

# Global Ionospheric Models, TEC, and Stochastic Structure

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#### **Abstract**

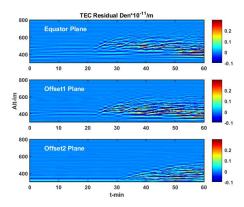
Global Ionospheric Models (GIMs) provide parametric functional descriptions of ionospheric observables. Total electron content (TEC) is the primary propagation diagnostic. TEC is a measure of the cumulative interaction of an electromagnetic wave with the ionosphere, formally a path integral of electron density. The modeling challenge is determining the ionization distribution along the propagation path. Propagation effects can be exploited, but more often they complicate the inversion process. Although GIMs provide a repository of our understanding of ionospheric dynamics and a starting point for studying the development of stochastic structure, a viable stochastic component has yet to be incorporated.

This talk will review some newly published results that connect configuration-space stochastic structure models directly to diagnostic measurements. The term scintillation, which is a frequency-dependent manifestation of ionospheric structure, has been avoided deliberately. The scintillation phenomenon is sufficiently well understood that progress is limited mainly by the lack of a unified structure model.

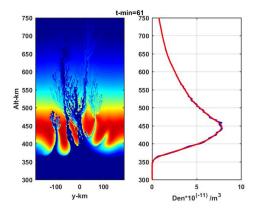
#### 1 Introduction

GIMs are commonly used to specify an ionospheric configuration, which is perturbed to initiate evolving structure. The initial evolution is quasi-deterministic in that it is determined by the initial conditions. However, at some point in the structure evolution stochastic structure develops. Figure 1 shows the evolution of simulated stochastic TEC structure derived from a high-resolution physics-based equatorial plasma bubble (EPB) model. The structure analysis is described in a recent paper [1]. The EPB simulation was performed by Tatsuhiro Yokoyama as described in the cited references. Figure 2 shows the EPB structure in the equatorial plane. Structure development is initiated abruptly at the point where a rising EPB penetrates the F-region peak. The structure maps along field lines from the equatorial plane to opposite hemispheres.

Stochastic structure is confined to two-dimensional slice planes that cut across magnetic field lines. In the paper [1] it was shown that the developed structure could be characterized by a one-dimensional two-component power-law of



**Figure 1.** Evolving stochastic TEC structure from EPB simulation.



**Figure 2.** EPB structure in equatorial plane at 1 hr.

the form

$$\Phi_{N_e}^{(1)}(q) \simeq C_s \begin{cases}
q^{-\eta_1} & \text{for } q \le q_0 \\
q_0^{\eta_2 - \eta_1} q_y^{-\eta_2} & \text{for } q > q_0
\end{cases}$$
 (1)

The defining parameters for modeling are turbulent strength  $C_s$ , the one-dimensional power-law indices  $\eta_n$ , and the break spatial frequency  $q_0$ , with the associated scale  $\sigma_b = 2\pi/q_0$ .

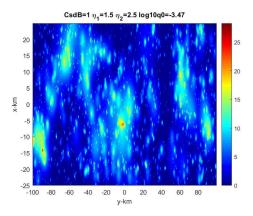
# 2 Configuration-Space Models

Ionospheric structure models generally proceed from the three-dimensional form of (1). A limitation of this modeling approach is that while structure realizations are statistically similar to real ionospheric structure, there is no physics-based connection to the parameters that characterize the structure.

In the paper [2] an alternative model was proposed that constructs realizations as summations of magnetic-field-aligned *striations*. By imposing bifurcation rules on the size distribution and peak electron density of the striations, the realizations support two-component inverse-power-law SDFs, thereby maintaining continuity with published diagnostic measures:

$$\Delta N(x_n, \vec{\rho}) = \frac{1}{N_s} \sum_{k=1}^{N_s} C(k) \sigma_k^{\gamma_k} p_{\perp} \left( \sqrt{(\vec{\eta} + \vec{\eta}_k)^2} / \sigma_k \right) \quad (2)$$

The parameters C(k),  $\gamma_k$ , and  $\sigma_k$  are configuration-space parameters that characterize the structure distribution. The variable  $\vec{\eta}$  is the cross field coordinate of the striation with  $\vec{\eta}_k$  the random location that defines the particular striation. A rotation transforms the data-space coordinates  $[x, \vec{\rho}]$  to the cross-field striation coordinates that characterize the striations. In this version of the model there is no variation along the field lines.



**Figure 3.** Configuration realization of developed EPB structure.

Figure 3 shows a configuration-space realization slice with parameters chosen to coincide with the one-dimensional SDFs associated with the structure altitudes shows in 2.

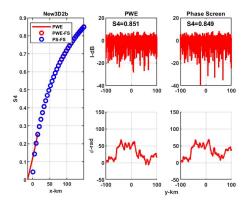
With the striation locations randomized, the characteristics that identify the structure as being initiated by an EPB is lost. However, as described in [1], intermingling of the large-scale variations and the statistically well defined smaller scale structure is preserved.

An important feature of the configuration-space model is data-space population with structure invariant along the magnetic field direction. Formally, a three-dimensional SDF derived from (2) will be singular along the direction corresponding to the magnetic field. However, because the data space is truncated, propagation calculation accommodate the formal singularities gracefully. Edge-effects are eliminated by avoiding truncation of individual striations. The geometric dependence of propagation simulations based on integration of the parabolic wave equation (PWE) is preserved.

## 3 Propagation Simulations

# 3.1 Equivalent Phase Screens

Configuration-space simulations are well suited for split-step integration of the parabolic wave equation (PWE). Figure 4 summarizes the results of a PWE integration for configuration structure with the magnetic field direction at  $45^{\circ}$  elevation and  $30^{\circ}$  azimuth. The left frame compares the full path integration results with propagation from an equivalent phase screen calculation with free-space propagation from the center of the structured region. From the exit plane of the structure the full diffraction and equivalent phase screen results are nearly identical. The middle and right frames compare the intensity and phase derived from a central scan of the structure. The phase-streen structure is shown in Figure 5.



**Figure 4.** Summary of PWE simulation for an L1 GPS signal intercepting structure at oblique incidence relative to the magnetic field direction.

## 3.2 Interpreting Diagnostic Measurements

Interpreting in-situ and propagation diagnostics requires a three-dimensional stochastic model. To this end, we note

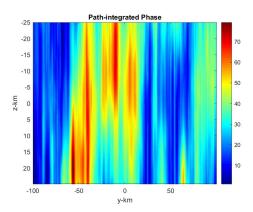


Figure 5. Path integrated phase.

that configuration-space three-dimensional structure is determined by the profile function argument in (2), which is transformed from data-space coordinates. For a uniform distribution of the striations, the three-dimensional SDF can be computed analytically. Arbitrary one-dimensional scan have SDFs that can be computed from a double integration of the three-dimensional SDF.

For interpreting propagation diagnostics phase-screen equivalence is used. If TEC is the primary diagnostic, calculation of the measured one-dimensional SDF is straightforward. If the primary diagnostic is the SDF of the signal intensity fluctuations, a theory is needed to compute the SDF as a function of the path-integrated SDF. The an analytic model can be constructed, but efficient computational procedures have been developed only for two-dimensional propagation ([3]). Alternatively, statistical equivalence can be carried one step further, by using Irregularity Parameter Estimation (IPE) as described in [3]can be applied directly.

### 4 Summary

A configuration space model has be introduced as a general model of ionospheric structure. The defining parameters can be derived from in-situ measurements or propagation diagnostics. Although the defining relation is an observable one-dimensional SDF, once defined the configuration-space model can be used to compute the associated three-dimensional or two-dimensional SDFs. If the ionospheric structure does not cause significant intensity structure, the path-integrated phase can be measured directly. No propagation-theory is needed. This is usually the case for L-band high-latitude scintillation.

If there is significant intensity scintillation, phase-screen equivalence proceed from path-integrated phase. The stochastic TEC structure that is typically discarded should be a structure model input. Ongoing analysis of auroral-zone measurements support this approach.

#### References

- [1] C. Rino, T. Yokoyama, and C. Carrano, "Dynamic Spectral Characteristics of High-Resolution Simulated Equatorial Plasma Bubbles," *Progress in Earth and Planetary Science*, Submitted for Publication, 2018.
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