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Diffraction of TE Polarised Electromagnetic Waves by a Nonlinear Metamaterial Waveguide

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Abstract

A numerical approach is proposed, justified and applied for the solution to the problem of diffraction of an electromagnetic TE wave by a metal-dielectric waveguide filled with nonlinear metamaterial medium. The problem is to find amplitudes of the reflected and the transmitted fields when the amplitude of the incident field is known. The numerical solution techniques are developed. Numerical results are presented.

1 Introduction

The electromagnetic wave diffraction by homogeneous [1] or inhomogeneous (see e.g. [2]) cylindrical metal-dielectric bodies filled with linear medium has been studied intensively since the 1940s. The case of nonlinear filling still constitutes an unsloved problem. A progress here is associated with recently developed techniques (see [3], [4], [5] and [6]) for the analysis of nonlinear boundary value problems for the Maxwell's and Helmholtz equations.

In 1967, Russian physicist V. G. Veselago predicted an extraordinary electromagnetic (EM) wave phenomenon which is related to materials with a simultaneously negative permittivity and negative permeability [7]. He hypothetically created a lossless meta-material and showed the extraordinary properties of this material which is not found in nature, in particular, negative group velocity, negative refraction, the reversal of the Doppler effect and Cherenkov radiation.

The present study focuses on the analysis of the diffraction of TE waves by an open waveguide, a Goubau line (GL) (see), with a nonlinear metamaterial medium. Metamaterial is an artificial material with negative permittivity and negative permeability [7, 8, 9]. The nonlinearity is expressed by the Kerr law [10, 11, 12]. The main task which we resolve is to elaborate mathematically correct problem statements for nonlinear differential equations that enable one to introduce and investigate the problem of diffraction. Such problems (the scattering by cylinders covered with nonlinear materials) finds some applications in cloaking devices, see [13] and [14].

The study of diffraction of electromagnetic waves by a nonlinear dielectric waveguide is presented in the following works [15, 16, 17, 18].

2 Statement of the problem

The cross section of the waveguide under study perpendicular to its axis consists of two concentric circles of radii r_0 and r (see Fig. 1): r_0 is the radii of the internal (perfectly conducting) cylinder, and $r - r_0$ is the thickness of the external (dielectric) cylindrical shell.

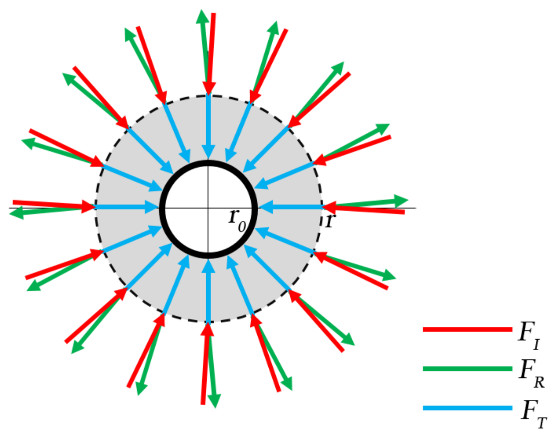

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Figure 1. Waveguide Σ, where r_0 and r are the radii of the internal (perfectly conducting) and external (dielectric) cylinders, respectively.

The complex amplitudes $\mathbf{E}$, $\mathbf{H}$ of the electromagnetic field satisfy Maxwell's equations

$$\begin{cases} \operatorname{rot}\mathbf{H} = -i\omega\varepsilon_0\widetilde{\varepsilon}\mathbf{E}, \\ \operatorname{rot}\mathbf{E} = i\omega\mu\mathbf{H}, \end{cases} \tag{1}$$

have zero tangential components of the electric field on the perfectly conducting surface $\rho = r_0$ and continuous tangential field components on the media interface $\rho = r$; here ω is the circular frequency.

We assume that the permittivity in the entire space has the form $\widetilde{\varepsilon}\varepsilon_0$, where

$$\widetilde{\varepsilon} = \begin{cases} -\varepsilon^2 + \widetilde{\alpha}|\mathbf{E}|^2, & r_0 \le \rho \le r, \\ 1, & \rho > r, \end{cases} \tag{2}$$

and $|\mathbf{E}|^2 = \left|(\mathbf{E}e^{-i\omega t}, \mathbf{e}_\rho)\right|^2 + \left|(\mathbf{E}e^{-i\omega t}, \mathbf{e}_\varphi)\right|^2 + \left|(\mathbf{E}e^{-i\omega t}, \mathbf{e}_z)\right|^2$;$(\cdot,\cdot)$ is the Euclidean inner product; $\varepsilon^2, \widetilde{\alpha}$ are real positive constants.

3 TE waves

Let us consider TE-polarized waves in the harmonic mode, according to [19],

$$\mathbf{E}e^{-i\omega t} = e^{-i\omega t}(0, E_\varphi, 0)^T, \ \mathbf{H}e^{-i\omega t} = e^{-i\omega t}(H_\rho, 0, H_z)^T,$$

where $\mathbf{E}, \mathbf{H}$ are complex amplitudes,

$$E_\varphi = \mathrm{E}_\varphi(\rho)e^{i\gamma z}, H_\rho = \mathrm{H}_\rho(\rho)e^{i\gamma z}, H_z = \mathrm{H}_z(\rho)e^{i\gamma z} \quad (3)$$

and γ is a given quantity.

Let $k_0^2 := \omega^2\mu\varepsilon_0$. Substituting components (3) into (1) and using the notation $u(\rho) := \mathrm{E}_\varphi(\rho)$ we obtain

$$\left(\rho^{-1}\left(\rho u\right)'\right)' + \left(k_0^2\widetilde{\varepsilon} - \gamma^2\right)u = 0, \quad (4)$$

where $\widetilde{\varepsilon}$ is defined by formula (2).

We assume that function u is sufficiently smooth,

$$u(\rho) \in C^1\left[r_0, +\infty\right) \cap C^2(r_0, r) \cap C^2(r, +\infty).$$

In the domain $r_0 \le \rho \le r$ equation (4) takes the form

$$\left(\rho u'\right)' - \left(\rho k_1^2 + \rho^{-1}\right)u = -\alpha\rho u^3, \quad (5)$$

where $\alpha := k_0^2\widetilde{\alpha}$, $k_1^2 := k_0^2\varepsilon^2 + \gamma^2$. In the domain $\rho > r$ equation (4) becomes

$$\left(\rho u'\right)' - \left(\rho k_2^2 + \rho^{-1}\right)u = 0, \quad (6)$$

where $k_2^2 := \gamma^2 - k_0^2$. For $\rho > r$, the solution to equation (6) must be written in the following form

$$u = \widetilde{A}I_1(k_2\rho) + \widetilde{C}K_1(k_2\rho), \ \ \rho > r. \quad (7)$$

The incident field is determined by

$$u_I(\rho) = \widetilde{A}I_1(k_2\rho), \quad (8)$$

where I_1 is the modified Bessel function (Infeld function) [20] and $F_I = \widetilde{A}I_1(k_2r)$ is the amplitude of the incident field (for $\rho = r$). The reflected field satisfies the radiation conditions of decay at infinity and therefore can be taken in the following form at $\rho > r$

$$u_R(\rho) = \widetilde{C}K_1(k_2\rho), \quad (9)$$

where K_1 is the modified Bessel function (Macdonald function) [20] and constant $F_R = \widetilde{C}K_1(k_2r)$ is the amplitude of the reflected field (for $\rho = r$). The total field in the region $\rho > r$ is a superposition of the incident, u_I, and reflected, u_R, fields,

$$u = u_I + u_R, \ \rho > r. \quad (10)$$

The amplitude of transmitted field F_T (for $\rho = r$) is a sum of amplitudes of the incident, F_I, and reflected, F_R, fields,

$$F_T = F_I + F_R.$$

Transmission conditions for the functions u and u' result from the continuity conditions for the tangential field components (E_φ and H_z) and have the form

$$\begin{aligned} [u]|_{\rho=r} = \left[u'\right]\big|_{\rho=r} &= 0, \\ u|_{\rho=r_0} &= 0, \end{aligned} \quad (11)$$

where $[v]|_{\rho=s} = \lim\limits_{\rho\to s-0} v(\rho) - \lim\limits_{\rho\to s+0} v(\rho)$ is the jump in the limit values of the function at a point s.

Formulate the diffraction problem (problem P): *to find amplitude F_R of the reflected field such that, for the given amplitude F_I of the incident field, there are nonzero function $u(\rho)$ defined by formula* (10) *for $\rho > r$ that solve the ordinary differential equation* (5) *for $r_0 < \rho < r$ and satisfy transmission conditions* (11).

4 Numerical method

For the numerical solution of Problem P a method based on the solution to the auxiliary Cauchy problem is proposed which makes it possible in particular to determine and plot the amplitude of the reflected field, F_R, with respect to the amplitude of the incident field, F_I.

Consider the Cauchy problem for the equation (6) with the following initial conditions

$$\begin{aligned} &u(r) = u_I(r) + u_R(r) = F_I + F_R, \\ &u'(r) = u_I'(r) + u_R'(r) = F_I\frac{I_1'(k_2r)}{I_1(k_2r)} + F_R\frac{K_1'(k_2r)}{K_1(k_2r)}. \end{aligned} \quad (12)$$

To justify the solution technique, we use classical results of the theory of ordinary differential equations concerning the existence and uniqueness of the solution to the Cauchy problem and continuous dependence of the solution on parameters.

Using the transmission condition on the boundary $\rho = r_0$, we obtain the following dispersion equation

$$\Delta(F_I, F_R) \equiv u(r_0) = 0, \quad (13)$$

where $\Delta(F_I, F_R)$ is determined explicitly and quantity $u(r_0)$ is obtained from the solution to the Cauchy problem for fixed values of F_I and F_R.

Thus for fixed value of F_I, when the number $F_R = \widetilde{F}_R$ is such that $\Delta(F_I, F_R) = 0$, then F_R is the solution of problem P which corresponds to the value of F_I.

In Figs. 2–5 the amplitudes of the reflected, F_R, and transmitted, F_T, fields calculated with respect to the amplitude of the incident field F_I are shown.

These simulation results describe the essential relationships between linear and nonlinear problems. Namely, the nonlinear reflected field can be predicted from that obtained

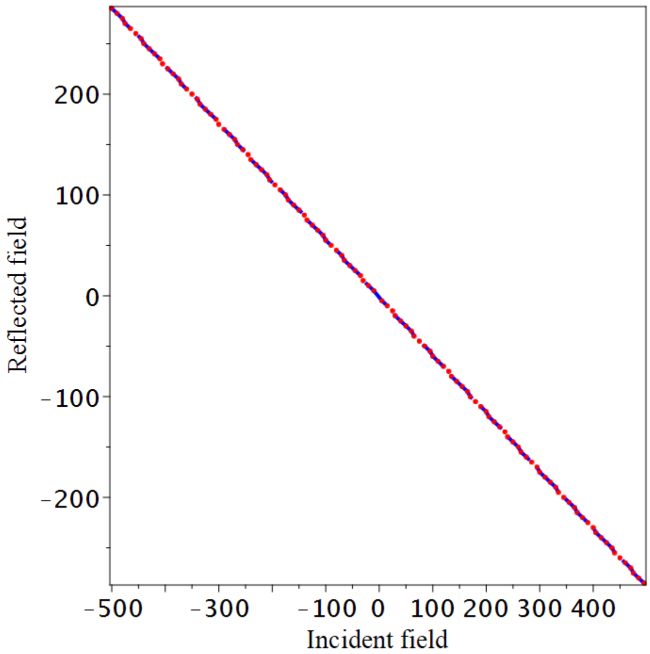

Figure 2. Amplitude of the reflected field F_R vs amplitude of the incident field F_I in the linear ($\alpha = 0$, blue) and nonlinear (red) cases. The values of parameters are $\gamma = 1.15$, $k_0^2 = 1$, $r_0 = 2$, $r = 4$, $\varepsilon^2 = 4$, $\alpha = 10^{-5}$.

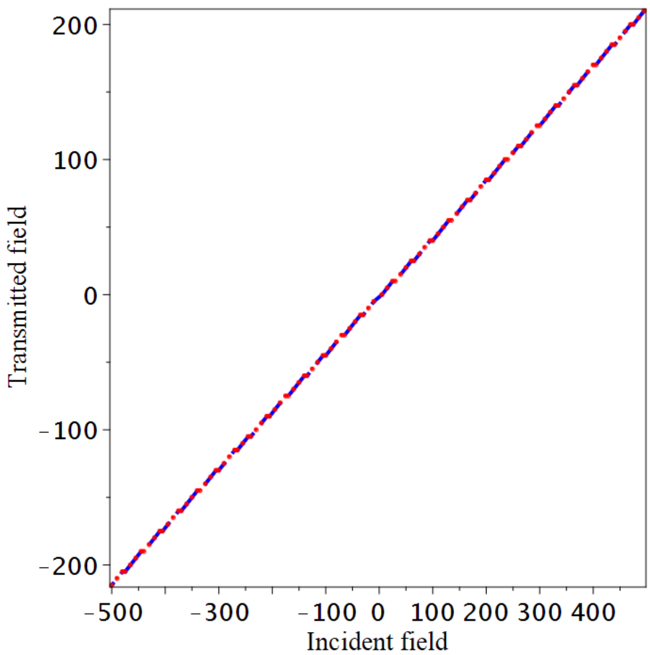

Figure 3. Amplitude of the transmitted field F_T vs amplitude of the incident field F_I in the linear ($\alpha = 0$, blue) and nonlinear (red) cases. The values of parameters are $\gamma = 1.15$, $k_0^2 = 1$, $r_0 = 2$, $r = 4$, $\varepsilon^2 = 4$, $\alpha = 10^{-5}$.

from the linear problem using the perturbation theory method (for small value of nonlinearity coefficient α). Uniqueness of the solution to the nonlinear problem is preserved, see Figs. 2 and 3. Note that the curves in Figs. 4 and 5 significantly different from linear curves. Uniqueness of the solution to the nonlinear problem is not preserved for the "big" value of nonlinearity coefficient α.

5 Conclusion

We have developed an analytical-numerical approach for the analysis of electromagnetic wave diffraction by a waveguide of circular geometry filled with nonlinear metamaterial medium. The method can be extended to more complicated nonlinearities and applied to numerical solution of the problems of diffraction by multilayered metal-dielectric structures with nonlinear media.

6 Acknowledgements

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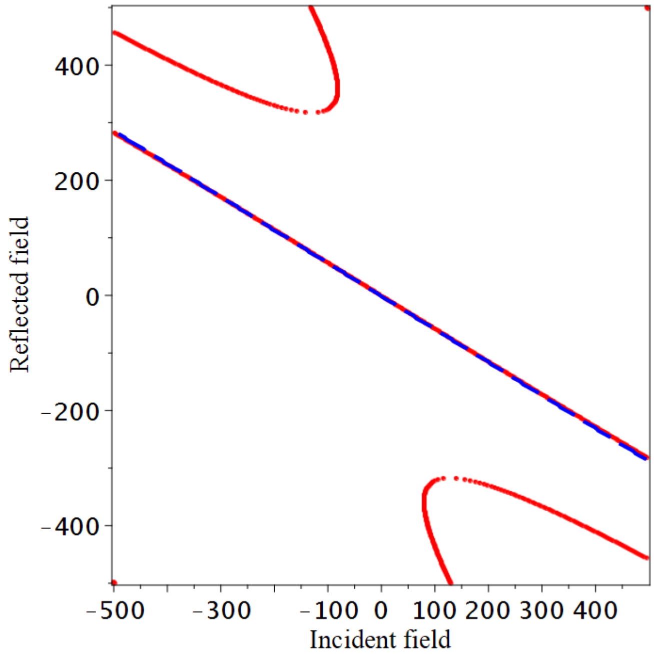

Figure 4. Amplitude of the reflected field F_R vs amplitude of the incident field F_I in the linear ($\alpha = 0$, blue) and nonlinear (red) cases. The values of parameters are $\gamma = 1.15$, $k_0^2 = 1$, $r_0 = 2$, $r = 4$, $\varepsilon^2 = 4$, $\alpha = 10^{-3}$.

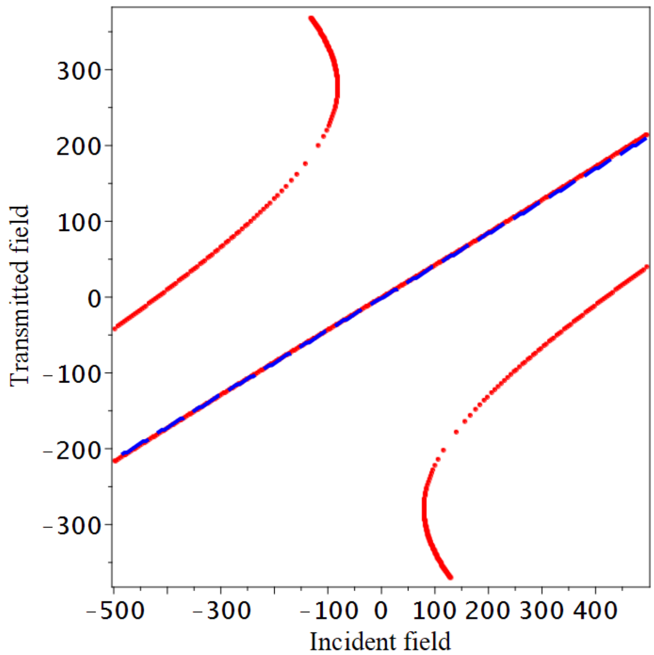

Figure 5. Amplitude of the transmitted field F_T vs amplitude of the incident field F_I in the linear ($\alpha = 0$, blue) and nonlinear (red) cases. The values of parameters are $\gamma = 1.15$, $k_0^2 = 1$, $r_0 = 2$, $r = 4$, $\varepsilon^2 = 4$, $\alpha = 10^{-3}$.