Quasi-Classic Approximation for High Frequency Wave Field through Transionospheric Stochastic Channel of Propagation

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Abstract

The numerical technique based on the quasi-classic approximation with complex-valued ray paths is presented for solving the appropriate Markov parabolic equation for the symmetric second order two-frequency and two-position coherence function of the high-frequency field propagating through a stochastic transionospheric channel.

1 Introduction

This quasi-classic approximation was first introduced in our papers [1, 2]. In these papers one may also find a sort of review of numerous papers devoted to different aspects of treatment of the high frequency wave field coherence properties, which propagate in the ionospheric reflection and transionospheric stochastic channels of propagation. The particular case of this quasi-classic approach for pure spatial case was also used in our relatively recent papers [3, 4] in order to construct the 4th order statistical moments of a high-frequency field propagating through the transionospheric stochastic channel and build the software simulator of the high-frequency stochastic signals in the transionospheric channel of propagation.

The quasi-classic approximation being discussed here may serve as a good tool for verifying and validating different pure analytic or semi-analytic techniques for solving the same problem. In particular, it is planned to use it to additionally validate recently developed in [5] analytical technique for describing the coherence properties of the high-frequency field in the transionospheric stochastic channel with the inhomogeneous background medium and anisotropic fluctuations of the electron density fluctuations.

2 Markov equation for coherence function

Following [3, 4, 5], Markov equation for the two-frequency, two-position symmetric coherence function $\Gamma_2(\boldsymbol{\rho}, z, \varphi)$ is reduced to the form

$$\frac{\partial F}{\partial z} + \frac{ik_d}{2k_1k_2}\nabla_T^2 F + \frac{k_1k_2}{8}D_\varepsilon(\boldsymbol{\rho}, z, \varphi)F = 0 \qquad (1)$$

so that the required function $\Gamma_2(\boldsymbol{\rho}, z, \varphi)$ is expressed through the solution of equation (1) for $\Gamma(\boldsymbol{\rho}, z, \varphi)$ as follows:

$$\Gamma_2(\boldsymbol{\rho}, z, \varphi) = F(\boldsymbol{\rho}, z, \varphi)exp\left[-\frac{A_N(0)k_d^2}{8k_1^2k_2^2}\int_0^z k_{pl}^4(z')dz'\right]. \qquad (2)$$

In equations (1, 2) $k_d = k_1 - k_2$ is the difference wave number (frequency); $k = \omega/c$, where ω is the transmission frequency and c is the light velocity in vacuum; z is the variable along the line of sight (ray path) and $\boldsymbol{\rho}$ is the transversal to the line of sight two-dimensional difference spatial variable; function $k_{pl}^2(z)$ in (1) is defined as

$$k_{pl}^2(z) = \frac{e^2 n(z)}{\varepsilon_0 m_e c^2} \qquad (3)$$

Here e is the electron charge, m_e is the mass of electron, and ε_0 is the dielectric permittivity of vacuum. $A_N(\boldsymbol{\rho})$ is the effective transversal correlation function of the fractional electron density fluctuations of the ionosphere defined as $N(z) = \delta n(z)/n(z)$ where $\delta n(z)$ represents the absolute fluctuations of the electron density and $n(z)$ is the electron density in the background ionosphere along the line of sight; fluctuations $N(z)$ are assumed to be statistically homogeneous.

In (1) the dependence on the transversal central variable is neglected as the characteristic scale of Γ on the central variable is expected to be much less than the characteristic scale of the background ionosphere.

The dependence of the effective transversal structure function of fluctuations of the dielectric permittivity of the ionosphere $D_\varepsilon(\boldsymbol{\rho}, z, \varphi)$ from (1) on the angle φ, indicates that equation (1) is written for a given geometry of propagation respectively the Earth's magnetic field, defined by the angle φ, so that the expected solution of equation (1) will also include this dependence.

In the following consideration, equation (1) is solved numerically for any given model of the background ionosphere as a function of z along a line of sight. The structure function of the dielectric permittivity $D_\varepsilon(\boldsymbol{\rho}, z, \varphi)$

is expressed through the effective transversal correlation function $A_N(\boldsymbol{\rho})$ as follows

$$D_\varepsilon(\boldsymbol{\rho},z,\varphi)=\big(A_N(0)-A_N(\boldsymbol{\rho})\big)\frac{1}{k_1^2k_2^2}\int_0^z k_{pl}^4(z')dz' \qquad (4)$$

Finally, for the correlation function of fractional electron density fluctuations the power law model is employed of the following form:

$$A_N(\boldsymbol{\rho},\varphi)=\frac{\sigma_N^2\,L_0}{\sqrt{2\pi}2^{((p-5)/2)}\Gamma((p-3)/2)}\cdot\left(\frac{2\pi}{L_0}f(\boldsymbol{\rho},\varphi)\right)^{\frac{p-2}{2}}K_{\frac{p-2}{2}}\left(\frac{2\pi}{L_0}f(\boldsymbol{\rho},\varphi)\right) \qquad (5)$$

In (5) σ_N^2 is the variance of the fractional electron density fluctuations, L_0 is the outer scale of fluctuations, Γ is the Gamma function, p is the three-dimensional spectral index, $f(\boldsymbol{\rho},\varphi)$ is the function of the transversal difference spatial variable, which depends on the anisotropy of fluctuations and on the angle φ between line of sight and the Earth's magnetic field.

3 Complex valued quasi-classic equations

The formal transition from equation (1) to its quasi-classic approximation is performed by tending the product k_1k_2 to the infinity: $k_1k_2\to\infty$. At this, the physically large dimensionless parameter of the problem appears to be as follows: $K=k_1k_2l_{min}^2$, where l_{min} is the minimal spatial scale of the structure function $D_\varepsilon(\boldsymbol{\rho},z,\varphi)$ of the anisotropic fluctuations of the ionospheric electron density in (1). Then, performing standard manipulations of the quasi-classic approximation [1, 2] and returning back to the physical variables, the main term of the solution to equation (1) is found in the form

$$\Gamma(\boldsymbol{\rho},z)=exp\big(k_1k_2\psi(\boldsymbol{\rho},z)\big)U_0(\boldsymbol{\rho},z). \qquad (6)$$

Here the complex-valued phase function (complex eikonal) $\psi(\rho,z)$ is governed by the eikonal equation

$$\frac{\partial\psi}{\partial z}+\frac{ik_d}{2}(\nabla_T\psi)^2+\frac{1}{8}D_\varepsilon(\boldsymbol{\rho},z,\varphi)=0, \qquad (7)$$

and the zero order amplitude U_0 obeys the main transport equation

$$\frac{\partial U_0}{\partial z}+ik_d(\nabla_T\psi\cdot\nabla_TU_0)+\frac{ik_d}{2}U_0\nabla_T^2\psi=0. \qquad (8)$$

To solve equations (7) and (8) the general method of characteristics is employed. Equation (7) is a Hamilton-Jacobi type equation, so that the appropriate Hamilton equations are written in the following form:

$$\frac{dz}{d\tau}=1,\qquad \frac{dp_z}{d\tau}=-\frac{1}{8}\frac{\partial}{\partial z}D_\varepsilon(\boldsymbol{\rho},z,\varphi),$$
$$\frac{d\boldsymbol{r}}{d\tau}=ik_d\boldsymbol{p},\qquad \frac{d\boldsymbol{p}}{d\tau}=-\frac{1}{8}\nabla_TD_\varepsilon(\boldsymbol{\rho},z,\varphi) \qquad (9)$$

Equations (9) determine complex-valued trajectories $\boldsymbol{r}=\boldsymbol{r}(z)$, which start at complex-valued points $(0,\boldsymbol{r}_0)$ on the initial surface $z=0$, where the boundary (initial) condition is stated, and arrive at the real-valued points of observation $(z,\boldsymbol{r})$ and are subject to the initial conditions (in the case of plane wave $\Gamma=1$ on the initial surface, hence $\boldsymbol{p}(0)=0$ and trajectories are orthogonal to the initial surface). The starting points are determined for each point of observation and for each value of the frequency difference. Once the complex trajectories have been constructed, the complex eikonal ψ and the main amplitude U_0 are found as the appropriate integrals along the corresponding complex valued trajectories, thus providing the solution to the problem under consideration given by equation (6).

It is worth pointing out that in the limiting case of a homogeneous background medium and quadratic structure function of the dielectric permittivity fluctuations the solutions to the main transport equation and eikonal equation together produce the rigorous well known solution to the spaced position and frequency coherence function [1].

4 Numerical results

In the applications, the dependence of the coherence function on the spatial separation and frequency difference is of importance. Generally, for an arbitrary orientation of the path of propagation with respect to the magnetic field direction, the problem is fully three-dimensional in space. However, there are two limiting cases when the problem may be reduced to the two-dimensional one. Firstly, this is the case of extremely anisotropic irregularities that are infinitely elongated in the direction of the magnetic field and propagation path is oriented transversal to this direction. This case corresponds to the conditions typical for the equatorial region. It is convenient to consider this case in 2D Cartesian variables (x,z) with z being the variable along the line of sight (direction of propagation) and x is the variable orthogonal to both line of sight and the Earth's magnetic field.

Secondly, this is the case of isotropic irregularities. This is to understand how important the effect of anisotropic shape of realistic random irregularities is. Here, due to the cylindrical symmetry of the solution in the plane perpendicular to the propagation path, the equations written in cylindrical coordinates do not depend on the angular variable, thus reducing the dimension. In this case of isotropic fluctuations it is convenient to introduce the 2D polar variables (r,φ) in the plane orthogonal to the same line of sight z. In this case the dependence on φ vanishes for spherically symmetric fluctuations.

Below the numerical results for both the cases are shown. For clarity, in the figures the particular case is presented of the horizontal magnetic field and vertical path of propagation, for which the angle is $\varphi = 90°$. The consideration for an arbitrary geometry, in principle, is straightforward. However, it appears to be much more cumbersome.

In the particular geometry of propagation, outlined above, in the plane perpendicular to the propagation path (as well as in the observation plane on the ground) and for any given frequency separation k_d, the coherence function for the anisotropic case (case 1) solely depends on x component of the difference spatial coordinate $\boldsymbol{\rho}$, which is orthogonal to the magnetic field direction and does not depend on the y component in the direction of magnetic field. In this case function f from equation (5) $f(\boldsymbol{\rho}, \varphi) = x$. For isotropic fluctuations (case 2) the coherence function is also isotropic in the spatial domain, and $f(\boldsymbol{\rho}, \varphi) = r$.

As a background ionospheric electron density profile in the simulations below, Chapman layer model is utilized with the height of maximum of the electron density of 350 km, the characteristic scale of the layer of h_m =100 km, and $f_{pl\,max}$ =10 MHz. The corresponding total electron content (TEC) is 49.7 TECu.

In the Figure 1, the coherence function on the ground is depicted as a contour plot in the domain $(x/L_0, y/L_0,)$ for both the case 1, corresponding to the extremely anisotropic (horizontal lines), and case 2, corresponding to isotropic (circle lines) fluctuations. Frequency separation $k_d = 0$. Expectedly, the coherence decays as the separation increases, tending to the constant value corresponding to the energy of the coherent component of a random field.

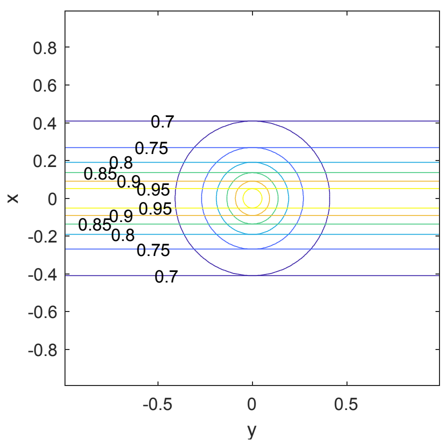

Figure 1. Contour plots on the ground of the coherence function for $k_d = 0$ and for both cases of isotropic and extremely anisotropic fluctuations. f_c=1 GHz, L_0=10 km, σ_N^2 =0.1

To demonstrate the dependence of the coherence on the frequency separation, in the Figure 2 the absolute value of the coherence function is presented as a contour plot in the domain $(k_d/k_c, r/L_0)$.

Unlike in spatial domain ($k_d = 0$), where the coherence tends to the constant value (as in the Figure 1), in the frequency domain it decays down to the small values as the frequency separation rises. The frequency bandwidth determined on the level 0.5 appears to be about 0.2.

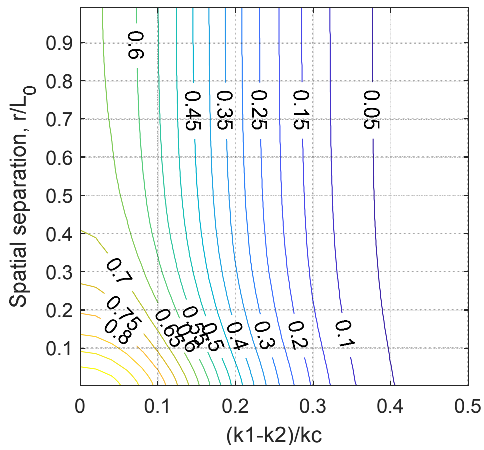

Figure 2. Contour plot of the absolute value of the coherence function in the domain $(k_d/k_c,\ r/L_0)$. f_c=1 GHz, L_0=10 km, σ_N^2 =0.1

The last Figure 3 demonstrates the same type contour plot as in the Figure 2, but obtained for the lower central frequency 400 MHz. In this case the coherence tends to zero in both spatial and frequency domains, and the frequency bandwidth reduces to about 0.07.

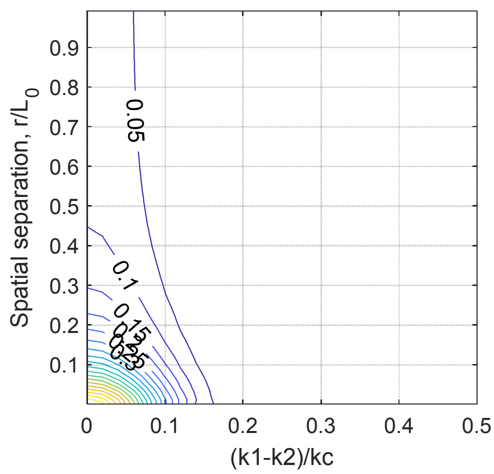

Figure 3. Contour plot of the absolute value of the coherence function in the domain $(k_d/k_c, r/L_0)$. f_c=400 MHz, L_0=10 km, σ_N^2 =0.1

5 Conclusion

The introduced earlier in [1, 2] quasi-classic technique for solving Markov equations for the statistical moments of the high-frequency random field, propagating through the stochastic transionospheric channel was presented and further discussed. Employing this technique, the effects of the inhomogeneous background ionosphere and anisotropic local random inhomogeneities of the ionospheric electron density on the field coherence properties were discussed. It is of use to also stress that the basic equation (1) of this consideration is of the same form as the non-stationary Schrödinger equation in quantum mechanics. However, when speaking in terms of quantum mechanics, here the "potential" appears to be pure imaginary.

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7 References

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