URSI GASS 2020, Rome, Italy, 29 August - 5 September 2020

Importance of modal analysis in vibratory microgyroscopes

J. Nazdrowicz[(1)], A. Stawiński* [(1)], and A. Napieralski[(1)]
(1) Lodz University of Technology, e-mail: jnazdrowicz@dmcs.pl; astawinski@dmcs.pl, napier@dmcs.pl

Abstract

In this paper some considerations regarding vibratory MEMS gyroscopes principle of operation and physical quantities which are related to vibrations are considered. Because this type of device uses energy comes from vibrations and transfers this energy with rotation presence to sensor, it is interesting how it behaves for different dimension parameters including geometrical dimensions. This work presents results of analysis eigenfrequency for all motion direction under different dimensions values.

1 Introduction

Vibratory microsensors, which the most famous representative is gyroscope belongs to large MEMS devices family.

Basically, vibratory microgyroscope is spring-mass-damper device with 2 degrees of freedom. It consists of seismic mass suspended on specific shape springs. The springs are anchored to substrate at the other end. Integral part of seismic mass are electrodes [1] which in combination with static electrodes create capacitors. These capacitor structures loaded with voltage can cause vibrations – for drive direction and transform vibration motion energy to electrical signal – for sense direction.

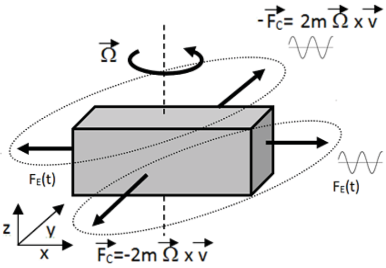

Figure 1. Gyroscope principal of operation.

2 Vibratory Gyroscope Theory of Operation

MEMS vibrating gyroscope operation is based on Coriolis force which is linearly proportional to rotational velocity. This force occurs when object rotates around axis perpendicular to surface determined by two axis – drive and sense and with non-zero linear velocity that occurs along one of these axis. (fig. 1) [3,4]. The response of system are motion along second axis.

It is worth to underline, that capacitance changes are detected by sensor capacitor which in turn can be transformed in corresponding ASIC to numerical value of angular velocity [2].

Fundamental equations governing MEMS gyroscopes can be expressed with second order differential equations:

$$\begin{cases} m_x \frac{d^2x}{dt^2} + c_x \frac{dx}{dt} + [k_x - m_x(\Omega_y^2 + \Omega_z^2)]x - 2m_x\Omega_z \frac{dy}{dt} + m_x\Omega_x\Omega_y y = F_x \\ m_y \frac{d^2y}{dt^2} + c_y \frac{dy}{dt} + [k_y - m_y(\Omega_x^2 + \Omega_z^2)]x - 2m_y\Omega_z \frac{dx}{dt} + m_y\Omega_x\Omega_y x = F_y \end{cases}$$

where Ω is angular velocity, v – linear velocity, m – mass of object.

Elements $2m\Omega\dot{y}$ and $2m\Omega\dot{x}$ are Coriolis force components induced by rotation. Coriolis force in fact are induced by dynamic coupling between actuator and sensor motion directions.

Assuming, that both modes are fully decoupled - k_{xy}, k_{yx}=0, F_y=0 and $2m\Omega\frac{dy}{dt} = 0$, system of equations can be simplified and finally, these equations can be written in the following form:

$$m_x \frac{d^2x}{dt^2} + c_x \frac{dx}{dt} + k_x x = F_D \sin(\omega t)$$

$$m_y \frac{d^2y}{dt^2} + c_y \frac{dy}{dt} + k_y y = -2m_y \frac{dx}{dt}\Omega$$

or

$$\frac{d^2x}{dt^2} + \zeta_x\omega_x \frac{dx}{dt} + \omega_x^2 x = \frac{F_D}{m_x}\sin(\omega t)$$

$$\frac{d^2y}{dt^2} + \zeta_y\omega_y \frac{dy}{dt} + \omega_y^2 y = -2\frac{dx}{dt}\Omega$$

where $\zeta_x = \frac{c_x}{m_x\omega_x}$, $\zeta_y = \frac{c_y}{m_y\omega_y}$, ζ is a damping ratio, $\omega_x = \sqrt{\frac{k_x}{m_x}}$, $\omega_y = \sqrt{\frac{k_y}{m_y}}$, $Q_x = \frac{m_x\omega_x}{c_x}$, $Q_y = \frac{m_y\omega_y}{c_y}$

Performed simulations shows that maximum displacement in y (sense) direction is about 1000 less than in x (drive) direction, resonance phenomena is commonly used in vibratory gyroscopes. Resonance requires proper and optimized frequency applied to actuator (x direction) to enable vibrations in y direction. Maximum displacement can occur in y direction when resonance frequency in that direction will be the same like in x direction. This fundamental rule is basics of mode-matching directions.

Because in case of MEMS vibratory gyroscope mode-matching achievement is crucial, modal analysis of such device is one of the most important steps during design.

3 Models of microgyroscopes

There were two planar vibratory gyroscope considered and simulated:

1. Including one common seismic mass for both drive and sense directions
2. Including seismic mass and inertial frame mass with strong separation of both drive and sense directions

In both cases masses were suspended with simple or complex serpentine springs. Complex serpentine springs consists of two simple ones and combined with one beam. The significant advantage of this type of suspension is to rigid transfer motion in one direction and flexibility in perpendicular direction.

In table 1 there is list of physical properties of Polysilicon – material used in simulations and in table 2 – list of geometrical dimensions particular devices.

Table 1: List of physical properties of Polysilicon.

Symbol	Quantity	Value
μ	Air viscosity	$1.8 \cdot 10^{-5}$Ns/m^2
ε_o	Permittivity coefficient	$8.854 \cdot 10^{12}$F/m,
ρ	Density (Polysilicon)	2328 kg/m^3
α	Thermal expansion coefficient	$2.9 \cdot 10^{-5}$ 1/K
β	Thermal coefficient of Young's modulus β (for ΔT=0)	-80*10-6[1/K]

Table 2: List of some important geometrical parameters.

Quantity	Value
Drive, Sense electrode count	30
Distance between electrodes	10^{-5}m
Spring length	$250 \cdot 10^{-6}$m
Proof mass width	$1000 \cdot 10^{-6}$m
Proof mass length	500, $1000 \cdot 10^{-6}$m
Device thickness	30^{-5}m
Central spring length	$140 \cdot 10^{-6}$m
Edge spring length	$250 \cdot 10^{-6}$m
Inertial frame height	$675 \cdot 10^{-6}$m

4 Results of simulations

To meet design and simulation purposes the following environments were used:

1. COMSOL Multiphysics – parametric sweep simulations:
   - solid expansion depending on temperature to calculate displacements in particular XYZ directions,
   - calculation eigenfrequencies depending on temperature (this step uses also solid temperature expansion results).
   - Calculation eigenfrequencies depending on some specific geometry dimensions
2. MATLAB/SIMULINK – mathematical model:
   - simulation (implementation of two Netwon's motion equations) to obtain device response in sense direction – displacement of moving part and vibration plots.

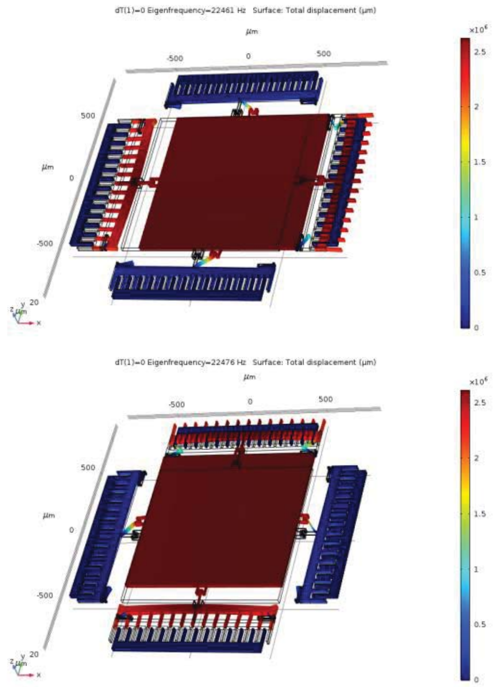

Figure 2. First and second mode of MEMS gyroscope (drive direction) – for first geometry.

Basically, two first modes reflecting vibrations along *x* (drive) and *y* (sense) directions were taken into considerations. Visual results of modal analysis, structure deformation, total displacements and eigenfrequencies (natural frequencies) values for both considered gyroscope geometries are presented in fig. 2 and fig. 3. Values of eigenfrequencies obtained from this analysis are

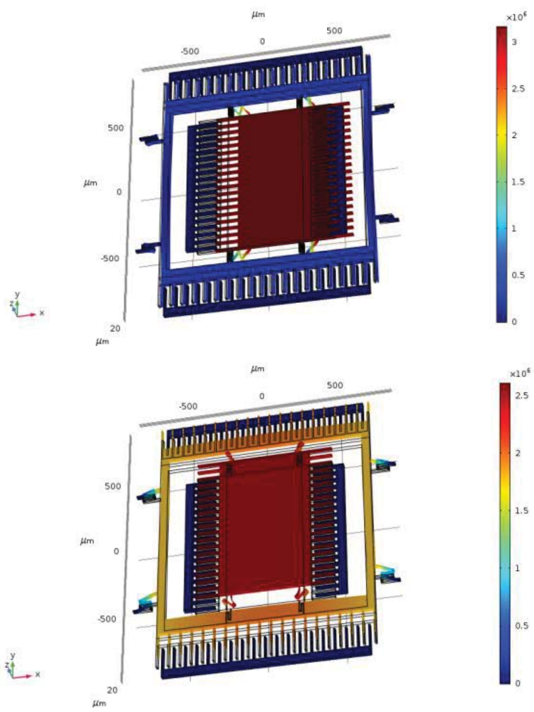

Figure 3. First and second mode of MEMS gyroscope (drive direction) – for second geometry.

fundamental for adjusting device vibration external frequency to get resonance effect in sense direction.
In fig. 4 and 5 there are shown plots of eigenfrequency dependency on two geometrical dimensions: width and length of springs on these seismic masses are suspended. In case of length growth spring constant drops rapidly (for small spring length) and then slightly along with suspension length increase. Modal analysis performed in COMSOL shows strong dependency eigenfrequency on this dimension (length) variation. In case of width increase eigenfrequency grows, moreover for short beams springs this dependency is stronger than for long ones.

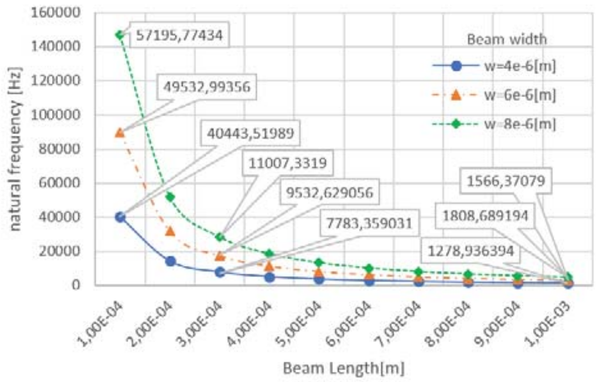

Figure 4. Natural frequency dependency on beam length for three different width dimensions.

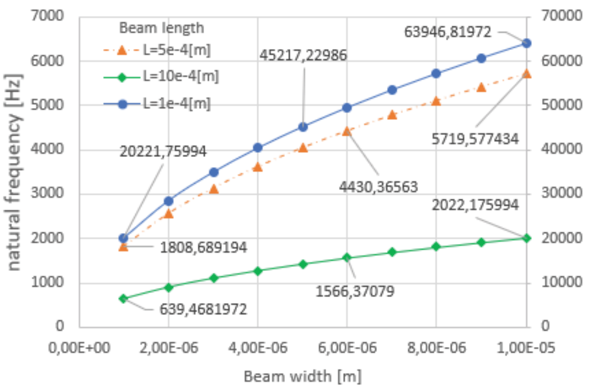

Figure 5. Natural frequency dependency on beam width for three different length dimensions.

In case of MEMS geometry with one common mass there is no problem with mass dimensions variations because for both directions changes are the same. More complex situation is for geometry with separate inertial and central masses. Comparing both device geometries we observe they have very similar masses (tiny differences come directly from spring masses differences that can be negligible). It is obvious, that natural frequency for 1st mode device (fig. 6) including complex serpentine springs is 2.125 times higher than device including edge springs. For 2nd mode (fig. 7)this coefficient is less and is equals 1.262. Frequency difference between both considered modes are 441Hz in case of central springs and 1206Hz for edge springs.
Natural frequency for microgyroscope including complex serpentine springs intensively grows for seismic frame thickness between 10-25um, next it decreases slowly along with frame thickness dimension grows to 125um. For second mode there is no drop of natural frequency along with thickness grows. When thickness grows natural frequency increase is not meaningful. For location of

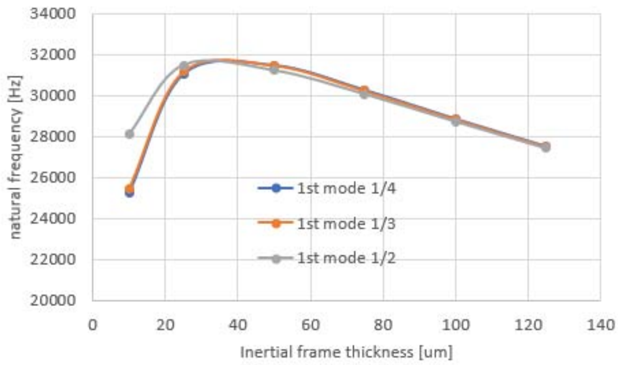

Figure 6. 1nd mode modal analysis of MEMS Gyroscope without inertial frame with inertial frame and central.

spring in 1/2 distance from symmetry axis we observe that for thin frame natural frequencies are much more higher than for 1/3 and 1/4 distance. In case of first mode situation is very similar.
Geometry details are one side adjusted by designer, but on

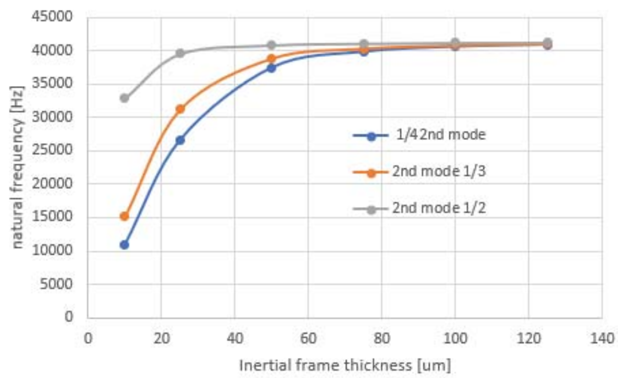

Figure 7. 2nd mode modal analysis of MEMS Gyroscope without inertial frame with inertial frame and central springs.

the other way can be unexpected effect of external factors like temperature.
Importance of modal analysis is visible in fig. 8 and 9 where magnitude and phase of transfer function are presented. For first considered gyroscope – fig.8 - (with one mass) we observe great mode-matching, because of small frequency difference (Δf). When vibrating gyroscope is mode-matched - in ideal situation - eigenfrequency of actuator perfectly matches eigenfrequency of sensor. This translates directly to improvement of performance especially sensitivity. In fig. 9 magnitude and phase of transfer function for device including seismic frame and central mass. In that case - larger angular difference between actuator and sensor - is about 0.3*10^{5} rad/s (4777Hz). Concluding, mode-matching is much worse than in case of single-mass configuration.

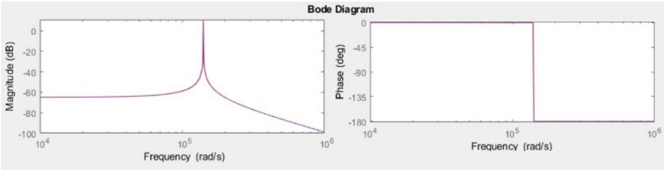

Figure 8. Magnitude and phase of transfer function for gyroscope with one inertial mass.

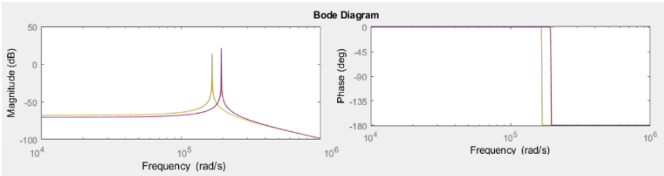

Figure 9. Magnitude and phase of transfer function for gyroscope with two masses.

5 Conclusions

In this article modal analysis of two MEMS gyroscope devices were presented. The simulations performed with COMSOL Multiphysics software show that dimensions change may influence on gyroscope output results like eigenfrequency.

Results obtained for two different geometries confirmed that value of natural frequency is dimension-sensitive. Change of one geometrical dimensions strongly requires individual approach and perform series of simulations to verify whether given configuration is optimal – mode matched. It should be kept in mind that even tiny natural frequencies differences (some Hz) may meaningfully influence on response of sensor (maximum displacement) and lose the matching.

Results show that less problem of mismatching is with configuration including one seismic mass, more – for device with separate seismic masses. Because natural frequency depends on mass and spring constant, the improvement of mode-matching can be achieved with adjustment of spring constants, being precise – by their dimensions.

Results obtained from simulations and presented here confirm, that problem of mode-matching is multidimensional and requires modal analysis of each device separately and individual approach to each structure.

7 References

1. P. Chandradip, P. MCCluskey, "Simulation of the MEMS Vibratory Gyroscope through Simulink", Conference: Device Packaging, March 2012, https://www.researchgate.net/publication/320170530_Simulation_of_the_MEMS_Vibratory_Gyroscope_through_Simulink
2. D. Xia, S. Chen, S. Wang, H. Li, "Microgyroscope Temperature Effects and Compensation-Control Methods", Sensors, *9*(10), pp. 8349-8376, 2009.
3. J. C. Fang, J.L. Li, W. Sheng, "Improved temperature error model of silicon MEMS gyroscope with inside frame driving". J. Beijing Univ. Aeronaut. Astronaut. 32, pp.1277–1280, 2006.
4. A. Lawrence, "Modern Inertial Technology: Navigation, Guidance and Control", .Springer Verlag, New York,1993.