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Diffraction by a Planar Junction between DPS and DNG Material Sheets: the UAPO Solution for Plane Waves at Skew Incidence

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Abstract

The plane wave diffraction by a planar junction between two lossy planar sheets is studied in this paper when the incidence direction is oblique to the rectilinear discontinuity of the structure. One slab consists of a standard double positive material, whereas an unusual double negative metamaterial is considered for the other slab. The uniform asymptotic physical optics approach is applied to find an efficient and user-friendly solution in the context of the uniform geometrical theory of diffraction. Such an approach is based on electric and magnetic equivalent sources radiating in the free space, and exploits an analytical process to obtain a closed form expression of the diffraction coefficients under the high-frequency assumption. A useful matrix representation is presented.

1 Introduction

The Uniform Asymptotic Physical Optics (UAPO) approach has been recently employed to obtain efficient solutions to the plane wave diffraction from simple and composite structures. Some of them consist of artificial engineered materials with exceptional characteristics at microwave and optical frequencies: the double negative metamaterials (DNG MTMs). Negative real parts of permittivity and permeability distinguish this MTM class. On the contrary, natural materials are identified by standard lossy dielectrics with positive real parts of both permittivity and permeability. Accordingly, they are referred as conventional double positive (DPS) materials.

Research papers [1]-[3] deal with the application of the UAPO approach to the interaction of plane waves with truncated DNG MTM sheets. In particular, the scattering from a lossless, isotropic and homogeneous DNG MTM slab was studied in [1] accounting for the Geometrical Optics (GO) field contributions and the UAPO diffracted field, which is excited by the linear edge of the half-plane modeling the object. Note that the knowledge of the reflection and transmission coefficients is important not only for the evaluation of the GO field, but also for determining the electric and magnetic PO equivalent surface currents, which are the foundations of the UAPO approach. This last provided a closed form and easy to handle solution in [1]. Numerical tests confirmed the ability of the corresponding UAPO diffracted field to compensate the discontinuities of the GO field at the shadow boundaries of the reflected and transmitted waves. Moreover, very good agreements with COMSOL MULTIPHYSICS® data assessed the accuracy of the UAPO-based scattered field. Model and formulas were modified to take into account the DNG MTM losses in [2] and a perfect electric conductor (PEC) backing in [3].

The target of this work is the analytical evaluation of the diffracted field due to the interaction of a plane wave with a planar junction of two lossy sheets having different characteristics: DPS material and DNG MTM sheets. The plane wave propagation is oblique to the straight-lined discontinuity of the configuration, thus opening a three-dimensional diffraction problem (see Fig. 1). The UAPO approach is applied to determine the corresponding diffraction matrix in the framework of the Uniform Geometrical Theory of Diffraction (UTD) [4]. Because of the linearity of the radiation integral, one of the advantages offered by the application of the UAPO approach is the possibility to obtain the solution by adding the individual contributions of the sheets. This peculiarity was already exploited in [5] with reference to a planar DPS - DNG MTM junction with PEC backing.

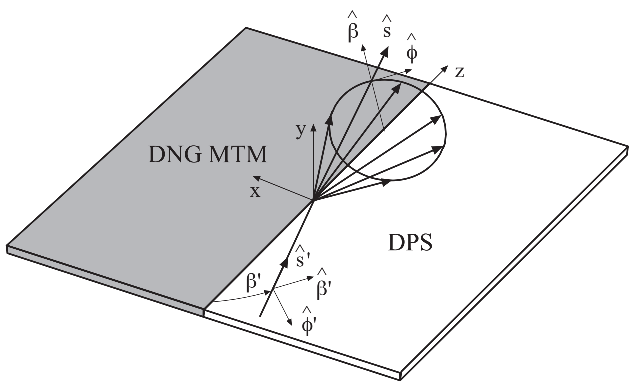

Figure 1. Plane wave diffraction from the planar junction between DNG MTM and DPS sheets.

2 The UAPO Solution

The geometry of the problem is depicted in Fig. 1. The sheets have thickness d and are characterized by the relative electric permittivity ε_r and the relative magnetic permeability μ_r. They are modeled by two half-planes joining at the z-axis of a reference co-ordinate system. The index 1 is associated to the DNG MTM half-plane $(y'=0, x'>0)$, whereas the co-ordinates $y'=0, x'<0$ define the points of the DPS material half-plane from this point on.

The UAPO approach allows one to isolate the high-frequency diffraction contribution from the evaluation of the scattered field due to electric and magnetic PO sources on the involved surface. According to the geometry in Fig. 1, the following electric ($\underline{J}_s$) and magnetic ($\underline{J}_{ms}$) equivalent surface currents at $\underline{r}'=x'\hat{x}+z'\hat{z}$ are assumed to radiate in the surrounding free space with impedance ζ_0 and propagation constant k_0:

$$\zeta_0 \underline{J}_{s_{1,2}} = \left[\left(1-R_{\perp_{1,2}}-T_{\perp_{1,2}}\right)E_\perp^i \cos\theta^i\, \hat{u}_\perp + \left(1+R_{\|_{1,2}}-T_{\|_{1,2}}\right)E_\|^i \left(\hat{y}\times\hat{u}_\perp\right)\right] \cdot \exp\left(jk_0\left(x'\sin\beta'\cos\phi' - z'\cos\beta'\right)\right) \quad (1)$$

$$\underline{J}_{ms_{1,2}} = \left[\left(1-R_{\|_{1,2}}-T_{\|_{1,2}}\right)E_\|^i \cos\theta^i\, \hat{u}_\perp + -\left(1+R_{\perp_{1,2}}-T_{\perp_{1,2}}\right)E_\perp^i \left(\hat{y}\times\hat{u}_\perp\right)\right] \cdot \exp\left(jk_0\left(x'\sin\beta'\cos\phi' - z'\cos\beta'\right)\right) \quad (2)$$

The above expressions contain the standard angle of incidence θ^i, the reflection (R) and transmission (T) coefficients for parallel (||) and perpendicular (⊥) polarizations, and the incident electric field $\underline{E}^i = \left[E_\perp^i\, \hat{u}_\perp + E_\|^i\left(\hat{u}_\perp \times \hat{s}'\right)\right]\exp\left(-jk_0\hat{s}'\cdot\underline{r}'\right)$, where $\hat{s}' = -\sin\beta'\cos\phi'\,\hat{x} - \sin\beta'\sin\phi'\,\hat{y} + \cos\beta'\,\hat{z}$ is the unit vector of the incidence direction and $\hat{u}_\perp = (\hat{s}'\times\hat{y}) / |\hat{s}'\times\hat{y}|$.

The scattered field $\underline{E}^S$ due to $\underline{J}_{s_{1,2}}$ and $\underline{J}_{ms_{1,2}}$ can be evaluated by means of the radiation integral:

$$\underline{E}^S \cong -jk_0 \iint_{S_1} \left(\underline{\underline{I}} - \hat{R}\hat{R}\right)\zeta_0 \underline{J}_{s_1} \frac{\exp(-jk_0R)}{4\pi R} dS + -jk_0 \iint_{S_1} \left(\underline{J}_{ms_1}\times\hat{R}\right)\frac{\exp(-jk_0R)}{4\pi R} dS + -jk_0 \iint_{S_2} \left(\underline{\underline{I}} - \hat{R}\hat{R}\right)\zeta_0 \underline{J}_{s_2} \frac{\exp(-jk_0R)}{4\pi R} dS + -jk_0 \iint_{S_2} \left(\underline{J}_{ms_2}\times\hat{R}\right)\frac{\exp(-jk_0R)}{4\pi R} dS \quad (3)$$

where $\underline{r}$ denotes the position vector of the observation point P, $\hat{R} = (\underline{r}-\underline{r}')/R$, $R = |\underline{r}-\underline{r}'|$ and $\underline{\underline{I}}$ is the 3x3 identity matrix.

Since the diffraction is confined to the Keller's cone, the approximation $\hat{R} \cong \hat{s}$ ($\hat{s}$ is the unit vector of the diffraction direction) can be used in the next step, thus obtaining the following matrix representation:

$$\underline{E}^s = \begin{pmatrix} E_\beta^s \\ E_\phi^s \end{pmatrix} \cong \left[\underline{\underline{M}}_1 I_1 + \underline{\underline{M}}_2 I_2\right] \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} \quad (4)$$

The above expression relates the scattered field to the incident one in useful local co-ordinate systems as in [4].

The matrices $\underline{\underline{M}}_{1,2}$ account for the adopted co-ordinate systems and the expressions of $\underline{J}_{s_{1,2}}$ and $\underline{J}_{ms_{1,2}}$ without the incident field terms, f.i.,

$$\underline{\underline{M}}_1 = \underline{\underline{N}}_1\left[\underline{\underline{N}}_2\underline{\underline{N}}_4\underline{\underline{N}}_5 + \underline{\underline{N}}_3\underline{\underline{N}}_4\underline{\underline{N}}_6\right]\underline{\underline{N}}_7 \quad (5)$$

with

$$\underline{\underline{N}}_1 = \begin{pmatrix} \cos\beta'\cos\phi & \cos\beta'\sin\phi & -\sin\beta' \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \quad (6)$$

$$\underline{\underline{N}}_2 = \begin{pmatrix} 1-\sin^2\beta'\cos^2\phi & -\sin\beta'\cos\beta'\cos\phi \\ -\sin^2\beta'\sin\phi\cos\phi & -\sin\beta'\cos\beta'\sin\phi \\ -\sin\beta'\cos\beta'\cos\phi & \sin^2\beta' \end{pmatrix} \quad (7)$$

$$\underline{\underline{N}}_3 = \begin{pmatrix} 0 & -\sin\beta'\sin\phi \\ -\cos\beta' & \sin\beta'\cos\phi \\ \sin\beta'\sin\phi & 0 \end{pmatrix} \quad (8)$$

$$\underline{\underline{N}}_4 = \frac{1}{A(\beta',\phi')}\begin{pmatrix} -\cos\beta' & -\sin\beta'\cos\phi' \\ -\sin\beta'\cos\phi' & \cos\beta' \end{pmatrix} \quad (9)$$

$$\underline{\underline{N}}_5 = \begin{pmatrix} 0 & (1-R_\perp-T_\perp)\sin\beta'\sin\phi' \\ 1+R_\|-T_\| & 0 \end{pmatrix} \quad (10)$$

$$\underline{\underline{N}}_6 = \begin{pmatrix} (1-R_{\|}-T_{\|})\sin\beta'\sin\phi' & 0 \\ 0 & -1-R_{\perp}+T_{\perp} \end{pmatrix} \quad (11)$$

$$\underline{\underline{N}}_7 = \frac{1}{A(\beta',\phi')}\begin{pmatrix} \cos\beta'\sin\phi' & \cos\phi' \\ -\cos\phi' & \cos\beta'\sin\phi' \end{pmatrix} \quad (12)$$

wherein $A(\beta',\phi') = \sqrt{1-\sin^2\beta'\sin^2\phi'}$.

The integrals $I_{1,2}$ in (4) contain only the propagation factor $\exp(jk_0(x'\sin\beta'\cos\phi' - z'\cos\beta'))$ multiplied by $-jk_0\exp(-jk_0R)/(4\pi R)$ and therefore do not possess information about the electric and magnetic characteristics of the structure. Such integrals are evaluated by using integral representations of the zeroth order Hankel function of second kind, the steepest descent method and the multiplicative method in the high-frequency approximation. Finally, the UAPO diffraction contributions $I_{1,2}^d$ are extracted from $I_{1,2}$ and expressed in terms of the UTD transition function $F_t(\cdot)$ [4]:

$$I_1^d = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} \frac{F_t\left(2k_0 s\sin^2\beta'\cos^2\left(\frac{\phi\pm\phi'}{2}\right)\right)}{\sin^2\beta'(\cos\phi+\cos\phi')} \quad (13)$$

$$I_2^d = -\frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} \cdot \frac{F_t\left(2k_0 s\sin^2\beta'\cos^2\left(\frac{(\pi-\phi)\pm(\pi-\phi')}{2}\right)\right)}{\sin^2\beta'(\cos\phi+\cos\phi')} \quad (14)$$

The sign + (–) must be used if $0<\phi<\pi$ $(\pi<\phi<2\pi)$.

According to the above procedure, the UAPO diffracted field can be formulated as follows:

$$\begin{pmatrix} E_\beta^d \\ E_\phi^d \end{pmatrix} = \left[\underline{\underline{M}}_1 I_1^d + \underline{\underline{M}}_2 I_2^d\right]\frac{\exp(-jk_0 s)}{\sqrt{s}}\begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} = \underline{\underline{D}}\frac{\exp(-jk_0 s)}{\sqrt{s}}\begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} \quad (15)$$

where s is the distance from the diffraction point to P.

3 Numerical Examples

The capability of the proposed UAPO solution to compensate the GO field jumps at the reflection and transmission boundaries is tested.

The reported figures refer to a junction with $d = 0.2\lambda_0$, where λ_0 is the free-space wavelength. The electric and magnetic parameters of the sheets are $\varepsilon_{r_1} = -2 - j0.01$, $\mu_{r_1} = -1 - j0.1$ and $\varepsilon_{r_2} = 4 - j0.001$, $\mu_{r_2} = 1$. The junction is illuminated by an incident plane wave characterized by $E_{\beta'}^i = 1$, $E_{\phi'}^i = 0$ and $\beta' = 45°, \phi' = 40°$. The point P moves on a circumference with radius equal to $5\lambda_0$, and crosses the reflection and transmission boundaries at $\phi = 140°$ and $\phi = 220°$, respectively. As expected, the corresponding discontinuities of the GO field (see Figs. 2 and 4 for the β- and ϕ-components, respectively) are compensated by the UAPO diffracted field, thus obtaining a continuous total field (see Figs. 3 and 5 for the β- and ϕ-components, respectively) on the observation domain.

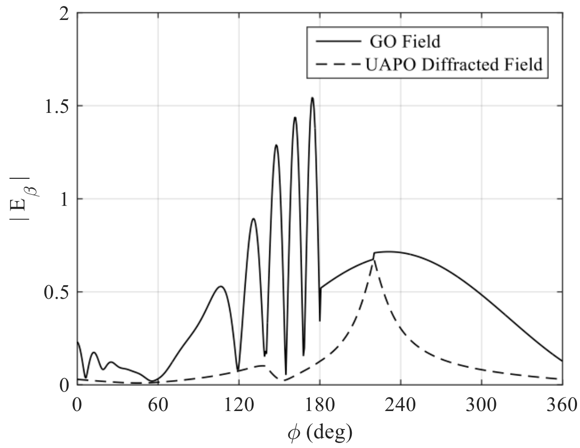

Figure 2. GO field and UAPO diffracted field. Amplitude of the β-component.

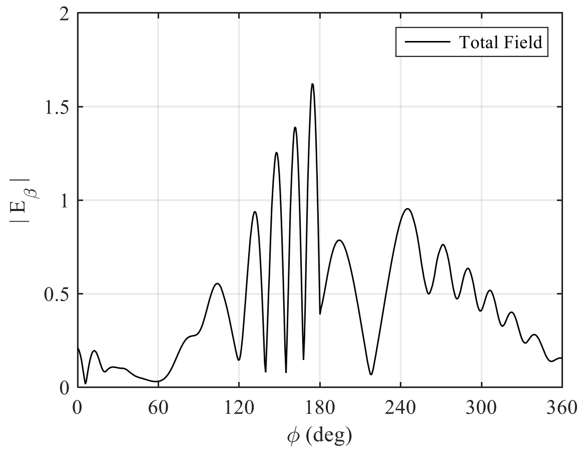

Figure 3. Total field. Amplitude of the β-component.

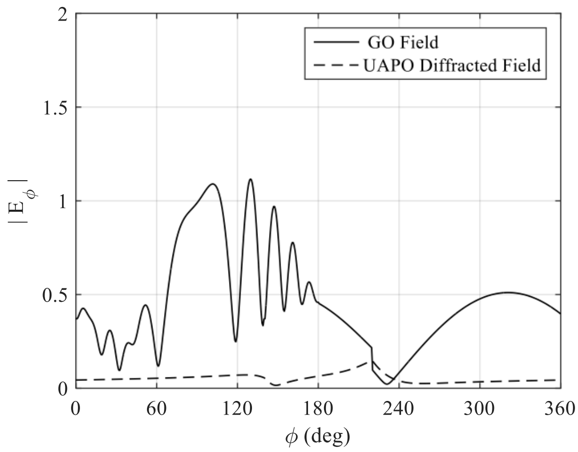

Figure 4. GO field and UAPO diffracted field. Amplitude of the ϕ-component.

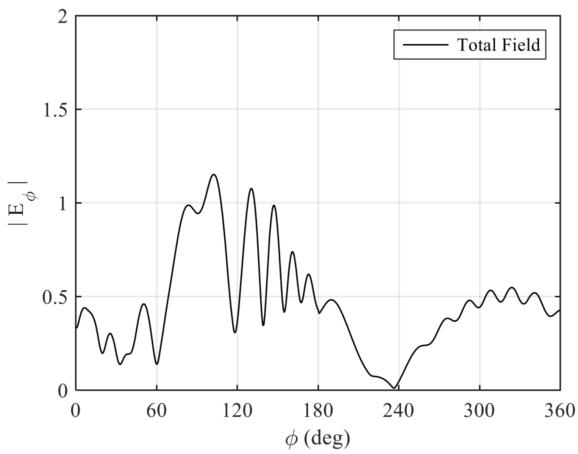

Figure 5. Total field. Amplitude of the ϕ-component.

4 Conclusions and Future Activities

The diffraction problem involving a planar junction between DPS and DNG material slabs has been solved by means of the UAPO approach. The resulting expression in matrix form can be applied in the UTD context. Such an expression is easy to handle and provides diffracted field values able to compensate the discontinuities of the GO field at the shadow boundaries. Future research works will be devoted to test the accuracy of the proposed UAPO solution by using a full-wave numerical tool.

5 References

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