The polarizability of an alternative sequence of isotropic and radially anisotropic multilayer sphere

S. Batool*[(1)], M. Nisar[(1)], F. Mangini[(2)], F. Frezza[(1)]
(1) Department of Information Engineering, Electronics and Telecommunications,
"La Sapienza" University of Rome, Via Eudossiana 18, 00184 Rome, Italy
(2) Department of Information Engineering, University of Brescia,
Via Branze 59, 25123 Brescia, Italy

Abstract

In this article, the polarizability of a multilayer sphere that consists of an alternative sequence of layers of isotropic and spherically radially anisotropic (SRA) material has been investigated. Within each SRA layer, components of the tensor permittivity have different values in radial and tangential directions. The mathematical treatment for extracting electrostatic polarizability has been formulated in terms of scattered potentials. Obtained results have been used to describe the behavior of a whole sphere as a function of number of the isotropic and SRA alternating layers using a numerical approach.

1 Introduction

The electromagnetic scattering by an inhomogeneous sphere is a simple and quite fundamental problem. It has become more crucial these days, when researchers is dedicated to the study of metamaterials due to their attractive properties. Multilayer inhomogeneous spheres find many application in metamaterials designing and their application, particularly for cloaking [1, 2]. Besides anisotropic materials with desired specifications specially permittivity value can be composed using alternating layers of dielectric or metals and testified for subwavelength imaging [3, 4].

Inhomogeneous multilayer spheres have been frequently studied by many researchers [5, 6, 7, 8]. Mangini et. al. proposed a model and presented limitations for how to consider a multilayer sphere as a single RU sphere [6]. Several other algorithms have been worked out to study EM scattering from multilayer spheres [9, 10, 11], most of them describe scattering for near field [11].

We have continued with the study of multilayer inhomogeneous sphere, that is made up of alternating sequence or array of different materials i.e, isotropic and anisotropic. We have continued towards obtaining generalized polarizability that also incorporates the effects of anisotropic or inhomogeneous permittivity. Radially anisotropic permittivity tensor in general dyadic form, irrespective of any coordinate system is [12]

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \left[\varepsilon_{rad}\mathbf{u}_{rad}\mathbf{u}_{rad} + \varepsilon_{tan}(I - \mathbf{u}_{rad}\mathbf{u}_{rad})\right] \tag{1}$$

where ε_0 is the permittivity of free space, ε_{tan} and ε_{rad} represents the relative tangential and radial components of permittivity, respectively. I is the unit dyadic, and $\mathbf{u}_{rad}$ represents the unit vector in the radial direction.

2 Formation

Let us consider a multilayer inhomogeneous sphere with radius a_k and consists of an alternative sequence of isotropic and SRA layers, such that the core is isotropic. Core is covered by SRA layer, the next layer is again isotropic and that is surrounded by another SRA layer and sequence continues as shown in Fig. 1. The radius of the outer layer is fixed and equal to a_1 and the internal radii may be written as [6]

$$a_k = \frac{N-(k-1)}{N} a_1 \tag{2}$$

The number of layers, N is arbitrary and $k = 1, 2, 3......N$.

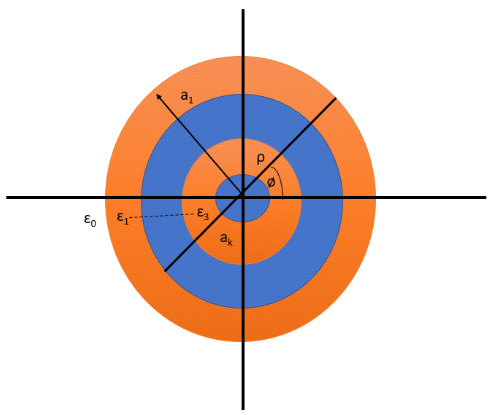

Figure 1. Geometry represents the alternative sequence of isotropic and anisotropic multilayer sphere immersed in free space.

With excitation from external electric field $\mathbf{E} = E\hat{\mathbf{z}}_0$, solution that satisfies Laplace's equation in an arbitrary k_{th}

isotropic layer can be written as [7]

$$\Phi_k = B_k r \cos\phi + C_k r^{-2} \cos\phi \quad (3)$$

whereas, for SRA layer it will be

$$\Phi_k = B_k r^{\nu} \cos\theta + C_k r^{-\nu-1} \cos\theta \quad (4)$$

where

$$\nu = \frac{1}{2}\left(-1 + \sqrt{1 + \frac{\varepsilon_t}{\varepsilon_r}}\right) \quad (5)$$

B_k and C_k are unknown coefficients representing the amplitudes of the constant field part and dipole-type contribution to the field respectively. ν represents the anisotropic ratio. Unknowns coefficients can be worked out using transmission-line method [8]. The boundary condition between any two adjacent layers k and $k+1$ must be

$$\begin{pmatrix} B_k \\ C_k \end{pmatrix} = [Q_k] \begin{pmatrix} B_{k+1} \\ C_{k+1} \end{pmatrix} \quad (6)$$

where

$$[Q_k] = \frac{1}{2\gamma_k \varepsilon_k} \begin{pmatrix} Q_{k11} & Q_{k12} \\ Q_{k21} & Q_{k22} \end{pmatrix} \quad (7)$$

and:

$$\begin{aligned} Q_{k11} &= [2\varepsilon_{k+1} + \varepsilon_{k+1}] a_{k+1}^{\nu_{k+1}-1} \\ Q_{k12} &= [2\varepsilon_k - (\nu_{k+1}+1)\varepsilon_{k+1}]\, a_{k+1}^{-\nu_{k+1}-2} \\ Q_{k21} &= [\varepsilon_k - \nu_{k+1}\varepsilon_{k+1}]\, a_{k+1}^{\nu_{k+1}+2} \\ Q_{k22} &= [\varepsilon_k + (\nu_{k+1}+1)\varepsilon_{k+1}]\, a_{k+1}^{-\nu_{k+1}+1} \end{aligned}$$

Since the core is isotropic and the total number of layers is $k = 1,2,3......N$, which implies that for every layer with odd number in entire sequence such that $k = 1,3......N-1$ we need to insert $\nu = 1$. For remaining layers $k = 2,4......N$, ν will have value as in Eq. (5).

It can be proved that when $\nu_k = \nu_{k+1} = 1$, all the above expressions can be reduced into the case of isotropic multilayer sphere.

When we take into account all layers, we obtain the following relationship

$$\begin{pmatrix} B_0 \\ C_0 \end{pmatrix} = \prod_{k=0}^{N-1} [Q_k] \begin{pmatrix} B_N \\ C_N \end{pmatrix} = [Q] \begin{pmatrix} B_N \\ 0 \end{pmatrix} \quad (8)$$

with

$$[Q] = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix} \quad (9)$$

$C_N = 0$, because the core does not contain any reflected field. The generalized expression of polarizability can be solved as [7]

$$\alpha_P = \frac{3V\varepsilon_0}{a_1^3} \frac{Q_{21}}{Q_{11}} \quad (10)$$

where, ε_0 is the vacuum's permittivity and V is the volume of the sphere, i.e., $V = \frac{4}{3}\pi r^3$. The effective permittivity can be derived with simple algebra and polarizability from Eq. 10

$$\varepsilon_{eff} = \varepsilon_0 + \frac{\frac{\alpha_P}{V}}{1 - \frac{\alpha_P}{3\varepsilon_0 V}} \quad (11)$$

As an illustration,we can write the polarizability by taking the case $N = 1$

$$\alpha_P = 3V\varepsilon_0 \frac{(\varepsilon_0 - \nu_1\varepsilon_1)}{(2\varepsilon_0 + \nu_1\varepsilon_1)} \quad (12)$$

For the case $N = 2$, we obtain the polarizability of two concentric isotropic and SRA spheres

$$\frac{\alpha_P = 2V\varepsilon_0 \left\{ (A_1 * A_2) + (A_3 * A_4) \right\} \left(a_2{}^3 / a_1^{2\nu_1+1} \right)}{\left\{ (A_5 * A_2) + (A_6 * A_4) \right\} \left(a_2{}^3 / a_1^{2\nu_1+1} \right)} \quad (13)$$

where

$$\begin{aligned} A_1 &= (\varepsilon_0 - \nu_1\varepsilon_1) \\ A_2 &= (2\varepsilon_1 + \nu_2\varepsilon_2) \\ A_3 &= (\varepsilon_0 + (\nu_1+1)\varepsilon_1) \\ A_4 &= (\varepsilon_1 - \nu_2\varepsilon_2) \\ A_5 &= (2\varepsilon_0 + \nu_1\varepsilon_1) \\ A_6 &= (2\varepsilon_0 - (\nu_1+1)\varepsilon_1) \end{aligned}$$

Similarly, we can find the polarizability of a multilayer inhomogeneous sphere with consecutive isotropic and SRA layers and consisting of an arbitrary number of layers.

3 Numerical Results and Discussion

Numerical results for the polarizability of inhomogeneous multilayer sphere as a function of number of the layers placed in free space have been described in this section. For this purpose equations (8)-(11) have been implemented using Matlab code. We have fixed the following parameters; radius of the inner core= $a_2 = 0.70$, outer shell $a_1 = 1$, the anisotropy ratio $\nu_1 = 1$, $\nu_2 = 4$ and vice versa such that $\nu = \{1,4\}$ for all cases. Permittivity has been taken as $\varepsilon = \{2,4\}$ for all cases, such that two concentric layers $\varepsilon_1 = 2$ and $\varepsilon_2 = 4$ and vice versa.

In Fig. 2, we can see that, for the case when permittivity of cover layer $\varepsilon_2 = 4$ has more weight as compared to the opposite case when permittivity of cover layer $\varepsilon_2 = 2$ initially. Then afterward with an increase in the number of layers in both cases polarizability approaches to zero. For the case

$N = 2$, the same trend is observed for outer layer permittivity, $\varepsilon_2 = 4$ and $\varepsilon_2 = 2$ respectively and has a constant value.

In Fig. 3, we investigated the behavior of inhomogeneous isotropic multilayer sphere. In this case, when permittivity of cover layer $\varepsilon_2 = 4$ and permittivity of inner layer $\varepsilon_1 = 2$, polarizability has more weight as compared to the other case, when permittivities of cover and inner layers are $\varepsilon_2 = 2$ and $\varepsilon_1 = 4$ respectively. Hence with an increasing number of layers in both cases polarizability approaches to a negative value. When $N = 2$, a similar pattern is noticed for both cases of outer layer permittivity, $\varepsilon_2 = 4$ and $\varepsilon_2 = 2$ respectively.

During our analysis, we have calculated a closed form of the polarizability of an inhomogeneous multilayer sphere having consecutive sequences of isotropic and SRA layers. The total number of layers is arbitrary and two alternating values of isotropic and anisotropic permittivity are employed. It has been detected that when layers of a sphere are alternatively isotropic and SRA, polarizability is zero for a higher number of layers. Whereas, when all consecutive layers are isotropic, polarizability is not zero rather it is near to zero. We will continue inspecting this model of inhomogeneous multilayer sphere for different applications.

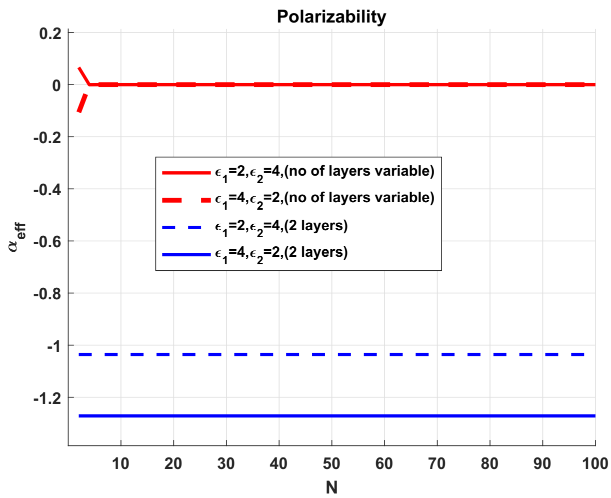

Figure 2. Normalized polarizability of alternative sequence of isotropic and SRA multilayer sphere as a function of the number of layers, with the following parameters: ε_1=2; ε_2=4; $\nu_1 = 1$ and $\nu_2 = 4$.

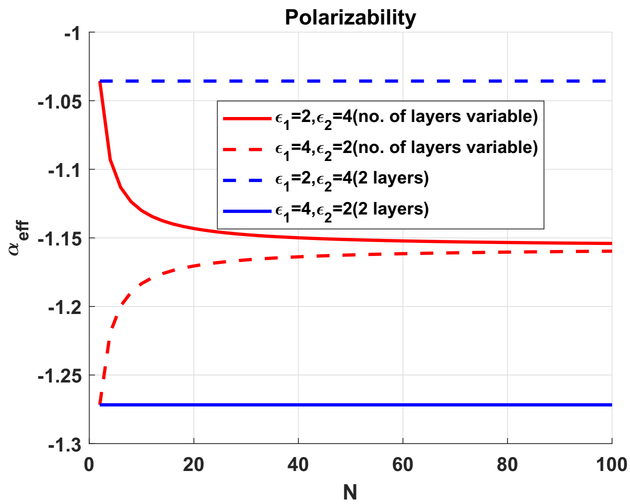

Figure 3. Normalized polarizability of an isotropic multilayer sphere as a function of the number of layers, with the following parameters: ε_1=2; ε_2=4; $\nu_1 = \nu_2 = 1$.

References

[1] C. W. Qiu, L. Hu, X. Xu and, Y. Feng, "Spherical cloaking with homogeneous isotropic multilayered structures," Phys. Rev. E, **79**, 2009, pp. 047602, doi:10.1103/PhysRevE.79.047602.

[2] H. Kettunen, H. Wallen, A. Sihvola, "Cloaking and magnifying using radial anisotropy. Journal of Applied Physics," **114**, 2013, pp. 0-8, doi.org/10.1063/1.4816797.

[3] B. Wood, J. B. Pendry, and D. P. Tsai, "Directed subwavelength imaging using a layered metal-dielectric system," Phys. Rev. B, **74**, 2006, pp. 115116, doi:10.1103/PhysRevB.74.115116.

[4] Z. Jacob, L. V. Alekseyev and E. Narimanov, "Optical Hyperlens: Far-field imaging beyond the diffraction limit," Opt. Express, **14**, 2006, pp. 8247-8256, doi:10.1117/12.736318.

[5] F. Mangini, N. Tedeschi, F. Frezza, and A. Sihvola, "Realization of a Radial Uniaxial sphere with a multilayer sphere," 31th URSI General Assembly and Scientific Symposium, URSI GASS, **28**, 2014, pp. 916-931, doi.org/10.1080/09205071.2014.896480.

[6] F. Mangini, N. Tedeschi, F. Frezza and A. Sihvola, "Homogenization of a multilayer sphere as a radial uniaxial sphere: features and limits," Journal of Electromagnetic Waves and Applications, **24**, 2014, pp. 916-31, doi:10.1080/09205071.2014.896480.

[7] A. Sihvola, I. V. Lindell, "Transmission line analogy for calculating the effective permittivity of mixtures with spherical multilayer scatterers," Journal of electromagnetic waves and applications, **8**, 1988, pp. 741-56, doi:10.1163/156939388X00044.

[8] F. Mangini, F. Frezza, "Electromagnetic scattering of an inhomogeneous elliptically polarized plane wave by a multilayered sphere," Journal of Electromagnetic Waves and Applications, **30**, 2016, pp. 492-504, doi.org/10.1080/09205071.2015.1121842.

[9] B.R. Johnson, "Light scattering by a multilayer sphere," Appl. Opt., **35**, 1996, pp. 3286-3296, doi:10.1364/AO.35.003286.

[10] W. Yang, "Improved recursive algorithm for light scattering by a multilayered sphere," Appl. Opt., **42**, 2003, pp. 1710-1720, doi:10.1364/AO.42.001710.

[11] K. Ladutenko, U. Pal, A. Rivera and O. Peña-Rodríguez, "Mie calculation of electromagnetic near-field for a multilayered sphere," Comput. Phys. Communication, **214**, 2017, pp. 225-30, doi:10.1016/j.cpc.2017.01.017.

[12] X. Yu, L. Gao, "Nonlinear dielectric response in partially resonant composites with radial dielectric anisotropy," Physics Letters A, **5**, 2006, pp. 16-22, doi.org/10.1016/j.physleta.2006.06.075.