\Omega$; results obtained for $n = 1$ and $n = 1.5$.

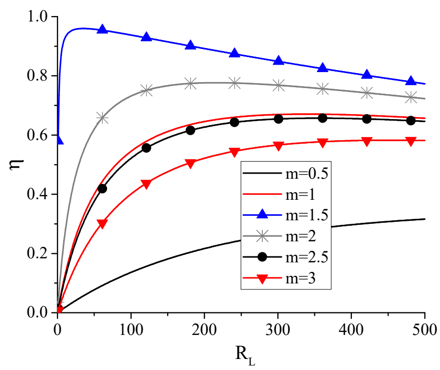

Figure 6. Results obtained for case B when two voltage generators are used at the input ports. The efficiency as function of the load is reported for different values of m defined as the ratio between the amplitudes of the input voltages ($m = |V_{G1}|/|V_{G2}|$).

by input currents satisfying amplitude and phase relations depending on the couplings of the link.

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